A Global View of Curriculum Issues on Mathematics with Technology

A Panel Discussion

Moderator: Wei-Chi Yang, USA (wyang@radford.edu)

Panelists: *(in alphabetical order)*
Douglas Butler, United Kingdom (debutler@argonet.co.uk)
Ivan Cnop, Belgium (icnop@vub.ac.be)
Masami Isoda, Japan (msisoda@ningen.human.tsukuba.ac.jp)
Peng Yee Lee, Singapore (pylee@nie.edu.sg)
Kaye Stacey, Australia (k.stacey@unimelb.edu.au)
Khoon Yoong Wong, Singapore (kywong@nie.edu.sg)

(1) Introductions. Why technology has changed math curriculum in some countries?

We will consider only computer-based, hand-held or calculator-based technology in this article. According to the *Principles and Standards* document by the National Council of Teachers of Mathematics (NCTM) in the U. S. (http://nctm.org/standards/pressrelease.htm), "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances student learning. It needs to be used wisely, by well-informed teachers, to support understanding." Thus, proper use of technology in mathematics education is essential. On the other hand, in the same article, it also addresses that adequate training on basic mathematics cannot be ignored either, "Students must be fluent in arithmetic computation -- they must have efficient and accurate methods, and understand them."

The mass media is also aware of the importance of ICT (do you want 'Technology' or 'IT' or 'ICT'?) in education. In Taiwan, an article in Lien Her Bao (a daily newspaper in Taiwan) on September 4, 2000, mentioned the master plan for education by the Ministry of Education in Taiwan. They hope that mathematics is accessible to 80% of students; complicated algebraic manipulations can be replaced by calculators or/and computers. It is known that many 'popular' software packages are being adopted by students and teachers from middle schools, high schools to universities in Taiwan, but the examination questions have not been modified to be more technology based. Another article published by Beijing Youth Daily on March 7, 2001, titled "The standard of the National Curriculum is set preliminarily." It stressed that the old traditional "fill the duck style" should be replaced by solving more real-life problems. Establish diverse standards of measuring students' success instead of basing on testing alone. The city of Shanghai was the first city in China requiring the use of certain approved 'scientific calculators' in the 'college entrance' examination in 2000, and many cities and provinces have looked into the 'Shanghai model'.

In Singapore, the Ministry of Education implemented its first Masterplan for IT in Education from 1997 to 2002. In 2003, it launches the Masterplan II for IT in Education (mp2 in short), which will run to the year 2007. The First Plan has provided the physical infrastructure in schools, teacher training, and learning resources. The Second Plan adopts a holistic approach to produce engaged learning by harnessing fruitful interactions among pupils, teachers, curriculum and assessment, and environment (at school, national and global level) in the use of ICT. Both plans aim to support and develop lifelong learners as part of the Singapore "Thinking Schools, Learning Nation" vision. Using ICT in mathematics education is an important component of these plans. Research into mathematics education and technology use in education form two of the three crucial components of the heavily funded Centre for Research in Pedagogy and Practice at the National Institute of Education, Singapore.

As we can see that the needs for changing mathematics curriculum in some of the Asian Pacific regions are urgent and somewhat geared toward the one in the U.S. In the meantime, many educators in the U.S. are concerned about the lack of basic computation skills? from a student if technology is not properly and timely introduced. Therefore, how much basic algebraic manipulation skills do we require from students and how students are able to manipulate technology to solve real-life applications will be an on-going investigation. We will see later how technology has impacted mathematics curriculum in some countries.

Schools in all states of Australia have revised the arithmetic curriculum very extensively over the past twenty years. Also most states of Australia have reduced the amount of algebraic manipulation, partly because of the switch to a mass education system but also partly in the expectation that long algebraic manipulations will be done by machine in the future. A project by Stacey and colleagues at the University of Melbourne [11, 12] is the first to investigate the use of computer algebra systems in school mathematics. An extended trial is now being conducted. Many schools use spreadsheets and many use graphing programs and statistics programs. The reasons are a general wish for the curriculum to stay relevant to the modern workplace and also to improve teaching and learning. There are many opportunities for improving concept formation with technology. Dynamic geometry is also becoming popular. In summary technology has changed the curriculum and offered new possibilities for teaching.

There are several reasons why technology has changed or will change mathematics curriculum in Singapore and also other countries. The changes may have started with non-mathematics curriculum like computer club activities, English (process writing), science (simulation) and so on.

1.1 Technology is a very visible tool and has high commercial value. Many companies big and small survive on selling hardware and software to schools and education institutes, which is big business in most countries. The push to have technology in the schools is often driven and initially "supported" by big companies like IBM, Microsoft, CASIO and TI.

1.2 Politicians and administrators believe that to have technology in the schools is essential to prepare pupils for the future within a knowledge-based economy, Internet connectivity, and so on. In Singapore, the IT Master Plan is developed with this purpose in mind. The physical environment is put in place (e.g., hardware in schools, fast Internet connections) so that teachers can use it for teaching. The implications of technology for teaching come later.

1.3 Mathematics educators come on board because the technology is there, initially for drill and practice, based on behaviorist theories and outcomes on mastery of skills. Later the tool and tutee modes become popular because these can promote higher order thinking. This trend is supported by psychological theories about information processing (e.g., using computer to simulate human algebraic thinking), constructivism (e.g., Papert's work), cooperative learning (e.g., Vygotsky), and metacognition. This paradigm shift is supported by advances in hardware (e.g., larger memory, faster processor, wireless technologies, down-sizing) and software (e.g., Dynamic Geometry Software such as Cabri and Geometer's Sketchpad etc., Computer Algebra System such as Maple, Mathematica, Derive and etc., and Java applets). With this paradigm shift, the objectives and topics of the mathematics curriculum should focus less on computational skills (e.g., division of fractions and algebraic manipulations are made simpler) and more on *exploratory* and *investigative* tasks.

In the sections below, we will concentrate on the situation of mathematics curricula in Europe. The situation in Europe has become more complex since over the last decades curricula and teaching methodologies have become more fragmented between countries. This may be a surprising result in view of the economic and monetary convergence, and the ongoing discussions about equivalence of tertiary degrees and qualifications in the European Union (the Bologna agreement). National decision bodies concentrate on national or short term priorities such as language issues, vocational training, integration of immigrants, drugs in schools, school finances, failing students having to redo their year, for example, and some of the convergence in education is still limited to paper statements and wishful thinking. Moreover it is difficult to compile overviews for countries in which the full education system, including language of education and mathematics curricula, are organized on a regional basis, sometimes in regions with a population of only a few million. Examples of such fragmentation are Germany (although its Bavaria region is larger than most countries in Europe), Spain and Belgium.

There is a consensus about the fact that technology cannot be avoided in mathematics education. Part of the motivation is external: use of computing technology in courses where mathematics is a service subject (physics, engineering, economics, and etc.) and pressure from industry and commerce. Curriculum changes have been slow and cautious, generally lagging behind the capabilities of technology. In some countries, introduction of technology has had a destabilizing effect on the teaching of mathematics. The uniform teaching methodology (fixed content, paper and pencil, rote learning, and etc.) has been replaced by a multitude of carriers, brand names, and teacher attitudes towards the new media. With each new product the industry has been turning out manuals that become obsolete with the introduction of the next product.

A lack of uniform guidelines, or their late arrival [1], has left many teachers insecure and has led to instability in content and assessment. The most recent example is the public outrage in the French newspapers (from teachers, pupils and their parents) after the 2003 baccalaureate examination. This examination included unexpected questions about three dimensions that figured in the new curriculum, were perfectly tractable by graphics technology, but had not been addressed by most teachers for unfamiliarity with this type of question, lack of time, or lack of experience with the technology.

Many teachers in East Asia, especially in Japan, believe that technology such as Dynamic Geometry Software, Graphing Software and Computer Algebra System are only educational aid. Indeed, the national curriculum document in Japan strongly recommended that teachers should use technology in classroom exploration (just in case it is useful). The national document looks refreshing and promising at a glance, but it implies that technology is only recognized as an educational aid which teachers do not need to use it. As a result, teachers in Japan will still prefer pencil and paper approach because there is no mandate, researchers use CAS to do their research because they can not live without it but most of them will not incorporate CAS in undergraduate level courses because they know it will take up lots of their time for preparations on lectures.

(2) What has been changed in teaching/learning when technology is used?

In Australia, calculators are widely used – four functions in upper primary school, scientific in lower secondary school and graphics calculators in senior secondary school. Dynamic geometry and spreadsheets are popular in some schools. Universities vary, but most students who use mathematics (e.g. engineers, economists etc.) use technology very heavily.

Many teachers really like the way in which graphs can be generated quickly and when students have graphics calculators, students have come to use graphical interpretations much more than before. The same phenomenon is observable with statistical and spreadsheet graphing – teachers can do more interpretation of results. Previously by the time students had drawn a graph, the class was nearly over – now they can talk about what it means. This is one instance of making certain representations of mathematical ideas more available in teaching. Graphing (algebra and data) has been the big success.

There is also some revitalization of geometry now observable when schools get dynamic geometry. Students and teachers are discovering exciting new conjectures by using Cabri, Geometer's SketchPad and etc. Therefore, it is predicable that technology has made a positive impact when it is used in classroom exploration. On the other hand, due to limitation of devices that are allowed in an examination, it is difficult to design appropriate questions to measure students' understandings. Fortunately, the new CASIO Class Pad 300 is a pen-based, handheld device which allows users to draw geometry figures quite easily. Encouraging such device in a classroom and examination may broaden the way we ask questions in new era. It is interesting to see in the state of Victoria, Australia, geometry has been in decline, but the new possibilities are capturing the imagination of some teachers and there may be a revitalization of this part of the curriculum. Some curriculum change directly uses the functional capability of technology – for example students now regularly use correlation coefficients and scattergrams even in low level senior courses. Before technological assistance was available, this topic was not covered at school.

2.1 In Singapore schools, not much has been changed in terms of contents and pedagogy. The rhetoric and visions have yet to transform the mathematics curriculum in a significant way. It is debatable how much should and could be changed.

2.2 Anticipated changes at primary level are quite minimal in Singapore because many topics have daily applications and are quite universal. The nature of the child brain is a limiting factor on how much can be changed. Unlike in some Western countries, basic calculators are rarely used in Singapore primary schools. The current debate is what to use basic calculators for: save computation time and effort, explore concepts, use in assessment.

2.3 In secondary schools, the Singapore mathematics curriculum has not changed much in the past twenty years. Minor changes such as less practice on fractions and manual graph plotting have taken place. The main reason is that the school curriculum is based on Cambridge O-level examination, which has remained quite stable over the past two decades.

2.4 The Singapore Ministry of Education recommends that 30% of curriculum time should involve use of ICT. Some mathematics teachers try to satisfy this by using PowerPoint as a presentation tool, which is usually not effective to teach pupils how to solve problems. Training courses in using ICT have been conducted since mid 1980s. In recent years, courses in graphing software, Excel and Geometer's Sketchpad are very popular, conducted by the Education Technology Division of the Ministry of Education and the National Institute of Education. However, their actual use to teach mathematics in schools is not well documented.

2.5 Use of multi-media courseware is also popular in Singapore. Although somewhat derided among some mathematics educators, well-designed courseware can help pupils to master basic skills, which should remain an important objective of school mathematics curriculum. To master skills, Singapore pupils spend considerable amount of time on practice and teachers also expend much time and effort in marking pupil homework. A Singapore company has developed an Internet-based system (called Math Explorer) that can mark multi-line, free-response solutions entered by the pupils. This should help reduce the amount of time spent by teachers on marking routine homework. The feedback on this system is quite positive.

2.6 Logo has entered mainstream mathematics teaching in some countries, but it has disappeared from Singapore schools in the past decade, although Logo was taught in some schools in the 1980s. Logo programming is supposed to help pupils apply mathematics, develop a more positive attitude toward errors, engage in group activities, and transfer problem solving in Logo to other contexts. Using different software seems to promote different mathematical thinking. For example, to draw a circle using Logo requires that the pupils use regular polygon of many sides (ideas of approximation and infinity are involved), whereas drawing a circle in Geometer's Sketchpad is through clicking such that the underlying mathematical construction is not evident. Such differences should be explored.

2.7 Controversies arise between those who believe that skills are of marginal values and should be eliminated (because they can be performed using technology as a black box) and those who believe that manually working through the skills is a pre-requisite for deep understanding of the mathematics. Research has not provided much insight into this issue.

2.8 It is relatively easy to train teachers to use various ICT tools, but it is much harder to help teachers integrate ICT into mathematics teaching and learning. Teachers have to change their conceptions of the nature of mathematics, the aims of mathematics education, and how mathematics is learned (especially contrasting practice of given skills versus construction of knowledge).

2.9 The Singapore mathematics curriculum spells out 6 ways that ICT can be used in mathematics education:

(a) Bridge the gap between abstract concepts and concrete experiences.
(b) Consolidate concepts and skills.

(c) Enjoy meaningful learning.
(d) Participate in cooperative work and broaden learning styles.
(e) Explore and attempt different approaches to tasks and problems, and hence observe a variety of consequences.
(f) Shift towards tasks and problems which require higher level of competencies.

It is not evident to what extent these objectives have been accomplished.

2.10 In Singapore, many ICT-based mathematics lessons are based on worksheets (also found in books and papers published elsewhere, e.g., NCTM). This "worksheet pedagogy" is supposed to guide pupils to focus on crucial mathematics, but in reality, some pupils just try to complete the worksheets as quickly as possible, without fully engaging their mind on the tasks. This mismatch should be investigated.

2.11 A typical "research" task is for pupils to search writings from various sources about a mathematics topic, use Word to write neat and colorful summaries, and Excel to draw charts or do calculations on real data. It is important that pupils should understand the mathematics that has been downloaded. Personal experience suggests that the average pupil does not learn well from reading mathematics text; in fact even educators who teach reading across curriculum areas may comprehend writings about mathematics, but they cannot apply such comprehension to do mathematics. Hence, getting pupils to search for articles in mathematics as a research task is not likely to be beneficial to the pupils in terms of enhancing mathematical understanding. There is a need to research pupils' own processing of mathematical information. Pupils have different learning styles and these should be considered when using technology.

2.12 To use technology effectively, pupils need to acquire basic technology skills. Who should be responsible to teach these skills to pupils? Mathematics teachers may not like to use mathematics lesson times to do so because this will reduce curriculum time for teaching mathematics.

(3) Can the mathematics curriculum be extended with technology? Or how technology influences mathematics content?

It is not possible to maintain the old curriculum in a technology environment. Time is lacking (and possibly still being reduced) to both cover all classical topics and the learning of good usage of technology. Some topics in mathematics remain relevant in their own right and some topics have become necessary to understand how the technology environment works. But many topics in traditional curricula can be handled much better or differently by a technology environment. For a new curriculum, a selection between topics has to be made along these criteria. On the other hand, innovative applications of mathematics have to be included in the curriculum. Technology can also shed new light or give a new representation of old topics [5]. Here are only a few examples of new viewpoints:

(a) The independence of problem size: technology handles inputs with large numbers and many decimals as easily as with small integers, or solves 10 by 10 systems of equations as fast as a 3 by 3 systems.

(b) The holistic approach to a function and its representation as a graph.
(c) Use representations in the plane or in space rather than point-wise or numeric computations;
(d) The choice of well-adapted approximations to functions: polynomials for local approximation to functions, trigonometric series for periodic phenomena, wavelets for hump-like graphs, transforms for smoothing, and etc.
(e) The increased importance of discrete mathematics and the interplay between discrete and continuous mathematics [8].
(f) The mathematical foundations of computer science.

There has been a lot of experimentation in the past few decades, some of which has stayed the test of time (for example: statistics), and some of which has faded away (for example: Set notation). Having access to modern software should allow the teaching process to be conducted more efficiently, with rates of understanding improved with access to visualization: for example, zooming in on a curve – local straightness. Improved efficiency and some time saved by shaving some of the more advanced drill and practice outlined above should leave room for a number of extensions and new threads. We outline the areas where mathematics curriculum can be modified when technology is introduced.

Statistics:
The study of statistics could be extended to include large, real data sets, and more involved tests. The important Central Limit Theorem is at the moment studied only by the more advanced students. Software methods can make it easier for younger students to understand it basic principles allow everyday statistics, such as opinion polls, to be better understood. For example, samples from a uniform distribution; varying the sample size. There is a good case for improving the statistical understanding of school aged pupils so that in time, the adult population can cope with data more intelligently than it does at present. A glaring example of this can have been seen most days in the newspapers: facts and figures are often cruelly misinterpreted by journalists who display a poor grasp of basis statistical concepts. For example: inappropriate extrapolation of hospital data.

Probability:
An understanding of this important discipline is central to many occupations, including insurance, and the provision of public services. At the moment only post 16 students have the opportunity to understand what is meant by a probability distribution, but ICT simulations are readily available to demonstrate the concept with ease to younger students. For example: 2 dice simulation, Poisson simulation.

Coordinate geometry and trigonometry:
This whole area comes to life with dynamic software, and is much more approachable to school-level students. The properties of conics can be re-introduced without resorting to heavy algebra. Example of conic constructions, and trig functions are much easier to understand if they are dynamic objects with variable amplitude, period, phase and etc.
Other graphic coordinate systems could regain some of their former prominence, for example, parametric (particularly useful in kinematics) and polar (obvious applications in radar). Both of these are much more approachable with modern dynamic software. For example: parametric equation of circle and family of polar graphs.

Calculus/ Advanced Calculus concepts:

With dynamic software and web-based java applets, the teaching of calculus has been transformed. 'Seeing' it make it much more accessible to both weaker and younger students. There is every possibility that the ideas of differentiation and integration could come earlier on the courses, thus introducing younger students to application of more advanced mathematics. Once the basic principles of differentiation and integration have been introduced, students could use symbolic algebra systems for more complex calculations, and move more towards numerical solutions to suit particular models, instead of becoming masters of the indefinite integral. The important principle of the Chain Rule could be understood visually, thus reducing the extent of routine drill and practice. For example: A dynamic area calculation to show that some areas are negative. Complex and abstracts in Advanced Calculus such as 'Uniform Convergence', 'Uniform boundedness' and etc. can be simulated by sketching a sequence of functions, see [10].

Modeling:

The concept of modeling could be freed up to include situations where the equation (or, at a higher level, the differential equation) only has a numerical solution. This should allow much more interesting situations to be included. This particularly gives opportunities to re-invigorate the teaching of Mechanics (post 16). If you add some of the science software titles (for example: Interactive Physics) to the dynamic mathematics software, the subject can really come alive. Some examples:

- Different ways to generate a parabola.
- A model where the resulting D.E. cannot be solved by elementary means, but the situation is well within the grasp of a school pupil.
- Goat on rope in circular pen – maximizing area, solved numerically in GSP.

Visualizing in 3D:

With the enormous advance in the use of 3D graphics in movies, games and architecture and design, this would be an appealing addition. At more advanced levels, the principles of 3D perspective and 2D representation of a 3D image could be considered, including parallel perspective and vanishing points. Modern dynamic software now makes this topic very approachable for school pupils, and much easier for the teachers to explain! Examples:

- 2 skew lines, 2 intersecting planes.
- 3 intersecting planes; the cross product.
- Some nice modern buildings?

Euclidean Geometry in 2D and 3D:

The dynamic software now available should enable a much more discovery-driven approach to Euclidean geometry, in both 2D and 3D. Geometry has been squeezed out of the teaching program in secondary schools just at a time when new software approaches should have seen it expanding. For example: 5 Platonic solids.

An appreciation of the history of Mathematics

Mathematics has an incredibly rich history that barely gets a mention at present. It might be viable to work with the History department. Resources abound on the web to help, and the huge benefit s that a historical aspect can be inserted transparently into a lesson (eg who was Pythagoras, where and when did he live – many pupils will not have much idea that it was as long ago as 500BC). Also the recent History is important, and Timelines can be a useful tool here. For example: the search for

Mercenne Primes – an example of the history of technology and mathematics – AND why they are useful.

An appreciation of the position of Mathematics in the grand scheme
Few pupils ever get to realize how central mathematics is to an understanding of structure, from quarks to the universe. Again, there are plenty of high quality and totally engaging web resources on hand to illustrate patterns and chaos in nature. For example: the multifarious applications of the Fibonacci sequence. For example: the beauty of fractal geometry.

An appreciation of the recreational aspect of Mathematics
Mathematical entertainment – two words that few school children would put together in the same sentence! Yet puzzles, fun and relevant problems to solve are all superbly resourced on the web, and so can be effortlessly slipped into lessons. For example: the game of NIM.

In many European countries, some guidelines have been issued regarding how technology should be incorporated into mathematics curriculum. However, these address simple techniques in mathematics and do not use the full potential of technology. For example, France is developing extensive learning material under Cabri II [9] but this covers few subjects from the final years of secondary. Delivering mathematical content over the internet has been slower than other content for several reasons: its highly graphical nature [6], the interactive programming it takes, the lack of a uniform representation of formulas (now resolved by MathML), formatting, and issues of security and copyright. All these issues are nonexistent or less sensitive in, say, language courses or history courses if they are written in Roman characters for use in Europe. Again, a few of the well-established mathematical websites (e.g. [2]) are more inclined to history and biographies. Other initiatives [French web site] concentrate on electronic delivery of existing textbooks with added hyperlinks between paragraphs. This involves copyright issues and takes a substantial effort to maintain. Such text databases are potentially useful for mathematical experts but fall short of the education goal of efficient introduction of mathematical concepts.

In many European countries the introduction of technology is paired to a reduction in teaching volume. The most extreme example is again France where curriculum volume was reduced from 9 hours to 4.5 (+ 1 optional) hours weekly. It is a fallacy to believe that the benefits of technology may compensate such reductions.

In the wake of technology introduction reforms that promise to solve most if not all problems, some countries in Europe have replaced their uniform curriculum by minimal objectives to be achieved by the average pupils and levels of content have a tendency to cover these very minimal standards. We note that this observation from many European countries does not apply to Singapore as it has provided differentiated mathematics curriculum at primary and secondary levels for pupils who are very capable, average or weak in mathematics, under its national streaming system. The Ministry of Education, Singapore, has set up a committee to revise its mathematics curriculum from primary to pre-tertiary levels to be implemented in the next decade. This review will consider changes to the mathematics contents and assessment procedures that are in line with availability of new technologies in instruction.

(4) How exam questions are different when technology is used?

If we wish ICT to have a real impact on the subject, then the method of assessment should embrace it too. Computer based technology is simply not appropriate for an examination environment (size of footprint, operating system crashes and power supply problems). However, new hand-held products are coming along. For example, the Casio ClassPad 300 and TI Voyage 200 seriously challenge Mathematics educators to rethink the content and its assessment, but there will always be two stumbling blocks: (a) there must be funds to equip the pupils and (b) both the pupils and the teachers need training in their use.

The national examination in Denmark offers different sets of series of exam questions for schools that have adopted graphics calculators or calculators with a CAS system. In this country, studies have appeared that compare student performance on these different sets of questions in different schools. In general these studies show that better students profit more from more advanced technology. In many countries, especially when no uniform technology is imposed, teachers try to make questions as technology-independent as possible. This is also motivated by the fact that there is no standard technology adopted in groups of schools. In some classes, students have wide variety of handheld devices. Few schools have symbolic software licenses although these can be obtained at a discount.

It is unfortunate that in many schools technology is merely being used for administration and for the production and grading of multiple choice questions in mathematics. This falls short of the potential of generating creativity in learners that modern technology has. Project work has its own problems of unwanted cooperation and cheating. It is possible to write open-ended exam problems that depend on a random parameter (such as the student's own birthday) so that the exam can be taken by a full class in a computer lab.

We give several examples below to demonstrate that how questions are different now when technology is introduced in a classroom. In the meantime, we will also see some traditional (pen and paper) problems should be modified if technology (calculators) are allowed in an exam. It should be noted that the degree of difficulty depends on the level of competency of students, and also depends on the amount of time allocated in an exam.

Example 1. Consider the graph of $y = \sin x$,
(a) how do we change the horizontal and vertical units on $y = \sin x$ so it represents the graph of $y = 2 \sin 3x$?
(b) make use of the answer from (a) to sketch $y = (-2\sin 3x) - 2$.
The previous example can be used as a problem which no graphics calculator is allowed. However, it is clear that if students have exposure to the use of technology in classroom exploration, they will be able to understand better conceptually.

Example 2. Given the graph of $y = f(x)$, investigate the graphs for $y = |f(x)|$, $y = f(|x|)$, and $|f(|x|)|$.

Example 3. Assume the function f has an inverse. If f is moved horizontally or vertically for some units, investigate what the corresponding inverse functions should move accordingly. For example, if $f(x) = arcos(x)$, then find the inverse functions for $f(x+p)$ and $f(x)+p$ accordingly.

Example 4. (1993 College Exam in Taiwan) Suppose $a_0 = 0, a_{n+1} = \sqrt{1+a_n}$, where $n=0,1,2\ldots$.

(a) Show $1 \le a_n \le \frac{1+\sqrt{5}}{2}$, where *n=0,1,2,....*

(b) Show that $a_n \le a_{n+1}$, where *n=0,1,2,....*

(c) Show a_n approaches to a fixed point as $n \to \infty$.

With the help of technology, students should be able to define a recursive sequence and use graphics devices to sketch the sequence and experiment the fixed point algorithm. If a sequence is convergent, one can define a function accordingly to show that the function *f(x) = x* has a solution, see [10] for more details. Another example is the sequence given by Ramanujan: find the value of $\sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+\ldots}}}}$ using a numerical approach, followed by an analytical one.

Example 5. [The National Center for University Entrance Examination, Japan, 2002]. Find the maximum and minimum values of $f(x) = -4x^2 + 4(a-1) - a^2$ with $a > 1$ and $-1 \le x \le 1$.

The problem itself is not hard, but most students with no training of using technology will solve this problem algebraically by finding the vertex of the parabola. However, this will be a good problem to use technology to explore. For example, one can generate an animation by varying the values for *a* (specifically, one can concentrate for *a* = 3 within 1 < *a* < 5), and see how it will affect the maximum of the parabola. One can also generate a 3D graph with variables *a* and *x*.

Example 6. [1995 College Entrance Exam Question in China, page 12] Given $y = \log_a(2-x)$ is an increasing function for *x*, then the range for *a* is (A) (0,2) (B) (0,1) (C) (1,2) (D) $(2,\infty)$.

Of course, no graphics calculators were allowed in answering this question. However, one better way to answer this question is to observe that if $f(x) = \log_a(2-x)$, then $f^{-1}(x) = 2 - a^x$. If the function f is increasing so is its inverse. Therefore, the answer is (B).

We note that when technology is introduced in a classroom teaching, this type of problem become excellent problems for explorations.

Both Examples 5 and 6 highlight a crucial distinction between what is being assessed at high stake public examinations and what is good for exploration in teaching.

Since ‘Entrance Exams’ are still very important for many Asian countries education systems, it is rather difficult for decision makers (higher authorities) in respective countries to make sudden changes. However, it is not advisable to make no changes at all either. It is advisable to gather crucial information regarding mathematics education reform *globally* before they solve their problems *locally*.

Readers can find many more examples from CASIO Exam Based Activities through ‘http://www.casio.co.jp/edu_e/’. Many CAS based questions that have been tested at the State of Victoria in Australia can be found from [11]. It is predicable that technology based exam questions will be more real-life applicable. In the mean time, the following concerns will come up, which we shall resolve accordingly:

(a) Are questions too hard because of the way (CAS) calculators present their solutions? We refer readers to [11] for examples.
(b) Are we (teachers) bound to ask certain questions due to limitations of a device? This can be resolved when technology improves. For example, it is now possible to use a handheld device

such as CASIO Class Pad 300 to answer questions involving numerical or symbolic interpretation or questions in geometry areas.

(5) What type of technology (calculators or computer) is allowed in an exam so far?

This depends on the values and objectives of the specific curriculum.

5.1 In the U.S., graphics calculators are allowed in doing part of questions of Advanced Placement (AP) Calculus exam since 1995. (Most colleges in the U.S. will accept AP exam grades for college credit). Calculators are not provided. Students must bring their own approved calculators. Sharing calculators is not allowed. About five of the fifteen multiple-choice questions in Section IB and portions of the free-response section will require the use of a calculator.

5.2 It is difficult to argue about "values" that a particular curriculum intends to promote. Research may shed some light on the feasibility of a particular decision, but research findings cannot tell policy makers what should or should not be included in a curriculum. Whether to allow a calculator or computer in exam may be a value-related issue as much as a research-based one.

5.3 Allowing a particular technology in exam may set off different chain reactions. For example, basic calculator $\rightarrow$ less skills tested $\rightarrow$ less practice on skills $\rightarrow$ less proficiency in manual skills. On the other hand, basic calculator $\rightarrow$ more "exploratory" items $\rightarrow$ investigative experience. Research should look at such chain reactions among different types of pupils. So how do we achieve a balance?

5.4 In Singapore, calculator is not allowed in primary school public examination (taken at the end of primary schooling). Scientific calculator is allowed in Paper 2 of O-level examination (end of secondary schooling) and in A-level examination (pre-university or matriculation). Graphic calculator is allowed in Further Mathematics at A-level. Computer (as desktop, laptop , PDA) is not allowed at the moment. On-line assessment under examination conditions at higher education level is also very rare. The inhibiting factor is access and equity.

5.5 In several European countries, exams are still written without advanced technology. In particular, graphics calculators and calculators carrying a CAS system are prohibited. Usage overview can be found in recent articles, see [3].

5.6 In [11], the author had thoroughly written the implementation of using CAS graphics calculators in classrooms and examinations in the State of Victoria, Australia. Many valuable advices of designing CAS based questions are mentioned there. Sample questions and guidelines for minimizing the effect of different brands and models of calculators on examination success are given in [12].

(6) How curriculum in mathematics has been modified since the introduction of technology?

Curriculum changes occur much slower than innovations in technology. Recently published curricula such as the SEFI Core Curriculum [4] for engineering (used in the United Kingdom, Scandinavia and several countries in Central Europe), still follow traditional content and leave choice of technology and level of introduction to individual institutions and lecturers. Such a curriculum outline for engineering will also influence mathematics curricula in other fields (including a mathematics degree), as do national exams for secondary and entrance exams for tertiary studies. Efficient use of technology is not tested in such exams. Again the reason is the wide diversity of devices and their capabilities.

Many efforts by teachers, educators and developers address the introduction of technology for the teaching of selected topics but lack an overview over the whole curriculum. As a consequence, good material has been developed that fits at some place in the old curriculum, but that could be handled more efficiently if its prerequisites and extensions were also handled in the same way. There is a need to replace the linear curriculum by a more hyper-linked curriculum in which the topics can be introduced in a way modern technology allows.

The main problem for introducing technology is the one of control by the student over his actions and his belief in outputs. The latter is linked to the necessity of proofs, but is not restricted to this problem. It has been a matter of debate if a computation in a symbolic software package constitutes a proof. Sometimes proofs on computer are more rapidly verified (by other computers or other software), than it would take a refereeing process to verify written proofs. Modern software packages [7] are reliable most of the time, but giving the same input (containing a slight error) several times will always produce the same (wrong) output with almost no possibility of checking how this result was obtained by the software.

A curriculum based on technology has to take into account these sensitive issues. Good practice means student control has to be learned stepwise along with the introduction of topics and the gradual increase in technology usage (or the increase in software complexity).

In [11], the followings are mentioned why the existing content should be modified in Australia. To create a sensible curriculum and to take advantage of new opportunities, there were several reasons for change:
(a) Some topics becoming more accessible when CAS is available;
(b) Including more topics to use curriculum freed by using CAS;
(c) Changes in the pragmatic value of topics when CAS is available;
(d) Changes in the epistemic value of topics when CAS is available.

In [11], it is also mentioned that “It is reasonable to expect that if students can use automated procedures to carry out certain routine calculations, then some curriculum time which would otherwise be spent on by-hand practice may be freed for other purposes: possibly to include more topics, to study topics in greater depth or to spend more time on applications of what has been learned. Since time needs to be allocated to learning how to use the machine and some by-hand work is essential for developing a strong understanding of each topic, the amount of curriculum time freed is limited, as has been observed by many authors (e.g. Guin and Trouche, 1999).”

(7) Suggestions/Comments for those countries where technology has not been considered in their curriculum

7.1 Adequate budget to purchase and maintain the hardware. Most hardware needs frequent servicing, which is expensive. Also, hardware becomes obsolete within 5 or fewer years. Even though good mathematics teaching can be carried out with simple software that runs on “old” machines, there is enormous pressure from various sectors to push schools to upgrade hardware.

7.2 Train teachers to use the relevant ICT software and hardware, at least up to intermediate level. Include courses on how to integrate technology into normal classrooms or in special laboratories; both versions should be explored for their advantages and disadvantages. The slogan "pedagogy-based" ICT use is very attractive, but it is necessary to unpack what constitutes good mathematics pedagogy, which has no "one correct" answer. A feasible approach is to start with good pedagogy (e.g., give examples from easy to difficult ones) and examine how technology can facilitate the implementation of this pedagogy. Some educators prefer to start with a given piece of technology (e.g., CAS) and examine how new pedagogies can be developed. Both approaches should be investigated.

7.3 Teachers must be given the time and support to experiment and reflect on their experimentation, preferably with collaborative action research. If there is too much emphasis on assessing teacher proficiency by the "number of passes or credits" on routine tests, this will discourage experimentation because there is always an initial dip in pupil "success" on routine tests with most changes in pedagogy.

7.4 Work out a balance between mastery of fundamental skills and active construction, or exploration of knowledge. Pupils need to be trained to adopt new learning strategies.

7.5 Balance between Assessment and Exploration. We can not rely on only one test or one type of technology based assessment to determine how students learn a subject. Students also need time to explore and solve more real-life problems, such as problems from modeling and many other open book projects.

7.6 Incremental introduction of technology does not mean that each year new technology is used. Only a limited number of technological methodologies should be introduced over the complete education cycle. A full mathematics curriculum can be supported by no more than two technologies: - a (plane) geometry package (including calculator) on handhelds or computer screen (or both) for ages 5 to 15, such as Cabri [9], Geometer's Sketchpad or CASIO ClassPad 300.

7.7 A (symbolic) computer algebra system on handhelds or computer screen (or both) for ages 12 to tertiary and beyond.

7.8 Special care should be taken at the introduction level by offering the right interface (touch screen, writing stylus, on-screen calculator, and etc.) to young pupils. For them, it is also necessary to select a large number of examples and problems covered by a limited set of commands. The set of commands is then extended as the mathematical content develops. Countries or institutions with limited finances should opt for simpler carriers, maybe slower or with smaller screens, but follow mainstream choices in software to fully profit from education material that has been developed elsewhere.

Wouldn't it be wonderful if technology will allow us to do the followings over the Internet:

(a) Display dynamic mathematics symbols and equations over the web with little fees.
(b) Hyperlink to a computation engine (from server or client site) whenever users desire.
(c) Deliver mathematics content (data, video and etc.) through broadband technology with portable devices (notebooks or PDAs).
(d) Cluster users will be able to use wireless technology and devices for communications.

Technology will evolve and certainly enable us to achieve items above in a very near future. Decision makers will need to keep up with all updated information *globally*, and get involved with the stages of mathematics reform *locally* (rather than being an outsider and relying on third party advices), and implement mathematics reform in stages. Last but not least is not let ‘mathematics education reform’ become a political issue. Politics and education reform are two different matters. The former can make wrongs be rights and vise versa; but the later will allow no room for errors.

In this report, we have outlined many important issues and have gathered views from different countries and regions. We are all grateful to be able to participate in this panel discussion, which addresses such important topics that will impact many institutions in various parts of the world. We hope this is just the beginning of our discussions and we welcome comments and recommendations. In fact, we encourage readers or ATCM participants to engage writings in these areas. We certainly need more experiments in the use of technology in various areas such as in teaching, assessment and etc. so we know what will work and what will not. We also would like countries where making changes is typically slow take notes on our finings now and in the future. In summary, we believe that ATCM is leading the mathematics community toward the right direction and will provide valuable information to mathematics education.

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