Early Development of Mathematical Concepts Using Dynamic Geometry

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Abstract:

The appropriate use of technology in mathematics teaching can greatly support the teaching and learning of mathematics. On the one hand technology can be supportive in learning how mathematical knowledge can be used, on the other hand it offers valuable support for the appropriate development of mathematical concepts.
Geometry is an important part of school mathematics - it is an area in which the students learn to build a mathematical (geometrical) model of real situations already in early school years. Therefore it is of great importance that the geometrical concepts are developed appropriately - in terms of basic mathematical concepts, which will be built upon in the next 8-10 school years. Dynamic Geometry Software (DGS) offers tools and possibilities, which can considerably improve the teaching of basic geometrical concepts in the primary (as well as in higher) grades. We use Cabri Geometre II to show, discuss, and analyze examples on how DGS can support the development of generic geometrical concepts in the first years of learning mathematics.

Dynamic Geometry Software

How to use different types of calculators and mathematical software in the classroom has become a central topic in mathematics education research during the last decade. Researchers focused on the use of numerical and graphical calculators, as well as on computer software like Computer Algebra Ssystems (CAS) and Dynamic Geometry Software (DGS). Based on empirical research about the use of such tools in mathematics classrooms and on recent theories of mathematical learning, some theoretical models about the use of technology for mathematics teaching were developed. Most of the research projects are centered on the use of CAS (undoubtedly this tool is widest used in mathematics classrooms) for mathematics teaching (Kutzler 2000, Artigue 2002, Cuoco 2002). But more and more attention is also put on the use of DGS in mathematics classrooms.
Dynamic Geometry Software (DGS) - this is a group of programs for doing "dynamic geometry" - is from the didactical point of view the most appropriate tool (group of tools) for supporting the development of geometrical concepts, in particularly concepts of Euclidean geometry, which are developed in the frame of teaching elementary mathematics.
The most popular representatives of DGS are: CABRI GEOMETRE (Baulac&Bellemain&Laborde 1988), THE GEOMETER`S SKETCHPAD (Jackiew 1992), GEOLOG(-WIN) (Holland 1993),

THALES (Kadunz&Kautschitsch 1993), THE GEOMETRIC SUPERSUPOSER (Yerushalmi &Schwartz 1993), EUKLID (Mechling 1994), AND CINDERELLA (Richter-Gebert&Kortenkamp, 1998).

The main characteristics of A DGS are:

- a dynamic modelling of the traditional paper and pencil (blackboard and chalk) teaching environment through the **drag mode**
- an option to condense a sequence of commands to form a "new command", a **macro**
- an option to visualize the paths of the movements of geometrical objects, a **locus**

These features allow a great support to the development of geometrical concepts, when the programs are used appropriately in the classroom.

There are studies which analyze the way DGS influences traditional Euclidean Geometry (Straesser, 2001). One of the central questions in these studies is: Do DGS lead to a different kind of geometry or to a different view of geometry? There are researchers which argue for a new view on geometry created by the use of DGS - from a pedagogical point of view we would rather say: a new cognitive model of geometry, for example a so called "CABRI-Geometry", (vgl. Hölzl, 1997) - which can be considered different from the traditional Euclidean geometry, because of some characteristics connected with the dynamic visualization of geometrical objects.

Not only influence DGS the geometry as such, but also its teaching and learning. How the use of DGS influences the teaching and learning of traditional geometry was investigated and analyzed. Results from a large number of research projects, looking at different aspects of the use of DGS for teaching and learning Euclidean geometry and focusing on changes in the teaching and learning of geometry, back up the hypothesis of a positive influence of DGS for teaching and learning geometry: "*from being a visual amplifier or provider of data towards being an essential constituent of the meaning of tasks and as a consequence affected the conceptions of the mathematical objects that the students might construct*" (Laborde, 2001, p.283). One of the most powerful and the most widely recognized didactical components of DGS is visualization (Kadunz, 1998). Numerous theoretically justified teaching suggestions are offered for using DGS on different levels of geometry teaching (Weigand 1988, Kokol-Voljc 1999, 2000).

When comparing algebra and geometry as two fields of school mathematics, DGS offer a quite comparable alternative to CAS (Laborde, 2003).

Geometry in school mathematics teaching

Euclidean Geometry is one of the basic areas of elementary school mathematics. On the early levels of teaching mathematics, beside elementary arithmetic, geometry plays a key role for developing mathematical thinking and communication skills within and about mathematics:

a) Geometry offers the most direct way to a mathematical model world. The students can directly and very early learn how to create geometrical/mathematical models of real situations and learn about the benefits of working within the model. Such as 7 apples in mathematics can be represented with the symbol 7, the triangular shape of a road sign can be represented by an equilateral triangle. Geometry is one of the most simple symbolical (sub)language of mathematics.

b) Geometry offers techniques to represent/visualize a large number of mathematical objects/concepts from different areas of mathematics: in school mathematics we could hardly do without lines and points as objects to represent (natural) numbers, nor could we do without coordinate system to represent graphs of function within them.

c) Geometry can be viewed as one aspect of mathematical concepts, namely the geometrical meaning of a mathematical concept (for example the geometrical meaning of the derivative).

Because of (a), (b), and (c), geometry and its objects are mostly used as a method for introducing and reasoning about mathematical concepts (e.g. introduction of negative numbers, geometrical reasoning about the Pythagorean Theorem, introduction of the derivative). As such, geometry is a very important part of school mathematics and it is essential that geometrical concepts are developed didactically appropriate to enable their best use within mathematics.
Since geometrical objects/concepts (like mathematical objects in general) are abstract generalizations of relations, they are not easily visualized within the limitations of a traditional paper&pencil teaching/learning environment. For example, in a paper&pencil environment it is very difficult to visualize the convergence of a sequence. And it is difficult to make experiments. We cannot even show that two points define a unique line.
The use of computer changes the situation considerably. While CAS significantly increase the possibilities to experiment and visualize in the areas of algebra, calculus, etc., DGS increase the possibilities of experimenting und visualizin in geometry. DGS become the number one tool of generalization (developing mathematical concepts) in geometry - but not only in geometry!

DGS-teaching examples

Already in very early phases of introducing geometrical concepts, DGS can be used as tools for developing the theoretical meaning of geometrical concepts - through the power of visualization and the mathematical background of the respective software feature.
In the following we give examples for early development of some geometrical concepts. We use the DGS Cabri Geometre II. (From software ergonomic and geometry content point of views this product is very appropriate to support the teaching of geometry.)

Example 1: Line, ray and segment

When introducing a ***line*** in the first year of primary school, the students has to draw a straight line using pencil and ruler. Using these tools, the students don't need to think much about prerequisites for the line (such as that the ruler must not be moved, and that the line can be drawn) - only the direction can be pointed out, but the importance of a "starting" point is not so evident.
When using Cabri Geometre for drawing a line, one first has to choose a point. Then the line appears, but the direction changes with the cursor. Hence, the direction of the line has to be fixed. The direction is chosen by selecting a second point of the line. Consequently, for having a line on the worksheet, one has to select exactly two pints (Figure1).
"*Two points determine a line*." says an axiom of Euclidean geometry - and this is the fact, which is practiced/learned when drawing a line using Cabri Geometre.
A very important aspect here is also some kind of *dynamic aspect*: the line changes its direction until the second point is fixed.

Using appropriate features of Cabri Geometre, students can similarly learn and visualize the characteristics of ***segments** and **rays*** and they learn about the differences between a line, a ray, and a segment (Figure 2).

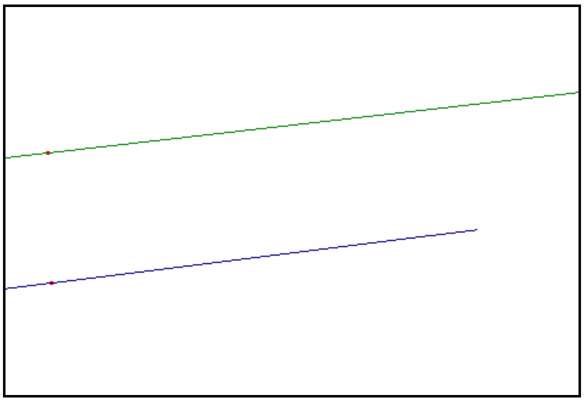

Fig. 1

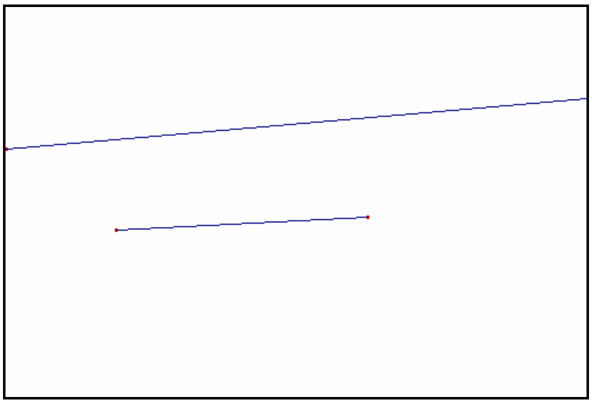

Fig. 2

Later the students can also experiment with moving the line (using drag mode - an important feature making the *dynamic aspect* of the performed activity).

Example 2: Plane figures

How do we draw a ***triangle*** with pencil and ruler? We hardly do it by choosing the three vertices ... If we use the respective feature of Cabry Geometre for drawing a triangle, we MUST select three points to obtain a triangle (Figure 3). Basically, a triangle is determined by three vertices - and this is the fact we learn when using DGS for drawing triangles.

The same is true when drawing a ***circle***.
Cabri Geometre offers two options for drawing a circle:

- The one tool for drawing a circle in Cabri Geometre is named "Circle": it demands to choose a point (the centre point) and a distance from the centre point to a point on the circle (the radius) to draw the desired object (Figure 4).

Therefore, through the use of Cabri Geometre, the students learn the essential conditions for a circle: to have the center point and the radius.

- The other Cabri Geometry tool for drawing a circle is named "Compass": it requires to choose a distance (by selecting two points!) as the radius and then to select a point which becomes the centre point - and the circle is given!

Both circle tools require/support to be aware, that a circle is determined through a point (the centre point) and a constant distance of the points on the curve (radius).

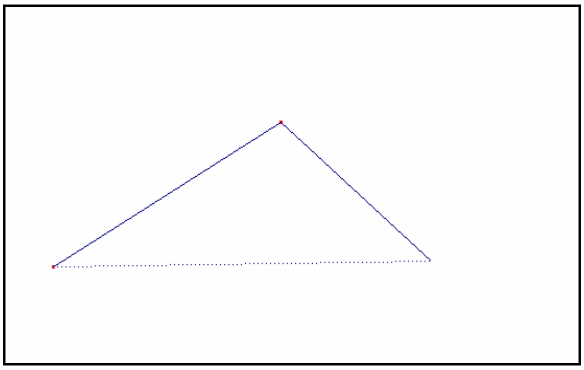

Fig. 3

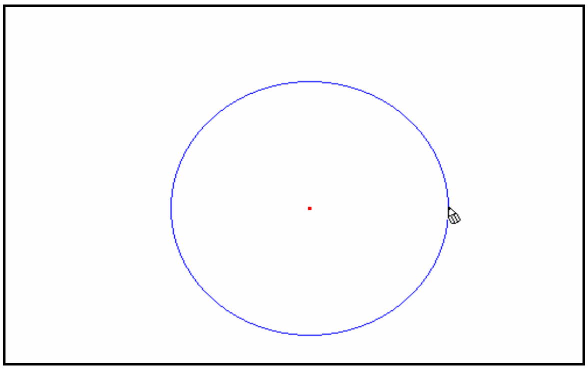

Fig. 4

Both, the triangle and the circle tool of Cabri Geometre support mathematically adequate concepts to be developed.

Example 3: Reflection

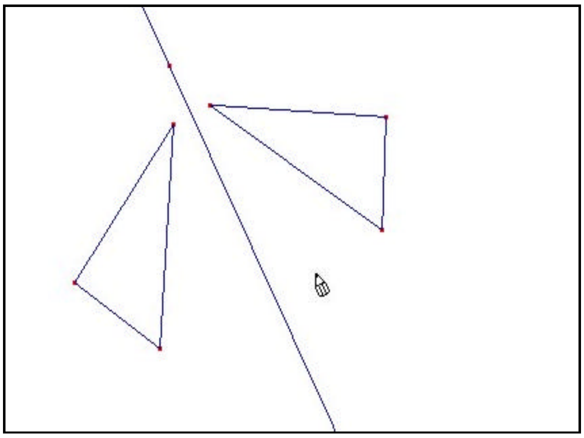

Fig. 6

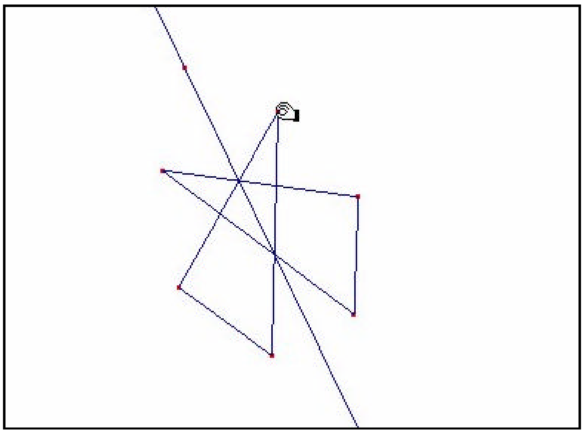

Fig. 7

Here is a concrete activity: A reflection in a mirror can be related to the respective operation in Cabri Geometre in order to develop the concept of a *reflection with respect to a plane*. Compared with a real reflection in a mirror, which contains only the *static aspect* of the reflection (to reflect a given set of points, Figure 6), the activity done on the computer also contains a *dynamic aspect* of the reflection (Figure 7): Using drag mode, the position of the original figure can be changed or the reflection line can be moved[1].

For the concept developing process the dynamic aspect is very important, as it significantly increases the possibilities for experimenting and generalizing.

[1] On the higher school levels the students can keep to the geometry and just experiment with other objects, or they can easily switch for instance to the calculus and experiment with the graphs of functions

Example 4: Tangent

As an example for early middle grade mathematics teaching, we take the concept of a ***tangent***. The definition of a tangent requests: "The tangent to a circle is a line having only one point in common with the circle." Through appropriate activities, performed with DGS, where the *dynamic aspect of the activity* is of great advantage, the students can develop the concept of a tangent, connecting the above definition with the main characteristic features of a tangent (Figure 8 and Figure 9).

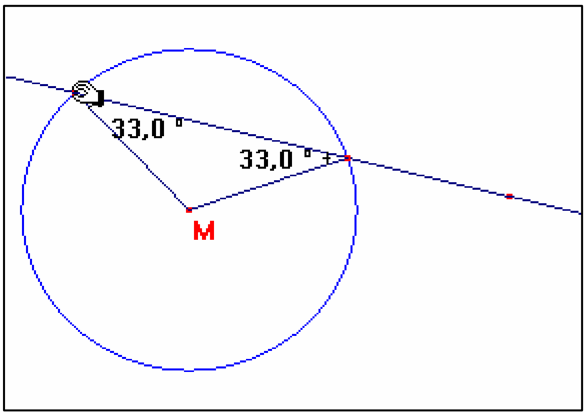

Fig. 8

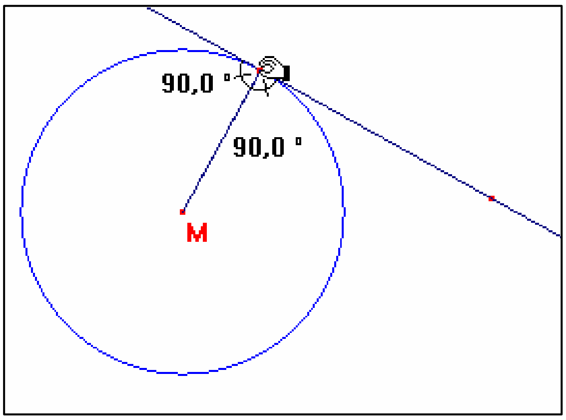

Fig. 9

The defining relationship between a line and a circle can be **visualized** when ***experimenting*** using drag mode. At the same time the student can observe one of the characteristic features of a tangent: *„The tangent is the line which is perpendicular to the radial segment in the point where line and circle touch."*

Dynamic schematizing

One of the main common characteristics of the examples shown in Section 3 is the *dynamic aspect* of the performed activities - I call this ***dynamic representation***.
In a traditional ruler/compass (or pair of compasses) geometry the sketch represents a *static representation* of the respective geometrical concept.
The *dynamic aspect* of the concept, which represents the relational aspect of the concept, remains hidden in a construction process when using a paper-pencil environment. And it is lost when looking only at the final product of the constructing/drawing activity.
The use of DGS changes the situation fundamentally: In the sketch itself and in the construction process, the *dynamic-relational aspect* of the respective concept retains (it would be kept through the drag mode). This is the main difference between a DGS produced sketch of a geometrical figure/object and a paper&pencil sketch of the same object.
A ***dynamic representation***, i.e. the ability of a DGS to represent dynamic processes and to make them an object of research, can be used for a dynamic construction (drag mode) or for the process

of constructing (macro, locus). The dynamic schematizing (Kokol-Voljc 1999) of geometrical objects can significantly improve the development of geometreical concepts at all levels of mathematics teaching.

Conclusion

DGS bring a new dimension into the teaching of geometry. Through making use of the dynamic features of DGS (***dynamic representation***), the geometrical concepts can be developed more efficiently and the graphical representations of the concepts - sketches - can be even more representative for the respective mathematical characteristics compared to doing the same in a paper-pencil environment.

The use of DGS should not replace the use of ruler and compass, but enrich and complement it.

There are great advantages of "ready-made" macro constructions - but for the students, an important part of the respective activity (the construction) would be lost. The concrete constructing activities are important also in a paper&pencil environment in the concept developing process. (This not only serves the purpose of a literation process.)

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