Analysis of Transient Ground Surface Displacements Due to a Point Sink in a Porous Elastic Half-Space

John C.-C. Lu[*] *and Feng-Tsai Lin*[**]

[*]Associate Professor
Department of Civil Engineering and Engineering Informatics
Chung-Hua University
Hsinchu 30012, Taiwan, R.O.C.
cclu@chu.edu.tw

[**]Instructor
Department of Naval Architecture
National Kaohsiung Marine University
Kaohsiung 81157, Taiwan, R.O.C.
ftlin@mail.nkmu.edu.tw

ABSTRACT

Based on Biot's three-dimensional consolidation theory of porous media, analytical solutions of the transient consolidation deformation due to a point sink in saturated isotropic porous elastic half-space are presented. Using the Laplace and Hankel integral transform techniques, closed-form solutions of the horizontal and vertical displacements of the ground surface are obtained. In the analysis, case of permeable half-space boundary is studied. The consolidation as effected by the consolidation parameters are illustrated and discussed.

INTRODUCTION

Land subsidence due to groundwater withdrawal is a well known phenomena.[1] As water pumps from an aquifer, the pore water pressure is reduced in the withdrawal region. This leads to an increase in the effective stress between the soil particles and subsidence of ground surface.

Biot's three-dimensional consolidation theory[2,3] is generally regarded as the fundamental theory for modeling land subsidence. Based on Biot's theory, Booker and Carter[4-7] presented solutions of subsidence due to pumping at a constant rate from a point sink embedded in a saturated elastic half-space. In their solutions, the flow properties are considered as isotropic[4] or cross-anisotropic[5-7] whereas the elastic properties of the soil are treated as isotropic. The half-space boundary is considered pervious. It was found that the anisotropic permeability has significant effects on the land subsidence due to fluid extraction. Nevertheless, transient closed-form solutions of the half-space due to fluid withdrawal were not obtained in the studies of Booker and Carter.[4-7]

In this paper, the soil mass is modeled as an isotropic saturated elastic half-space with a pervious ground surface. Using the Laplace and Hankel transform techniques, transient horizontal and vertical displacements of the ground surface due to a point sink are obtained. Results are then illustrated and compared to provide better understanding of the time dependent consolidation settlement due to pumping.

MATHEMATICAL MODELS

Basic Equations

Figure 1 presents a point sink buried in a saturated porous stratum at a depth *h*. The soil mass is considered as a homogeneous isotropic porous medium with a vertical axis of symmetry. The constitutive behavior of the elastic soil skeleton for linear axially symmetric deformation in the cylindrical coordinates (r,θ,z) are expressed by

$$\tau_{rr} = \frac{2G(1-\nu)}{1-2\nu}\frac{\partial u_r}{\partial r} + \frac{2G\nu}{1-2\nu}\frac{u_r}{r} + \frac{2G\nu}{1-2\nu}\frac{\partial u_z}{\partial z} + p\,, \tag{1a}$$

$$\tau_{\theta\theta} = \frac{2G\nu}{1-2\nu}\frac{\partial u_r}{\partial r} + \frac{2G(1-\nu)}{1-2\nu}\frac{u_r}{r} + \frac{2G\nu}{1-2\nu}\frac{\partial u_z}{\partial z} + p\,, \tag{1b}$$

$$\tau_{zz} = \frac{2G\nu}{1-2\nu}\frac{\partial u_r}{\partial r} + \frac{2G\nu}{1-2\nu}\frac{u_r}{r} + \frac{2G(1-\nu)}{1-2\nu}\frac{\partial u_z}{\partial z} + p\,, \tag{1c}$$

$$\tau_{rz} = G\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right), \tag{1d}$$

where τ_{rr}, $\tau_{\theta\theta}$, *etc.*, are the total stress components; p is the excess pore water pressure of the soil mass; u_r, u_z are the displacements in the radial and axial directions, respectively; ν and G are the Poisson's ratio and shear modulus of the stratum, respectively. The shear stress components $\tau_{r\theta}$ and $\tau_{\theta z}$ vanish by locating the vertical *z*-axis through the point sink.

The total stresses must satisfy the following equilibrium relations

$$\frac{\partial \tau_{rr}}{\partial r} + \frac{\tau_{rr} - \tau_{\theta\theta}}{r} + \frac{\partial \tau_{rz}}{\partial z} + b_r = 0\,, \tag{2a}$$

$$\frac{\partial \tau_{rz}}{\partial r} + \frac{\tau_{rz}}{r} + \frac{\partial \tau_{zz}}{\partial z} + b_z = 0\,, \tag{2b}$$

in which $b_i(i=r,z)$ denote the body forces. By using equations (1a)-(1d), the equilibrium equations for axially symmetric problem without body forces b_i can be expressed in terms of displacements u_i and excess pore water pressure p as follows:

$$G\nabla^2 u_r + \frac{G}{1-2\nu}\frac{\partial \varepsilon}{\partial r} - G\frac{u_r}{r^2} + \frac{\partial p}{\partial r} = 0\,, \tag{3a}$$

$$G\nabla^2 u_z + \frac{G}{1-2\nu}\frac{\partial \varepsilon}{\partial z} + \frac{\partial p}{\partial z} = 0\,, \tag{3b}$$

where the Laplacian operator ∇^2 can be expressed as $\nabla^2 = \partial^2/\partial r^2 + 1/r\,\partial/\partial r + \partial^2/\partial z^2$ and $\varepsilon = \partial u_r/\partial r + u_r/r + \partial u_z/\partial z$ is the volume strain of the porous medium.

A third relation between u_r, u_z, and p can be obtained from the conservation of mass:

$$\nabla \cdot [n(\boldsymbol{v}_w - \boldsymbol{v}_s)] + n\beta \frac{\partial p}{\partial t} + q = 0 , \tag{4}$$

where n is the porosity of the porous medium; $\boldsymbol{v}_w$ and $\boldsymbol{v}_s$ are the velocities of pore water and solid matrix, respectively; β is the compressibility of pore water; q is the rate of water extracted from the porous medium per unit volume. Assuming that the pore water is governed by Darcy's law, we have

$$n(\boldsymbol{v}_w - \boldsymbol{v}_s) = -\frac{k_r}{\gamma_w}\frac{\partial p}{\partial r}\boldsymbol{i}_r - \frac{k_z}{\gamma_w}\frac{\partial p}{\partial z}\boldsymbol{i}_z , \tag{5}$$

in which k_r and k_z denotes the permeability of the soil mass in the horizontal and vertical directions, respectively; γ_w is the unit weight of pore water.

Let us consider a point sink of constant strength Q located at point $(0,h)$. Substituting (5) into (4) yields

$$-\frac{k_r}{\gamma_w}\left(\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r}\right) - \frac{k_z}{\gamma_w}\frac{\partial^2 p}{\partial z^2} + n\beta\frac{\partial p}{\partial t} + \frac{Q}{2\pi r}\delta(r)\delta(z-h)u(t) = 0 , \tag{6}$$

in which $\delta(x)$ and $u(t)$ are Dirac delta and Heaviside unit step function, respectively. Eqs. (3a), (3b) and (6) constitute the basic governing equations of the time-dependent axially symmetric poro-elastic responses of a saturated porous medium.

Boundary Conditions

Consider the half-space surface, $z = 0$, is a traction-free boundary for all time $t \geq 0$. From Eqs. (1c) and (1d), the boundary conditions are expressed in terms of u_r and u_z by

$$\frac{2G\nu}{1-2\nu}\left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r}\right) + \frac{2G(1-\nu)}{1-2\nu}\frac{\partial u_z}{\partial z} = 0 \quad \text{for} \quad z = 0, \tag{7a}$$

$$G\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right) = 0 \quad \text{for} \quad z = 0, \tag{7b}$$

An additional condition is provided by considering the half-space as pervious. The mathematical statements of the flow condition at the boundary $z = 0$ is given by

$$p = 0 \quad \text{for} \quad z = 0. \tag{7c}$$

Initial Conditions

Assuming that there have no changes in displacements and seepage of the stratum initially, the initial conditions at time $t = 0$ of the mathematical model can be treated as

$$u_r = 0 , \; u_z = 0 , \; p = 0 . \tag{8}$$

ANALYTIC SOLUTIONS

Laplace and Hankel Transforms Solutions

The governing partial differential equations (3a), (3b) and (6) can be reduced to ordinary differential equations by performing appropriate Laplace and Hankel transforms[8] with respect to the time variable t and the radial coordinate r, we obtain

$$\left(\frac{d^2}{dz^2}-2\eta\xi^2\right)\tilde{u}_r-(2\eta-1)\xi\frac{d\tilde{u}_z}{dz}-\frac{1}{G}\xi\tilde{p}=0, \tag{9a}$$

$$(2\eta-1)\xi\frac{d\tilde{u}_r}{dz}+\left(2\eta\frac{d^2}{dz^2}-\xi^2\right)\tilde{u}_z+\frac{1}{G}\frac{d\tilde{p}}{dz}=0, \tag{9b}$$

$$\frac{k_r}{\gamma_w}\xi^2\tilde{p}-\frac{k_z}{\gamma_w}\frac{d^2\tilde{p}}{dz^2}+n\beta s\tilde{p}+\frac{Q}{2\pi s}\delta(z-h)=0, \tag{9c}$$

where ξ and s are Hankel and Laplace transform parameters; $\eta=(1-\nu)/(1-2\nu)$; and the symbols $\tilde{u}_r$, $\tilde{u}_z$, $\tilde{p}$ are defined as

$$\tilde{u}_r(z;\xi,s)=\int_0^\infty rL\{u_r(r,z,t)\}J_1(\xi r)dr, \tag{10a}$$

$$\tilde{u}_z(z;\xi,s)=\int_0^\infty rL\{u_z(r,z,t)\}J_0(\xi r)dr, \tag{10b}$$

$$\tilde{p}(z;\xi,s)=\int_0^\infty rL\{p(r,z,t)\}J_0(\xi r)dr, \tag{10c}$$

in which $J_n(x)$ represents the first kind of Bessel function of order n and the Laplace transformations with respect to u_r, u_z and p are denoted by

$$L\{u_r(r,z,t)\}=\int_0^\infty u_r(r,z,t)\exp(-st)dt, \tag{11a}$$

$$L\{u_z(r,z,t)\}=\int_0^\infty u_z(r,z,t)\exp(-st)dt, \tag{11b}$$

$$L\{p(r,z,t)\}=\int_0^\infty p(r,z,t)\exp(-st)dt. \tag{11c}$$

The general solutions of equations (9a)-(9c) are obtained as

$$\begin{aligned}\tilde{u}_r &= C_1\exp(\xi z)+C_2 z\exp(\xi z)+C_3\exp(-\xi z)+C_4 z\exp(-\xi z)\\ &+C_5\exp\left(\sqrt{\frac{k_r}{k_z}\xi^2+\frac{s}{c}}z\right)+C_6\exp\left(-\sqrt{\frac{k_r}{k_z}\xi^2+\frac{s}{c}}z\right)\\ &+\frac{Q\gamma_w}{8\pi\eta Gk_z}\frac{1}{s}\frac{1}{\left(\frac{k_r}{k_z}-1\right)\xi^2+\frac{s}{c}}\exp(-\xi|z-h|)\end{aligned}$$

$$-\frac{Q\gamma_w}{8\pi\eta G k_z}\frac{1}{s}\frac{1}{\left(\frac{k_r}{k_z}-1\right)\xi^2+\frac{s}{c}}\frac{\xi}{\sqrt{\frac{k_r}{k_z}\xi^2+\frac{s}{c}}}\exp\left(-\sqrt{\frac{k_r}{k_z}\xi^2+\frac{s}{c}}\left|z-h\right|\right), \tag{12a}$$

$$\tilde{u}_z=\left(-C_1+\frac{2\eta+1}{2\eta-1}\frac{1}{\xi}C_2\right)\exp(\xi z)-C_2 z\exp(\xi z)$$

$$+\left(C_3+\frac{2\eta+1}{2\eta-1}\frac{1}{\xi}C_4\right)\exp(-\xi z)+C_4 z\exp(-\xi z)$$

$$-\frac{1}{\xi}\sqrt{\frac{k_r}{k_z}\xi^2+\frac{s}{c}}C_5\exp\left(\sqrt{\frac{k_r}{k_z}\xi^2+\frac{s}{c}}z\right)+\frac{1}{\xi}\sqrt{\frac{k_r}{k_z}\xi^2+\frac{s}{c}}C_6\exp\left(-\sqrt{\frac{k_r}{k_z}\xi^2+\frac{s}{c}}z\right)$$

$$\pm\frac{Q\gamma_w}{8\pi\eta G k_z}\frac{1}{s}\frac{1}{\left(\frac{k_r}{k_z}-1\right)\xi^2+\frac{s}{c}}\exp\left(-\xi\left|z-h\right|\right)$$

$$\mp\frac{Q\gamma_w}{8\pi\eta G k_z}\frac{1}{s}\frac{1}{\left(\frac{k_r}{k_z}-1\right)\xi^2+\frac{s}{c}}\exp\left(-\sqrt{\frac{k_r}{k_z}\xi^2+\frac{s}{c}}\left|z-h\right|\right), \tag{12b}$$

$$\tilde{p}=2\eta G\frac{1}{\xi}\left[\left(\frac{k_r}{k_z}-1\right)\xi^2+\frac{s}{c}\right]C_5\exp\left(\sqrt{\frac{k_r}{k_z}\xi^2+\frac{s}{c}}z\right)$$

$$+2\eta G\frac{1}{\xi}\left[\left(\frac{k_r}{k_z}-1\right)\xi^2+\frac{s}{c}\right]C_6\exp\left(-\sqrt{\frac{k_r}{k_z}\xi^2+\frac{s}{c}}z\right)$$

$$-\frac{Q\gamma_w}{4\pi k_z}\frac{1}{s}\frac{1}{\sqrt{\frac{k_r}{k_z}\xi^2+\frac{s}{c}}}\exp\left(-\sqrt{\frac{k_r}{k_z}\xi^2+\frac{s}{c}}\left|z-h\right|\right). \tag{12c}$$

The parameters, $C_i(i=1,2,\cdots,6)$, are functions of the transformed variables ξ and s which must be determined from the transformed boundary conditions; the parameter $c=k_z/n\beta\gamma_w$; the upper and lower signs in equation (12b) are for the conditions of $(z-h)\geq 0$ and $(z-h)<0$, respectively.

Transformed Boundary Conditions

Taking Hankel and Laplace transforms for Eqs. (7a)-(7c) yields

$$\frac{d\tilde{u}_r}{dz}-\xi\tilde{u}_z=0, \tag{13a}$$

$$\eta\frac{d\tilde{u}_z}{dz}+(\eta-1)\xi\tilde{u}_r=0, \tag{13b}$$

$$\tilde{p}=0, \tag{13c}$$

where $\tilde{u}_r$, $\tilde{u}_z$ and $\tilde{p}$ follows the definitions of Eqs. (10a)-(10c).

The constants $C_i\left(i = 1,2,\cdots,6\right)$ of the general solutions can be determined by the transformed half-space boundary conditions at $z = 0$ and the conditions at $z \to \infty$, where the effect of the point sink must vanish. Finally, the desired quantities u_r, u_z and p can be obtained by applying appropriate inverse Hankel and Laplace transformations with the help of mathematical handbook[9] and Mathematica through tedious inversions.

Expressions for Ground Surface Displacements

The horizontal and vertical displacements of the ground surface, $z = 0$, due to a point sink are interested in this paper. The transformed ground surface displacements can be found, from Eqs. (12a)-(12b), and expressed as

$$\tilde{u}_r\left(0;\xi,s\right) = \frac{Q\gamma_w}{2\pi(2\eta-1)Gk_z}\frac{1}{s}\frac{1}{\left(\frac{k_r}{k_z}-1\right)\xi^2+\frac{s}{c}}\exp(-\xi h)$$

$$-\frac{Q\gamma_w}{2\pi(2\eta-1)Gk_z}\frac{1}{s}\frac{1}{\left(\frac{k_r}{k_z}-1\right)\xi^2+\frac{s}{c}}\exp\left(-\sqrt{\frac{k_r}{k_z}\xi^2+\frac{s}{c}}h\right), \tag{14a}$$

$$\tilde{u}_z\left(0;\xi,s\right) = \frac{Q\gamma_w}{2\pi(2\eta-1)Gk_z}\frac{1}{s}\frac{1}{\left(\frac{k_r}{k_z}-1\right)\xi^2+\frac{s}{c}}\exp(-\xi h)$$

$$-\frac{Q\gamma_w}{2\pi(2\eta-1)Gk_z}\frac{1}{s}\frac{1}{\left(\frac{k_r}{k_z}-1\right)\xi^2+\frac{s}{c}}\exp\left(-\sqrt{\frac{k_r}{k_z}\xi^2+\frac{s}{c}}h\right). \tag{14b}$$

For simplicity, only isotropic permeability with $k_r = k_z = k$ are discussed in this paper. Using the Hankel inversions formula defined as following

$$u_r(r,z,t) = \int_0^\infty \xi L^{-1}\{\tilde{u}_r(z;\xi,s)\}J_1(\xi r)d\xi, \tag{15a}$$

$$u_z(r,z,t) = \int_0^\infty \xi L^{-1}\{\tilde{u}_z(z;\xi,s)\}J_0(\xi r)d\xi, \tag{15b}$$

$$p(r,z,t) = \int_0^\infty \xi L\{\tilde{p}(z;\xi,s)\}J_0(\xi r)d\xi, \tag{15c}$$

in which the Laplace inversions are defined as

$$L^{-1}\{\tilde{u}_r(z;\xi,s)\} = \frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty}\tilde{u}_r(z;\xi,s)\exp(st)ds, \tag{16a}$$

$$L^{-1}\{\tilde{u}_z(z;\xi,s)\} = \frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty}\tilde{u}_z(z;\xi,s)\exp(st)ds, \tag{16b}$$

$$L^{-1}\{\tilde{p}(z;\xi,s)\}=\frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty}\tilde{p}(z;\xi,s)\exp(st)ds \,. \tag{16c}$$

With the help of Eqs. (15a)-(15c) and (16a)-(16c), the transient horizontal and vertical displacements $u_r(r,0,t)$ and $u_z(r,0,t)$ of the ground surface due to a point sink are obtained as follows:

$$u_r(r,0,t)=\frac{Q\gamma_w}{2(2\eta-1)\pi Gk}\left\{\frac{ctr}{\left(h^2+r^2\right)^{3/2}}\right.$$

$$\left.-\int_0^{ct}\frac{(ct-\tau)hr}{16\tau^3}\exp\left(-\frac{r^2+2h^2}{8\tau}\right)\left[I_0\left(\frac{r^2}{8\tau}\right)-I_1\left(\frac{r^2}{8\tau}\right)\right]d\tau\right\}, \tag{17a}$$

$$u_z(r,0,t)=\frac{Q\gamma_w}{2(2\eta-1)\pi Gk}\left\{-\frac{cth}{\left(h^2+r^2\right)^{3/2}}\operatorname{erf}\left(\frac{\sqrt{h^2+r^2}}{2\sqrt{ct}}\right)+\frac{h}{h^2+r^2}\sqrt{\frac{ct}{\pi}}\exp\left(-\frac{h^2+r^2}{4ct}\right)\right.$$

$$\left.-\frac{h}{2\sqrt{h^2+r^2}}\operatorname{erfc}\left(\frac{\sqrt{h^2+r^2}}{2\sqrt{ct}}\right)\right\}. \tag{17b}$$

The long-term ground surface horizontal and vertical displacements can be found as following by letting $t\to\infty$:

$$u_r(r,0,\infty)=\frac{Q\gamma_w}{4(2\eta-1)\pi Gk}\frac{hr}{\sqrt{h^2+r^2}\,(\sqrt{h^2+r^2}+h)}, \tag{18a}$$

$$u_z(r,0,\infty)=-\frac{Q\gamma_w}{4(2\eta-1)\pi Gk}\frac{h}{\sqrt{h^2+r^2}}\,. \tag{18b}$$

NUMERICAL RESULTS

Of particular interest is the settlement of the stratum at each stage of the consolidation process. Defining the average consolidation ratio U as following:

$$U=\frac{\text{settlement at time } t}{\text{settlement at end of compression}}\,. \tag{19}$$

Then U can be found as bellow:

$$U=\frac{2ct}{h^2+r^2}\operatorname{erf}\left(\frac{\sqrt{h^2+r^2}}{2\sqrt{ct}}\right)-\frac{2}{\sqrt{h^2+r^2}}\sqrt{\frac{ct}{\pi}}\exp\left(-\frac{h^2+r^2}{4ct}\right)+\operatorname{erfc}\left(\frac{\sqrt{h^2+r^2}}{2\sqrt{ct}}\right). \tag{20}$$

Figure 2 shows the average consolidation ratio U at $r=0$. Note that U initially decreases rapidly but the rate of settlement then slows. Since U approaches 1 asymptotically, theoretically consolidation is never achieved.

The profiles of normalized vertical and horizontal displacements at the ground surface $z=0$ are shown in Figures 3 and 4, respectively. The ground surface has significant horizontal displacement. For example, Fig. 4 shows that the maximum surface horizontal displacement is about 30% of the maximum ground settlement.

CONCLUSIONS

Closed-form solutions of the transient consolidation due to pumping from a pervious elastic half-space were obtained by using Laplace and Hankel transformations. Not only study on the vertical settlement, but also the ground surface horizontal displacement was investigated.

Based on the numerical results, we found that the maximum surface horizontal displacement is about 30% of the maximum surface settlement. From the average consolidation ratio U at $r = 0$, we found that U initially decreases rapidly but the rate of settlement then slows. It is concluded that horizontal displacement should be properly considered for better prediction of the transient settlement induced by groundwater withdrawal.

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SYMBOLS

b_r, b_z	Body forces
c	Parameter, $c = k_z / n\beta\gamma_w$
G	Shear modulus of the isotropic porous medium
h	Pumping depth
$\boldsymbol{i}_r, \boldsymbol{i}_z$	Unit vector parallel to the radial/vertical direction
$J_n(x)$	First kind of the Bessel function of order n
k	Permeability of the isotropic porous medium
k_r, k_z	Horizontal/vertical permeability
n	Porosity of the porous medium
p	Excess pore fluid pressure
$\tilde{p}$	Hankel and Laplace transforms of p, Eq. (10c)
q	Rate of water extracted from the ground per unit volume
Q	Strength of the point sink
(r, θ, z)	Cylindrical coordinates system
s	Laplace transform parameter
t	Time variable
$u(t)$	Heaviside unit step function
$u_r(r,z,t), u_z(r,z,t)$	Radial/axial displacement of the porous medium
$\tilde{u}_r(z;\xi,s), \tilde{u}_z(z;\xi,s)$	Hankel and Laplace transforms of u_r and u_z, Eqs. (10a)-(10b)
U	Average consolidation ratio
$\boldsymbol{v}_w, \boldsymbol{v}_s$	Velocity of fluid/solid
β	Compressibility of pore water
γ_w	Unit weight of pore water
$\delta(x)$	Dirac delta function
ε	Volume strain of the porous medium
η	Parameter, $\eta = (1-\nu)/(1-2\nu)$
ν	Poisson's ratio for the isotropic porous medium
ξ	Hankel transform parameter
τ_{ij}	Total stress components of the porous medium

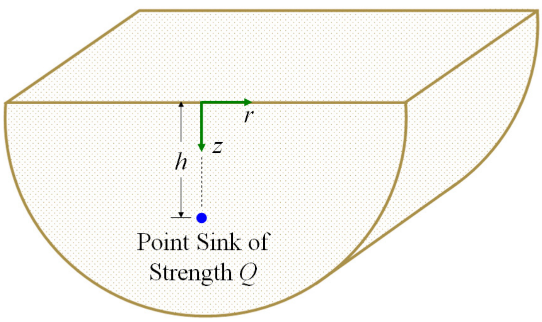

Figure 1. Point sink induced land subsidence problem.

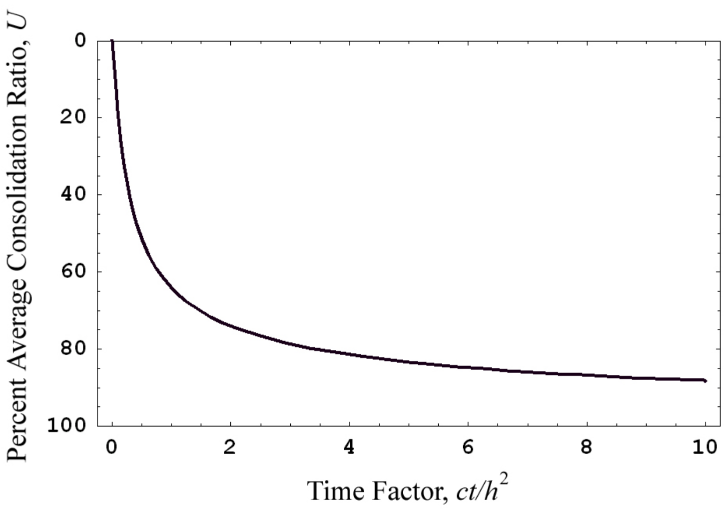

Figure 2. Graphical interpretation of average consolidation ratio U at $r = 0$.

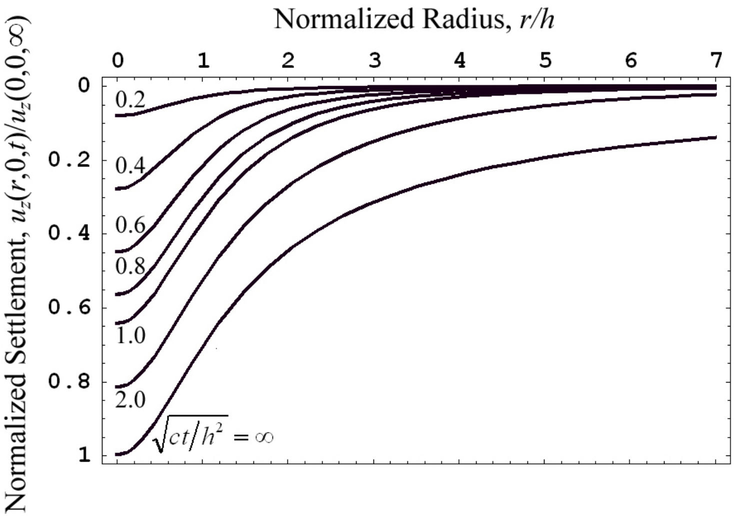

Figure 3. Normalized vertical displacement profile at the ground surface $z = 0$.

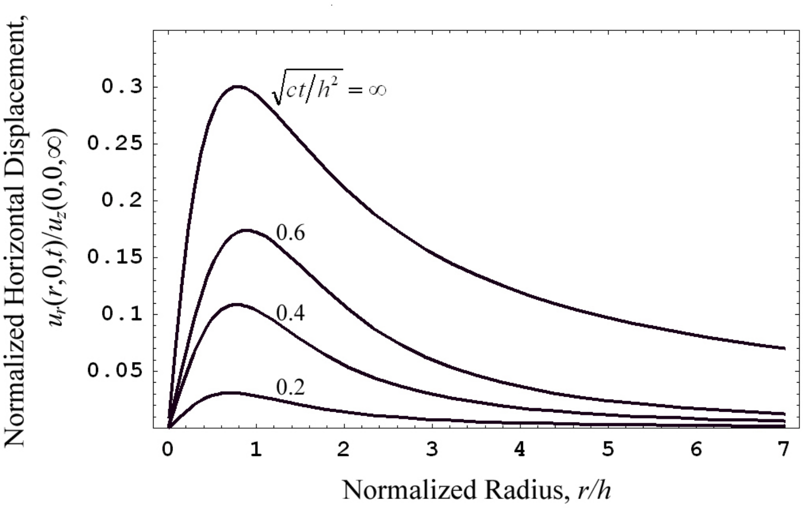

Figure 4. Normalized horizontal displacement profile at the ground surface $z = 0$.