Development of Proof Abilities Through Working Backward with Graphing Calculator

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Abstract

The axiomatic method used in Euclid's Elements in BC 3rd century has been introduced to improve students' logical thinking abilities for a long time. The working forward method to deduce a conclusion from given conditions has a same order as teachers explain proof processes appeared in textbooks. It does not show mathematical activities such as imagination, intuition, experiment, thoughtful guess, trial and error, mistake etc which were necessary in generating the proof. As a result, students have few meaning in the proof explained by teachers and lose confidence in learning mathematics eventually. To improve students' proof abilities, an "active justification" to find the proof method in their own position should be required rather than a "passive justification" through teachers' explanation.

Mathematical heuristic related with a proof method goes back to AD 3rd century. The heuristic so called "analysis method" was systemized by Greek mathematician Pappus. The analysis method assumes what is sought as if it were already done and inquire what it is from which this results and again what is the antecedent cause of the latter and so on, until by so retracing the steps coming up something already known or belonging to the class of first principles. As same as Euclid's Elements, current geometry textbooks introduce only the synthesis as the reverse of the analysis. The analysis also should be introduced in order to develop students' proof abilities. However, it might be very difficult to apply it in the paper and pencil environment because various dynamic operations such as manipulating geometric figures are required.

This study is to investigate experimentally whether the graphing calculator with dynamic geometry is a good environment for Korean 10th grade students to apply the analysis method as a working backward strategy in solving construction problems by compasses and ruler. This study describes students' processes to find their own construction method by using the analysis method with the graphing calculator as a

working tool and to justify the method deductively by using the synthesis method. Each of six experimental classes with four students consists of the four phrases: “understanding” to recognize problem conditions and goals clearly, “analysis” to assume what to be solved is done and to find the construction method by using the analysis, “synthesis” to construct a deductive proof as a reversed process of the analysis, and “reflection” to reflect on whole problem solving process. All activities on the graphing calculator were captured as moving figures and students’ dialogs were recorded with audiotapes. Data analysis was conducted by using the materials and an informal interview with students before and after each class.

Development of students’ proof abilities through graphing calculator with dynamic geometry were evaluated by the following points: Drawing, dragging, measuring and transformation functions helped students find the series of steps toward the construction method by themselves. Students appreciated the synthesis as the reverse of the analysis and accomplished it well in the dynamic geometry. Students confirmed dynamically that the construction method is valid. Students recognized the importance and value of the proof through the active justification.