Design of virtual manipulatives for mathematical explorations using Flash ActionScript

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Abstract: Recently, Moyer, Bolyard & Spikell (2002) proposed the concept of "virtual manipulatives" for developing mathematical knowledge. This approach is an innovative medium for use by elementary school children. Virtual manipulatives are often exact visual replicas of concrete manipulates, and are placed on the Internet as applets. Children can use computer mice to manipulate the images. These creations are based on innovations in computer technology that allow programmers to generate electronic objects. Flash is well known to be a highly effective means of creating animations, which are available all over the Internet. Flash can be used to design innovative animations on the Internet. Additionally, Flash ActionScript can be used to generate highly interactive software. In this study, Flash ActionScript is used to design virtual manipulatives for mathematical explorations. Two examples are discussed. The first is an instructional medium for use by teachers. Rather than manipulating real concrete objects in class, teachers can use it as an instructional tool to introduce basic number concepts. The second example is a pattern exploration tool. Students were asked to explore possible results of figures by changing methods of banding and the number of points on a circular lattice. The development of this software reduces the manipulation time; students have much time to focus on finding patterns of numbers. Results of this study suggest that Flash ActionScript is a highly effective means of designing virtual manipulatives for mathematical learning. Ideas from various classroom activities can be translated into designing software.

1. Introduction

Technology contributes significantly to mathematical learning, and the nature and extent of its contribution depends largely on the technology adopted (Clement and Sarama, 2005). However, teachers often experience difficulty when using technology to teach mathematics, possibly because of a lack of effective methods and instructional models. Recently, Moyer, Bolyard & Spikell (2002) proposed the feasibility of generating "virtual manipulatives" to present opportunities for constructing mathematical knowledge. This innovative technology is highly promising for use in learning and teaching mathematics. Virtual manipulatives are often exact visual replicas of concrete manipulates, and are placed on the Internet as applets. Restated, they are virtual images on the computer and can be dynamically manipulated in the same manner as a concrete manipulative. Children can use computer mice to manipulate the images. These developments are owing to innovations in computer technology that enable programmers to generate electronic objects (Moyer, Niezgoda, & Stanley, 2005).

Flash is known to be a highly effective means of generating animations for Internet. *Flash* can be used to design innovative animations on the Internet. With its extensive functionality, *Flash ActionScript* can be used to develop an interactive, Web-based virtual representation of dynamic objects. In this study, two virtual manipulatives were generated using *Flash ActionScript* to enhance the teaching and learning of mathematics. These tools are designed based on classroom activities. Via these tools, users can manipulate and alter representations to study concepts and construct meaning. The following two sections discuss in detail the two tools.

2. Tool 1: *Magic Board*

For elementary students in their first few years of school, teachers use physical objects that are manipulated to introduce basic number concepts, such as counters , Cuisenaire rods, pattern blocks, geometric solids, and base-ten blocks. If the physical objects are inadequately large, manipulating objects on the blackboard is often unclear when discussed in front of the entire class. Also, the making and cleaning of these concrete instructional tools are time-consuming, and they are not easily preserved for a long time. Research has demonstrated that manipulating shapes and objects on a computer can be just as, or more effective, in facilitating mathematical learning (Char, 1989; Clement and McMillen, 1996). Previous studies have demonstrated that virtual manipulatives can provide children with flexible representations to support mathematical explorations (Moyer, Niezgoda, & Stanley, 2005).

Observations of classroom teaching activities stimulated the idea of designing the *Magic Board* tool. Rather than manipulating real objects in class, teachers use it to introduce basic number concepts. The *Magic Board* is a collection of objects that teachers can use to present number concepts and design learning activities. During the development of this instructional tool, school-teachers were invited to use it and provide feedback to help to improve the function of the software. Many teachers offered suggestions regarding the use of this tool, which was finally completed accordingly. For instance, some teachers suggested that adding different objects and scenes could help them tell stories to help teach mathematics. The components and functions of *Magic Board* are discussed in detail below.

2.1 Counters

Different objects on the *Magic Board* can be selected before manipulating. They are fruits, vegetables, animals, fish, toys, food, flowers, trees, clothes, cubes, Cuisenaire rods, money, geometric figures, and others. The user first selects a type of object, and then drags the selected object into the manipulation area. Most objects can be used as counters to teach basic number concepts, including practicing counting skills, introducing the concept of place value and explicating the meanings of basic operations. A number can be written on the manipulation area if the writing function is selected. A pen can be used to make marks on the manipulation area. Clicking the “clear” bottom erases the writing. Selecting the object and dragging it to the trash throws it away. Figure 1 displays the interface of the *Magic Board*.

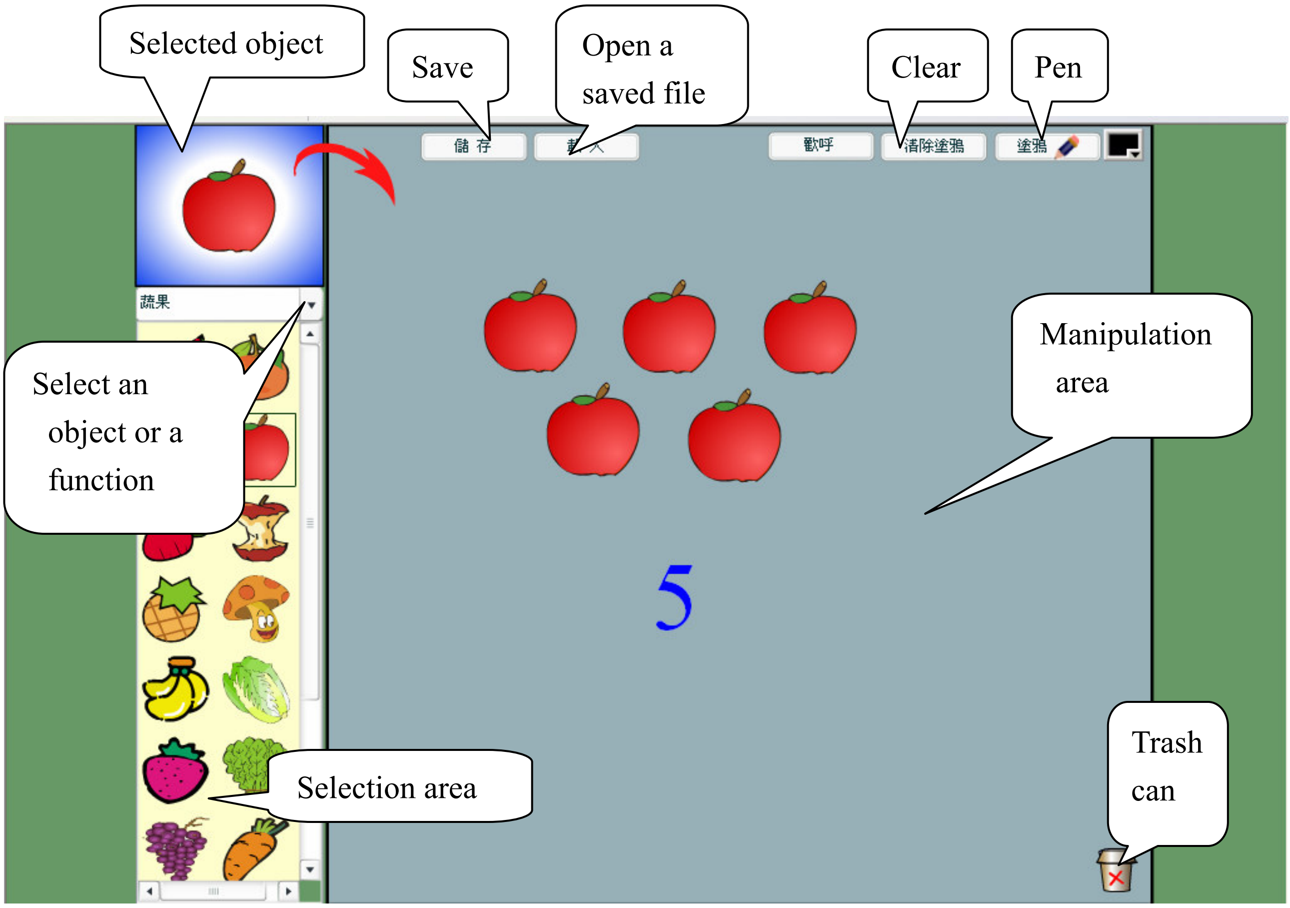

Figure 1 Interface of Magic Board

2.2 Ten-frames

Helping children relate to such numbers as five or ten is vital in teaching number concepts. The conventionally adopted model for this relationship is the ten-frame model. A teacher can give students a number in a particular representation, such as a picture or a number, before showing the number in a ten-frame form. On the selection area, the user can select a ten-frame with the expected dots on it. Also, the user can select an empty ten-frame and fulfill it with expected dots by dragging dots from the selection area. Figure 2 presents an example of this explanation.

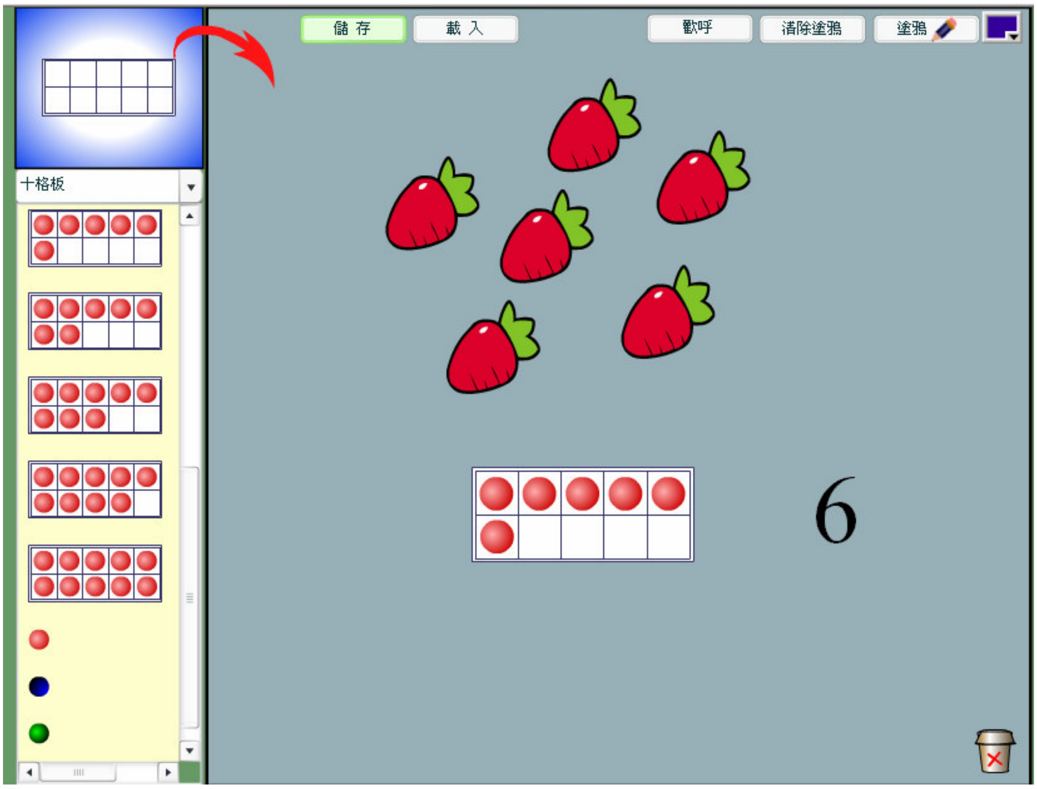

Figure 2 Activity using ten-frame

2.3 Number boards

More thoroughly understanding the properties of, and the relationships among numbers is important in elementary school. The *Magic Board* is an effective means of creating number boards (of dimensions n by m, where $4 \le n, m \le 10$). A 10 by 10 board is called a virtual hundred board, which provides a visual means of highlighting and presenting various patterns and relationships among numbers less than 100. The squares have five possible colors, and teachers can use these functions to highlight the number patterns for exploration. For instance, if all multiples of nine are colored blue, and all multiples of six are colored red, then some of the numbers (common multiples) can be colored in both blue and red. If these numbers are colored green, then these numbers are clearly multiples of 18. Figure 3 shows the results of such an exploration. On the bottom of this tool, the give squares with no number can be used to hide the numbers. Teachers can use them to design a guessing game or a test. Addition and multiplication tables can also selected to introduce basic facts about numbers. (See Figs. 4 and 5.)

Figure 3 Using a hundred board to explore common multiples of six and nine

Figure 4 Addition Table

Figure 5 Multiplication Table

2.4 Scenes

Teachers can also select different scenes to motivate learning, as was suggested by teachers. When this function is used in class, students participate actively and enjoy learning. (See Fig. 6 for an example of a designed scene.) Most young students enjoy listening to stories. This function provides teachers an easy way to solve design problems that can motivate learning.

Figure 6 Example of designed scene

The *Magic Board* was first used in a first-grade classroom during the end of the second semester in 2005. Students were pleased to learn from this tool and look forward to learning more in the future. Most of the teachers who tried to use the *Magic Board* thought that they would use it again in the future. A small proportion of elementary-school teachers use computers to teach mathematical concepts (Weiss, 2000). The *Magic Board* supports this approach. Using it in mathematical instruction is hoped to increase students' conceptual understanding.

3. Tool 2: *Circle Geoboard*

Recognizing and understanding patterns are essential to mathematical learning. The software design was based on a class activity (Perl, 2004). Students were asked to explore possible results of figures by changing methods of banding and number of points on a circular lattice, which can be achieved by asking students to draw on a recording sheet or use rubber bands on a real board. This is a time-consuming procedure, and students may spend time on meaningless tasks. The development of this tool reduces the manipulation time; students have much time to focus on finding the patterns of numbers.

3.1 *Circle Geoboard*

When the *Circle Geoboard* is opened, two icons appear – (1) select the number of points on a circle; (2) create a new circle on the board. The user can create a circular board with between 2 and 50 points. When a circular board is created, the user can begin to connect red segments between points, similar to locating a rubber band on the circle. Red segments can be connected either by dragging the mouse between points or selecting a number, say 5, to connect automatically for every 5 points until the original point 0 is connected again. The user can generate as many circles as possible simultaneously for different explorations. Figure 7 shows an example of four circles with different points (12, 20, 30 and 40). Each circle was connected with red segments for every other points. Clearly, various polygons were created. Records of the banding can be opened by clicking the icon "…" and a window is opened at the right of the circular board. According to our results, as the points were connected differed, or the number on the circle changed, the results changed (Fig. 8).

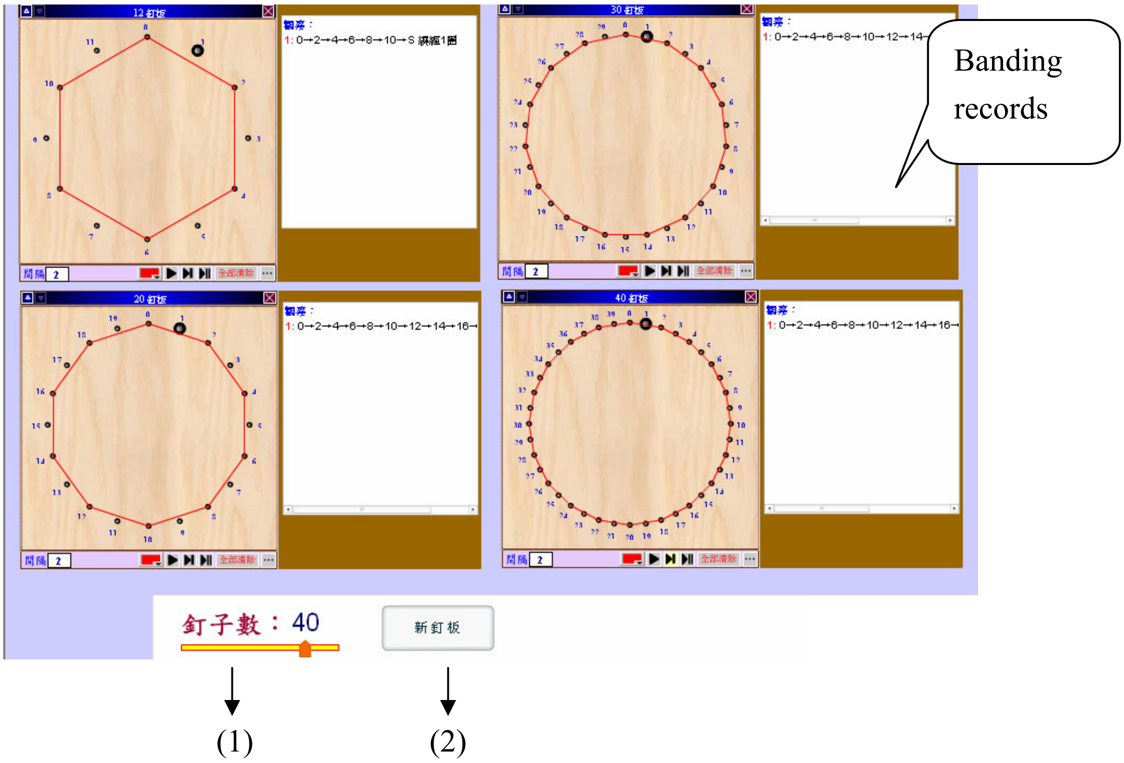

Figure 7 Creating four circles on the Cicler Geoboard

After just a few clicks, the results appear much more quickly than when a teacher uses physical materials or draws on paper in the classroom. A teacher who is using the *Circle Geoboard* can design record sheets on which students can record data, allowing them to spend more time to find interesting patterns. The following section presents an example.

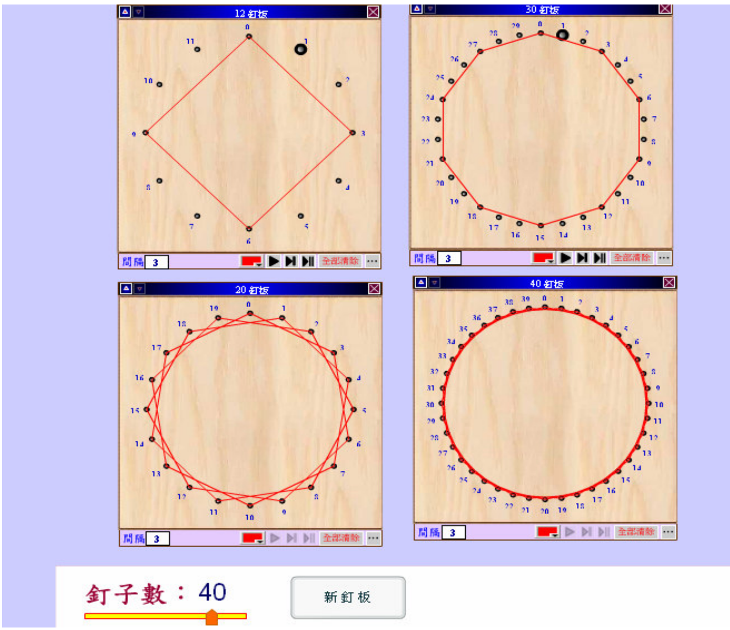

Figure 8 Circles banded for every three points

3.2 Star or polygon ?

When a circular board with 12 points is created, what figures can be created in various patterns to connect the points? Can any pattern be found in these figures? What conjectures are reasonable? Answering these questions may take a long time. However, the *Circle Geoboard* greatly facilitates the exploration of this study. Figure 9 shows all possible results. These results can be applied to other situations, including those circles with different number of points.

Figure 9 shows that the figures that corresponds to number 5 and 7 differ from other numbers. They are the same and look like stars. All other numbers, except that which corresponds to 6, are polygons. One and 11, 2 and 10, 3 and 9, 4 and 8 correspond to the same results. Twelve can be divided by six; when connecting for every six points, it is clearly a line. Eleven is one less than 12, so like the number 1, it will be a figure connecting every point on the circle. Based on these observations, two conjectures are proposed. First, two numbers that are complement of 12 have the same figures. Second, if $(n, 12)=1$ and $n \neq 11$, the figure is in the shape of a star. Can these two conjectures be true in all the other cases? This question can be answered by considering other numbers. These two conjectures are surprisingly true for 24 points. Figure 10 shows the results for numbers which are relatively prime to 24 (except 23). They are all stars and all complements of 24 have the same figures.

Students use the *Circle Geoboard* to explore this problem by changing the number of points on the circular board. The results are clear. Moreover, their conjectures can be checked quickly. The students may find proving these conjectures difficult, but this tool provides students an opportunity to explore patterns and relationships among numbers.

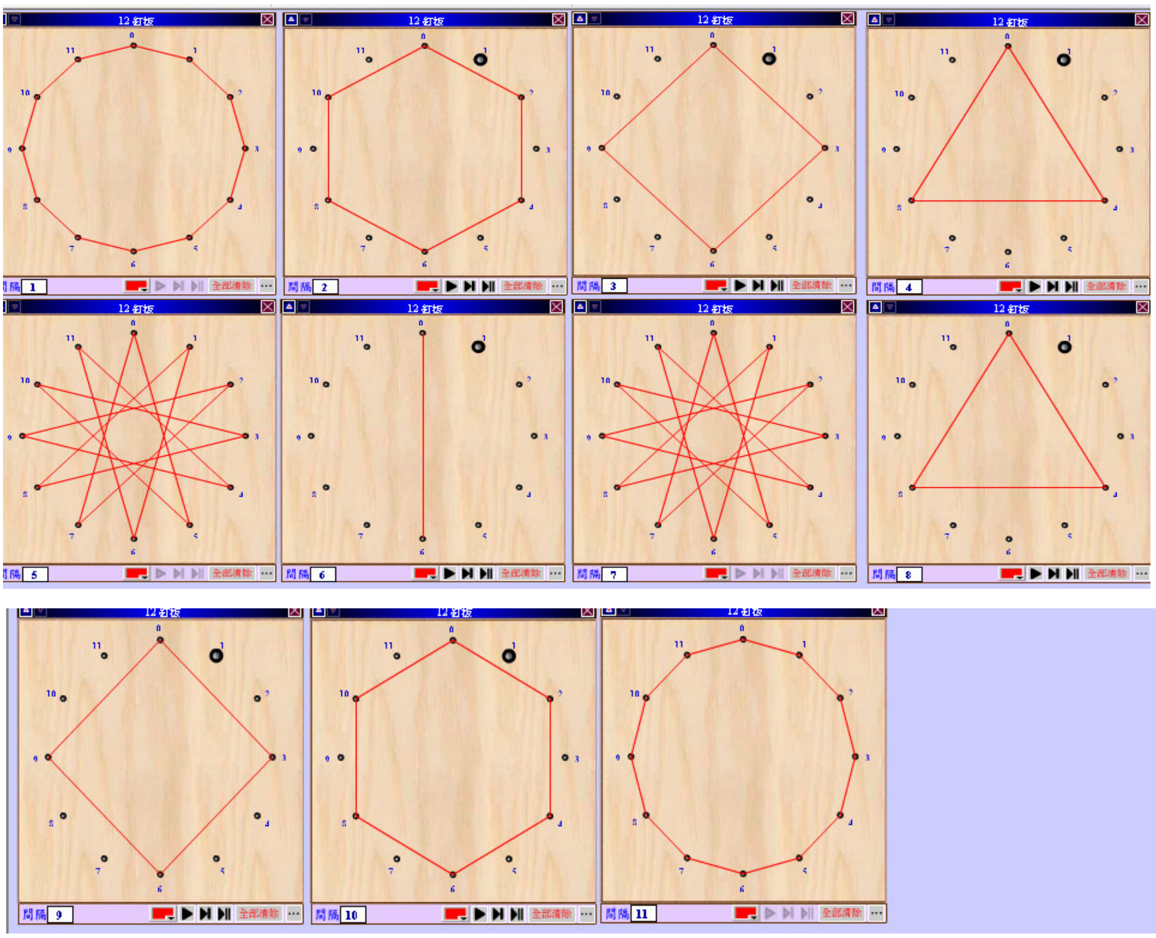

Figure 9 Results in case of 12 points on the circle

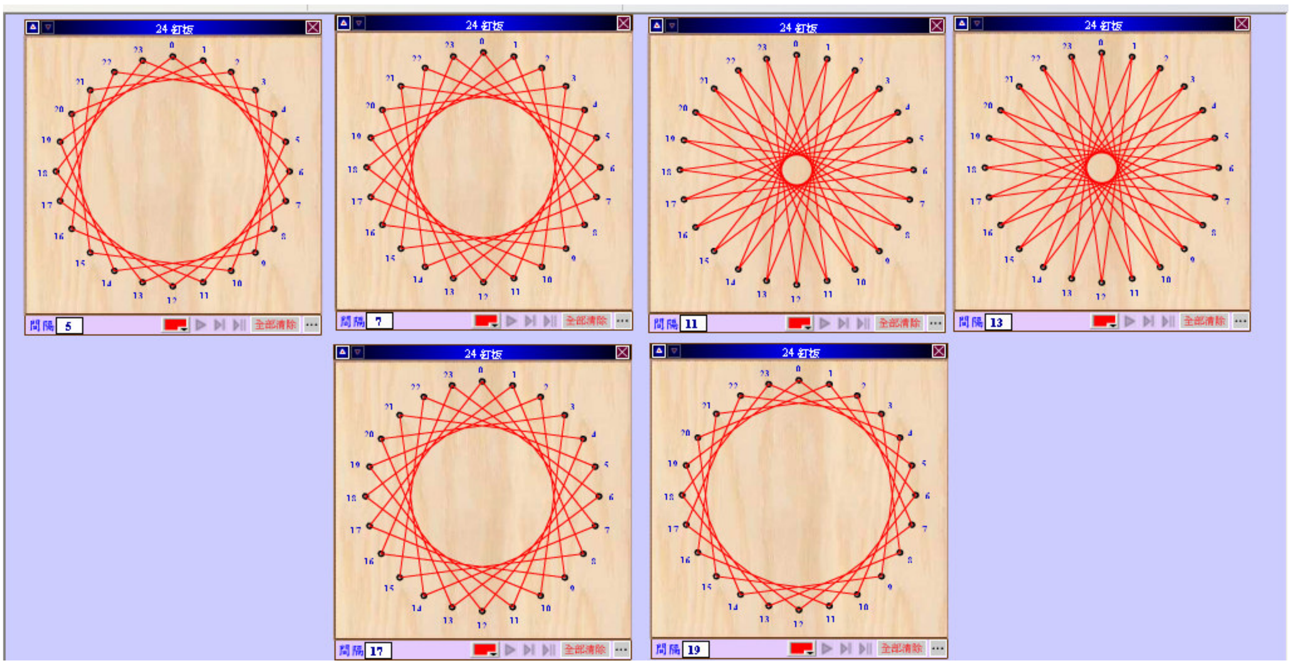

Figure 10 Star figures from circles with 24 points

4. Conclusion

Capable of helping teachers to present mathematical concepts clearly, virtual manipulatives can be used to provide interactive learning environments in order to help students study mathematical concepts and improve their conceptual understanding (Terry, 1996). Ideas from various classroom activities can be translated into designing software. In this study, the designs of these two tools are based on real classroom activities, and some limitations are overcome in a virtual technology environment. For instance, when the *Magic Board* is used, distribution objects and cleanup are easy; when the *Circle Geoboard* is used , teachers and students can save much time in banding. Results of this study suggest that *Flash ActionScript* can be useful in designing instructional tools for mathematical learning. However, "good ideas, not necessarily new technological developments, guide the way" (McNergney & Kent, 1999, p50). Finding effective ways to use innovative means of teaching mathematics is essential for all students. If schools are equipped with appropriate technological equipment and such useful tools, teachers can easily integrate technology into mathematical teaching.

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