

Virtual Lab Sessions for First year Undergraduate Math Courses

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Abstract: In this paper, I will share my experience of designing the virtual lab sessions for one of mathematics courses that I taught in National University of Singapore (NUS). Computer lab sessions have been part of the first year Calculus and Linear Algebra courses in NUS for many years. Students taking these courses are required to go to a physical computer lab at a scheduled timeslot to complete a worksheet using computer aided software such as Maple, MATLAB or Derive. The worksheets will then be manually marked by lab assistants. The main aim of the lab sessions is to enhance students' understanding of mathematical concepts through visualization and experimentation. Since the enrolment for the first year math courses are usually very large, a lot of administrative work is therefore required to coordinate the booking of labs, assigning of lab assistants and marking of lab quizzes. Recently, I have designed virtual lab sessions to replace some of these regular lab sessions. The new virtual lab allows students to access the lab online anywhere anytime. Each lab comprises two components: lab activities and a lab quiz. In the lab activities, students will use Web tools or applets to explore and visualize concepts taught in lectures. Breeze presentations are also developed to demonstrate how to use the various Web tools. Discussion forums are also set up for students to discuss among themselves as well as to communicate with both the lecturer and lab assistant. The online lab quizzes are designed using the university course management system. Students are given a deadline to complete each quiz. These quizzes are marked automatically by the computer programme. Consequently, a lot of administrative works has been saved. Survey has been conducted to collect students' feedback on the new format of lab sessions.

1. Introduction

Computer lab sessions have been part of the first year Calculus and Linear Algebra courses that I taught in National University of Singapore (NUS) for many years. Students taking these courses are required to go to a physical computer lab at a scheduled timeslot to complete a worksheet using computer aided software such as Maple, MATLAB or Derive. The worksheets will then be manually marked by lab assistants. The main aim of the lab sessions is to enhance students' understanding of mathematical concepts through visualization and experimentation. Since the enrolment for the first year math courses are usually very large, a lot of administrative work is therefore required to coordinate the booking of labs, assigning of lab assistants and marking of lab quizzes. Recently, I have designed virtual lab sessions to replace some of these regular lab sessions. In this paper, I will share my experience of designing these virtual lab sessions.

The idea of web-based instruction for math courses is not new. Other than hyperlinks, many of these online courses also include applets or web tools to make the learning process more interactive and interesting. Some excellent examples are: Engineering Math Course (18.013A) at MIT (see [1]), Calculus course (Math 1501) at Georgia tech (see [2]), the Connected Curriculum Project (many courses) at Duke University (see [3]), Visual Calculus by University of Tennessee (see [4]), Project Links at Rensselaer Polytechnic Institute (see [5]). However, the concept and format between my virtual lab and these online courses are quite different. For the virtual lab, the content of the course is not included. Rather, it contains hyperlinks to the relevant parts of course webpage.

Furthermore, there is an assessment component in each virtual lab that is related to the lab activities.

2. Objectives

The main objective of the virtual lab sessions remain the same as that of the physical lab sessions, that is, to supplement students' learning of the various topics with visualization and exploration. The syllabus of the course is mainly multivariable calculus which includes topics like space curves, multivariable functions and vector fields. For students who encounter these topics for the first time, these may not be easy to understand. If they can interpret these topics graphically, then they will be able to grasp the concepts better. Another objective of having the virtual lab session is to overcome the space and time constraints that physical lab sessions are bounded by. With virtual lab, students are able to access the lab sessions online anywhere anytime. This facilitates students of different capabilities to learn at their own pace.

3. Preparation

The first thing is to decide on the web-based platform. Initially some well known and not so well known Computer Algebra Systems were sought after. These CAS, though powerful, incur costs on purchasing the license and maintenance of the software. Some of them can be quite expensive and funding could be a problem. After evaluation on the systems, it is felt that, for the purpose of my course, the full feature of the system is not required.

Therefore, instead of using a web-based CAS, I decided to search for individual interactive tools available on the web that are relevant to the topics of my course. To begin with, I had to come up with a list of topics or concepts that I want my students to explore during the virtual lab. Then, based on the list of selected topics, I started to do my "web-shopping". The web tools that I have acquired include:

1. "**Tracés animés**" from WWW Interactive Multipurpose Server (WIMS) (see [6])
2. "**Functions of two variables**" applet developed by MIT (see [7])
3. "**Vector field analyzer**" applet developed by Matthias Kawski of Arizona State University (see [8])

The criteria for selecting the web tools are as follow:

Firstly, the tools have to be interactive so that students can do experimentation by changing parameters. For example, by typing in different functions and ranges, **Tracés animés** allows users to plot the graphs of various plane and space curves as well as surfaces.

Secondly, the tools should have several options that link the various concepts that are associated to a mathematical object. For example, in the **Functions of two variables** applet, users not only can input the function and view the surface generated by the tool, they can also switch to the "level curves", "partial derivatives" and "directional derivative" modes to visualize these concepts. In this way, the students can better understand how the concepts are inter-related to one another.

Thirdly, the tools should be user-friendly and preferably self-explanatory. The functionality and various options should be easily understood. There should not be a steep learning curve in learning how to use the tool itself. Otherwise, the users may have difficult or even be discouraged from using the tool.

Last but not least, it is preferable that the access to the web tool is not restricted to a single web server. This is to minimize the possibility that a certain web tool is not available due to temporary server down or other reasons. For example, **Tracés animés** has plenty of mirror sites. In fact, the owner of WIMS generously allows his software to be freely downloaded.

4. Design

Once the topics and the web tools have been finalized, I carried out the process of designing the virtual lab. The whole idea is to let the students carry out some guided hands-on activities on their own using the web tools to visualize and explore the topics. The format of each lab session consists of six sections:

- a. Topic and Objective
- b. Web Tool
- c. Activities
- d. Quiz
- e. Discussion Forum
- f. Help

Topic and Objective

The first section gives an overview of that particular lab session. The purpose and expected learning outcome are spelt out. The topics and concepts that the students are going to explore are also being related to their lectures.

VIRTUAL LAB 1 (by Feb 5, 2005)

[Topic & Objective](#) : [Web Tool](#) : [Activities](#) : [Quiz](#) : [Lab Forum](#) : [Need Help?](#) : [Lab Home](#)

T o p i c & O b j e c t i v e

This lab session let you visualize the **space curves** associated to various **vector functions in one variable** you learned in chapter 1.

Work through [activities](#) 1 through 5 using the [web tool](#) below to study how changing the attributes in the vector functions affect the shapes of the space curves.

Complete the [lab 1 quiz](#) by **Feb 5, 2005**. It is advisable that you work through the lab activities before you attempt the quiz.

Refer also to the gallery on [spiralling curves](#) and [interesting curves](#) to learn more about the effect of vector functions have on space curves.

Figure 1 Topic and Objective

Web Tool

The second section introduces the web tool(s) to be used in the lab activities. It consists of the links to access the tools. If available, links to the mirror sites are also included. To facilitate the students

in carrying out the lab activities, demonstration of how to use the tools are also being presented here. The presentation is made using Macromedia Breeze.

W e b T o o l
The lab activities in this session require plotting of space curves. You will be using the web tool Tracés animés on the WWW Interactive Multipurpose Server (WIMS) to do the plotting.
Click here to access Tracés animés . Then select the button "Parametric curves 3D". In case the above server is down, you may access Tracés animés via any one of the following mirror sites: cochise site ecnu site u-psud site unimib site leidenuniv site
How to plot curves using Tracés animés : demonstration Click to view a presentation on how to use the web tool.

Figure 2 Web Tool

Activities

The third section is the main components of the lab session. It consists of a number of activities associated to the topics under studied. Instructions are given clearly in steps for students to carry out the activities. Certain parts required students to apply what they have learned in class in order to get the desired outcome. Some hints and references are provided as and when necessary.

A c t i v i t i e s
ACTIVITY 1 (STRAIGHT LINES)
(a) Use Tracés animés to plot the following straight lines (i) and (ii) on the same axes. First use the range $-0.5 < t < 0.5$. Then use the range $-2 < t < 2$. (View the demonstration if you need help with plotting) (i) $r(t) = t \mathbf{i} + t \mathbf{j} + t \mathbf{k}$ (ii) $r(t) = t^3 \mathbf{i} + t^3 \mathbf{j} + t^3 \mathbf{k}$ [Type t^2 for t^2 , t^3 for t^3 etc.] What do you observe? Explain your observation. [You may discuss this in lab 1 forum .]
(b) Plot the line that passes through the points $(-3, 2, -3)$ and $(1, -1, 4)$. [You need to work out the vector equation of the line first. Refer to section 1.4 of notes.]
(c) Plot a line that lies in the xy -plane [Hint: The vector function of any line in the xy -plane always has the k -component equal to 0. Why?]

Figure 3 (i) Activities

ACTIVITY 2 (HELICES)
(a) Plot the standard helix $\mathbf{r}(t) = \cos(t) \mathbf{i} + \sin(t) \mathbf{j} + t \mathbf{k}$ (use the range $0 < t < 8\pi$) [Type pi for π , 2pi for 2π , etc.]
(b) Plot the helix $\mathbf{r}(t) = a \cos(t) \mathbf{i} + a \sin(t) \mathbf{j} + t \mathbf{k}$ with the above standard helix on the same axes where $a = 2, 3, 4$. Observe the effect of the vector function on the space curve as the value of a changes.
(c) Now plot the following two helices (i) and (ii) together with the helices in (a) and (b) on the same axes (use different colors to differentiate): (i) $\mathbf{r}(t) = a \cos(t) \mathbf{i} + \sin(t) \mathbf{j} + t \mathbf{k}$ (ii) $\mathbf{r}(t) = \cos(t) \mathbf{i} + a \sin(t) \mathbf{j} + t \mathbf{k}$ using one of the values of a in (b). Observe and compare the shape of the curves. [You may want to set the vertical viewing angle to 90 for clearer view.]
(d) Plot the helix $\mathbf{r}(t) = \cos(bt) \mathbf{i} + \sin(bt) \mathbf{j} + t \mathbf{k}$ with the standard helix in (a) on the same axes where $b = 2, 3, 4$. Observe the effect of the vector function on the space curve as the value of b changes. [You may want to set the vertical image size larger for clearer view.]

Figure 3 (ii) Activities

Quiz

The fourth section is the other main component of the lab session. It is an online quiz aiming to assess the student learning outcome of the lab activities. The online quiz is developed using the in-house course management system (IVLE) of NUS. The quiz consists of multiple choice questions. There is a deadline set for each lab quiz.

Q u i z
Important Note:
(1) The online quiz consists of 5 multiple choice questions.
(2) You can only attempt the quiz once.
(3) Do not click the "submit" button until you have answered all the questions.
(4) You may save your session and continue later.
(5) No time limit for you to try out the session.
Click here to enter online quiz for lab 1 .
Deadline: Feb 5, 2005

Figure 4 Quiz

The following is a sample question from the online quiz.

Straight lines
 Different vector equations may give the same space curves. In this question, you will identify the various vector equations of a same straight line.

1) Which of the following vector functions **does not** give the same straight line as the other three? (assuming no restriction on the range of the parameter t , i.e. $-\infty < t < \infty$)

a $(12+4t^3) \mathbf{i} - (6+2t^3) \mathbf{j} + (3+t^3) \mathbf{k}$
 b $4t \mathbf{i} - 2t \mathbf{j} + t \mathbf{k}$
 c $64t^3 \mathbf{i} - 8t^3 \mathbf{j} + t^3 \mathbf{k}$
 d $(4+8t) \mathbf{i} - (2+4t) \mathbf{j} + (1+2t) \mathbf{k}$

Questions : [< 1 >](#) [2](#) [3](#) [4](#) [5](#)

Max Marks
1

Figure 5 Quiz Question

Discussion Forum

In the regular lab sessions, students are able to do face to face discussion with each other and with the lab assistant. In order to facilitate discussion in virtual lab, an online discussion forum is created using IVLE for each lab session. Students are free to post anything related to that particular lab session except the answers to the online quiz.

L a b F o r u m

Click here to enter [discussion forum for lab 1](#).

Figure 6 Discussion Forum

Help

In the absence of physical lab assistant in the virtual lab, some form of support need to be made available online to the students. An email account has been set up for students to submit their queries when they need help. A lab assistant is also assigned to manage the account.

N e e d H e l p ?

If you encounter any problem regarding this lab session, you may send an email to 1506lab@nus.edu.sg.
 Alternatively, you may also post your questions on the Lab forum above.

Figure 7 Help

5. Implementation

There are all together 4 virtual lab sessions throughout the course. Students are given about 2 to 3 weeks to go through the lab activities and online quiz for each session. The lab activities are meant to be self-exploratory with some guided instructions. They will not be assessed. However, it will prepare the students better in attempting the online quiz. The quiz serves as an assessment as well as a mean to track students' progress. The questions in the quiz are randomized. This makes it very difficult for students to copy answers. They have to submit the quiz before the deadline. The quiz will then be auto-graded by the course management system.

6. Response and Feedback

Survey has been conducted to collect students' feedback on the new format of lab sessions. The general feedback are very positive.

The following are the response to the questionnaire from over 400 students.

Question	A	B	C	D	E
1. Do you find the virtual lab session help in understanding concept better?	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
	42	241	121	36	11
2. Do you find the virtual lab interesting?	Very interesting	Quite interesting	Neutral	Not that interesting	Boring
	59	244	91	38	16
3. Do you find the web tool useful?	Very useful	Quite useful	Not very useful	Useless	Other
	126	252	51	10	7
4. Do you need to view the demonstration in order to use the web tool?	Whole	Partly	No	Other	
	120	218	98	4	-
5. Do you need to use the web tool in answering the quiz problems?	Yes, all	Yes, some	No		
	184	219	33	-	-
6. Do you find the lab quiz difficult?	Very difficult	Difficult	OK	Easy	Very easy
	24	107	286	21	5
7. Did you work through the lab activities before taking the quiz?	Yes, all	Yes, some	No		
	229	147	63	-	-
8. Do you find the lab activities help in answering the quiz problems?	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
	84	209	112	23	6
9. Do you prefer online lab sessions or regular lab sessions (computer lab)?	Online	Regular	No preference	Neither	
	349	26	41	31	-

The following are some feedback from the students:

- I will prefer online lab sessions as it saves the hassle of us to be physically present during regular lab sessions.
- With online lab sessions, students can do them at their own leisure instead of a fixed time. They can explore as much as they want to without any time limit as opposed to regular sessions. But it may be good to have a lab session in school which is not compulsory, but for students with any queries to get help.

- The virtual lab tools are very self explanatory and handy, no necessity to attend the actual laboratory session unless we have difficulties in understanding the concepts.
- There is no pressure on the students, thus increasing learning value tremendously. Students can do the lab as and when it suits their timetable, thus reducing workload and stress.
- A shorter learning curve can be achieved with the help of TAs.
- Virtual labs give students better visualization of the graphs. I personally find it very useful.
- Online lab session allows different people to do the lab and answer the questions with different rates. Fast does not need to wait for the slower ones and vice versa.
- Since we are given a deadline it requires our discipline to finish the lab on time which is what a tertiary education should be.
- Most of us were unsure about the regular lab session last semester and we were mostly just following the instruction on the lab manual. The online lab session this semester is clearer and easier to use. Most importantly, it enhances our understanding of the various curves and surfaces.
- It also allows for discussion with friends... Learning is done more effectively.
- Can we restrict our activities to a few tools so that we do not need to learn how to use each one every time we need to do a lab?
- Online sessions are a bit more worrisome because when there are things that we are unsure about there is no one to physically ask and sometimes putting it on forum is difficult.
- The layout of the virtual lab is very clear. It starts with an overview of what the question is about, and I think that is very good to get us on the right direction.
- It saves a lot of hassle and the questions are mostly rather stimulating and help to understand the topic better.
- No pressure to complete on time. Fun to use.
- We can get technical support for the web tool instantly.
- Online lab sessions are more flexible and it focuses more on self-learning.
- Web tools are much better and more user friendly.
- It helps me learn more as I can do more activities than we are supposed to do.
- Regular lab sessions allow us to interact with fellow peers. It might also be easier for everyone to discuss together, diagrams can also be drawn to facilitate discussion. (this might be harder to accomplish for virtual lab sessions)

7. Concluding Remarks

We have seen that the Virtual Lab is an integration of a number of technologies. First of all, students access the virtual lab through a webpage, which host all the components of the lab session. In the lab activities, students use various web tools to explore and visualize concepts taught in lectures. Breeze presentations are developed to demonstrate how to use the various Web tools. Discussion forums are also set up for students to discuss among themselves as well as to communicate with both the lecturer and lab assistant. The online lab quizzes are designed using the university course management system to assess students' learning outcome.

Virtual lab allows students to access the lab online anywhere anytime. However, it cannot completely replace regular lab sessions. It would still be beneficial for math students to have some hands-on experience with CAS like Maple or MALTAB at some stage of their education. With CAS, users can write programmes to solve specific problems. In contrast, the computations that can be performed by web tools are rather limited, given the current state of technologies. Hopefully,

with the advancement of technologies, more powerful web tools will be available and the effectiveness of the Virtual lab can be further improved.

References

- [1] <http://www-math.mit.edu/18.013A/HTML/index.html>
- [2] <http://www.math.gatech.edu/~bourbaki/math1501/html/>
- [3] <http://www.math.duke.edu/education/ccp/index.html>
- [4] <http://archives.math.utk.edu/visual.calculus/>
- [5] <http://links.math.rpi.edu/index.html>
- [6] <http://wims.unice.fr/wims/wims.cgi?module=tool/geometry/animtrace.en>
- [7] <http://www-math.mit.edu/18.02/applets/FunctionsTwoVariables.html>
- [8] <http://math.la.asu.edu/~kawski/vfa2/>