## Mathematical Activities with CASIO CFX9850 with Discussion of International Trends and Didactical Concerns on Calculator Use

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### Abstract

Technologies are assumed to serve as tools to deepen and enrich mathematics activities and mathematical thinking. Calculators are more affordable to students because of their relatively cheap price compared to computers. Since the 1980s, calculators have been introduced to mathematics classes and much research has been carried out in this area.

According to the recent TIMSS-R(Third International Mathematics and Science Study) report, there is an enormous variation of calculator use in mathematics classes across countries and a remarkable trend in the emphasis on calculator use in mathematics classes: Except Hong Kong and Singapore, the ratio of high emphasis on calculators among eighth graders is much less than the international average in South and East Asian participant countries; and there is a shift toward less frequent use of calculators classes between 1995(TIMSS) and 1999(TIMSS-R).

The lower level calculator use in some Asian countries may be explained from several perspectives: financial limitation, cultural characteristics, and evaluation system. The international decreasing trend of high emphasis on frequency in calculator use may result from the quality of mathematics activities dealt with in the mathematics classes and the policy of MOE related to calculator use might be the factors of such trend.

In this paper, the current trends of calculator use in mathematics classes shown in the TIMSS-R report will be summarized focusing on Asian countries and the results briefly analyzed. Then some sample mathematics activities with the graphic calculator CASIO CFX9850 at the secondary school level will be presented with didactical concerns. It is assume that the improper use of calculators may cause the trend of calculator use shown in the TIMSS-R report. Calculator-based activities for alternative approaches to Korean traditional mathematics curriculum contents or for exploration of new types of meaningful mathematics problems will be presented.

### Introduction

Computers and calculators have changed the world of mathematics. Technologies have been assumed as tools to deepen and enrich mathematics activities and mathematical thinking and professional curriculum documents across the world strongly urged for the importance of developing calculator use in schools (Ruthven, 1996). The recent NCTM's Standards (NCTM, 2000) suggested the 'technology principle' as one of the six curriculum guidelines for the new millennium mathematics education. The technology principle assumes that technology can help mathematics learning and affect the contents of mathematics to be learned. The National Curriculum of England also strongly recommends the provision of opportunities to students to

apply and develop their ICT capability through the use of ICT tools to support their learning in all subjects (DEE, 2000).

They can also affect the contents and teaching sequences of school mathematics and change the relative importance among mathematical concepts as well as teaching methodology (Bright et.al., 1994; Cornu, 1992; NCTM, 1996). Using technology effectively "requires objectives for mathematics education that are aligned with the mathematical needs of the information age" (NRC, 1990, p.18).

Calculators are easy to carry and are called hand-held technology. They are more affordable to students because of their relatively cheap price compared to computers. Since the 1980s, calculators have been introduced to mathematics classes and much research has been carried out in this area. The unequivocal results of research in empirical effects of calculator use on mathematics achievements (Ellington, 2003; Ruthven, 1992) indicate that using technologies does not automatically guarantee a higher quality of education. Hence teachers should also know when and how to use technologies.

Even though the recognition of the importance of calculator use in mathematics classrooms, according to the recent international study (Mullis et. al., 2002), there is an enormous variation of calculator use in mathematics classes across countries. Among Asian countries, Singapore and Hong Kong are shown to be exceptionally active in using calculators in mathematics classes. Most of the Asian countries are not so active in using them in mathematics classes and a shift toward less frequent use of calculators was shown in the international average between 1995 and 1999.

In this paper, the current trends of calculator use in mathematics classes shown in the TIMSS-R report will be summarized focusing on Asian countries and the results briefly analyzed. Then some sample mathematics activities with the graphic calculator CASIO CFX9850 at the secondary school level will be presented with didactical concerns. Some discussion about calculator use will be followed.

# **1.** Trends in Frequency of Calculator Use in Mathematics Classes shown in International Studies

### 1. Frequency of Calculator Use in Mathematics Classes

According to the TIMSS-R (Third International Mathematics and Science Study-Repeated) report (Mullis et. al., 2000), there is an enormous variation of calculator use in mathematics classes across countries. In 1999, the top 7 countries in the percentage of students at high level index of Emphasis on Calculators in Mathematics Classes (ECMC) are the Netherlands, Singapore, Australia, England, Canada, New Zealand, and Hong Kong in that order (p.214). High level indicates the students reported using calculators in lessons 'almost always' or 'pretty often, and the teacher reported students use calculators at least once or twice a week for any of the tasks(Mullis et. al., 2000, p. 214). From 95% to 75% of the eighth graders in the top seven countries were in high level ECMC. But in other countries including Japan, Korea, Malaysia, Taiwan, Thailand, Iran, and Turkey, the percentage of the students in high level ECMC is no more than 3%. Except Hong Kong (75%) and Singapore (79%) in South and East Asian participant countries, the ratio of high level emphasis on calculators among eighth graders was no more than 6% which is much less than

32%, the international average (Mullis, et. al., 2000, p.214, Exhibit 6.16). The diagram of international diversity in index of ECMC is shown in Figure 1.

	Percentage of Students at High Level of Index of Emphasis on Calculators in Mathematics Class (ECMC)
Netherlands	
Singapore	0
Australia	0
England	0
Canada	0
New Zealand	0
Hong Kong, SAR	
Israel	0
United States	
Italy	0
South Africa	0
Finland	
Slovak Republic	
Beigium (Hemish)	
Czech Kepublic	
Hundoor	
Moldava	
Morocco	
chile	
Latvia (LSS)	
Cyprus	
Macedonia, Rep. of	
Jordan	
Slovenia	0
Bulgaria	
Philippines	
Indonesia	
Tunisia	
Romania	-0
Turkey	-0
Iran, Islamic Rep.	-0
Thailand	-0
Chinese Taipei	-0
Malaysia	0
Korea, Hep. or	-
Japan	
Lititiania	ŧ
International Avo.	ê

Figure 1. Index of Emphasis on Calculators in Mathematics Class (ECMC) (from Mullis, et.al. (2000) p.215 Exhibit 6.16 Continued)

### 2. Trends in Frequency of Calculator and Computer Use

There is a remarkable trend in the emphasis on calculator use in mathematics classes between 1995(TIMSS) and 1999(TIMSS-R): a shift toward less frequent use of calculators. Table 1 represents the trends in frequency of calculator use in Asian countries and other countries showing significant changes between 1995 and 1999. The black upright and inverse triangles indicate the significant increase and decrease, respectively.

As shown in Table 1, the percentage of students at the high level of emphasis in two countries, Belgium and Thailand, has been increased from 20% to 39% and from 1% to 2%, respectively. In five countries such as Czech, Latvia, England, Russia, Slovak, there were significantly fewer students at the high level emphasis of calculators in 1999 than in 1995. The international average at the high level emphasis of calculator use shifted 4% lower than before.

The international average percentage of students in lower level emphasis on calculator use in mathematics classrooms was 20% each and did not change during the period. The average percentage of those in medium emphasis increased 3%, from 33% to 36%, and the increment seems to be from the decrement of the high level emphasis group.

### Table 1. Trends in Index of Emphasis on Calculators in Mathematics Class(ECMC)

	High EC.MC Percent of Students			Medium ECMC Percent of Students			Low ECMC Percent of Students		
	1995	1999	1995-1999 Difference	1995	1999	1995-1999 Difference	1995	1999	199 Diff
ium (Flemish)	20 (3.2)	39 (2.7)	19 (4.2) 🔺	43 (3.9)	54 (2.7)	11 (4.8) 🗉	37 (4.7)	7 (2.6)	-30
zech Republic England	59 (3.8) 90 (1.3)	35 (3.2) 80 (2.3)	-24 (5.0) V -10 (2.7) V	38 (3.7) 10 (1.3)	60 (3.5) 19 (2.2)	23 (5.1) ▲ 9 (2.6) ▲	3 (1.8) 0 (0.0)	5 (2.0) 1 (0.7)	1
ng Kong, SAR	76 (4.2)	75 (1.9)	-1 (4.6)	18 (3.5)	25 (1.8)	7 (3.9)	6 (2.4)	0 (0.2)	-5
n, islamic kep. r	1 (0.4)	2 (0.5)	1 (0.6)	49 (4./)	42 (3.9)	-/ (6.1)	50 (4.7)	56 (4.2)	6
Israel T	63 (57)	60 (2.8)	6 (6 3)	32 (51)	30 (2 7)	-3 (5 8)	5 (2.8)	1 (0.8)	-4

23 (3.2)

25 (3.7)

42 (3.0)

44 (2.8)

20 (2.1)

32 (2.8)

55 (3.8)

33 (5.2)

33 (0.7)

21 (3.2)

29 (3.3)

53 (3.6)

60 (2.1)

15 (1.6)

55 (3.3)

62 (3.4)

39 (3.4)

36 (0.6)

-3 (4.5)

3 (4.9)

11 (4.7)

-5 (2.6)

24 (4.3)

7 (5.1)

6 (6.2)

3 (0.9)

16 (3.5) 🔺

76 (3.3)

74 (3.7)

9 (2.5)

7 (1.8)

1 (0.1)

1 (0.6)

32 (4.4)

66 (5.2)

20 (0.6)

79 (3.2)

71 (3.3)

31 (3.4)

12 (2.4)

0 (0.0)

3 (1.7)

29 (3.9)

59 (3.6)

20 (0.6)

1995-1999 Difference -30 (5.4) ▼

1 (2.7)

1 (0.7)

-5 (2.4)

6 (6.4) -4 (2.9)

3 (4.6)

-3 (5.0)

22 (4.2)

5 (3.0)

-1 (0.1)

3 (1.8)

-4 (5.8)

-7 (6.3)

1 (0.8)

.

(edited from Mullis, et. al. (2000) p.216 Exhibit 6.17)

() Standard errors appear in parentheses.

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0 (0.2)

0 (0.1)

49 (3.7)

50 (3.0)

79 (2.2)

68 (2.8)

13 (2.1)

1 (0.2)

47 (0.6)

0 (0.1)

0 (0.3)

16 (2.2)

29 (2.3)

85 (1.6)

41 (3.1)

10 (1.6)

2 (0.3)

43 (0.5)

0 (0.2)

0 (0.3)

-33 (4.4) 🔻

-21 (3.8) 🔻

6 (2.7)

-26 (4.2) 🔻

-3 (2.6)

1 (0.4)

-4 (0.8) 🔻

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Belg

C

Ho

Ira

Japan

Korea, Rep. of

Russian Federation

Slovak Republic

International Avg. §

Latvia (LSS)

Singapore

Slovenia

Thailand †

Table 2.Trends in Frequency of Computer Use in Mathematics Class<br/>(edited from Mullis, et. al. (2000), p.218 Exhibit 6.19)

	Almost Always or Pretty Often		Once ii	n a While	Never	
	Percent of Students 1999	1995-1999 Difference	Percent of Students 1999	1995-1999 Difference	Percent of Students 1999	1995-1999 Difference
Canada	8 (0.7)	4 (0.8)	25 (1.5)	12 (1.9) 🔺	67 (1.6)	-15 (2.2) 🔻
Cyprus	6 (0.4)	-5 (0.9) 🔻	13 (0.7)	-3 (1.1)	81 (0.8)	8 (1.2) 🔺
Hong Kong, SAR	8 (0.5)	4 (0.7)	18 (0.8)	11 (0.9)	75 (1.1)	-16 (1.3) 🔻
Iran, Islamic Rep. r	1 (0.3)	-4 (0.6) 🔻	4 (0.3)	0 (0.5) =	96 (0.5)	3 (1.0) 🔺
Korea, Rep. of	3 (0.3)	2 (0.4) 🔺	13 (0.7)	8 (0.8) 🔺	83 (0.8)	-10 (1.0) 🔻
Latvia (LSS) s	2 (0.3)	-2 (0.5) 🔻	3 (0.6)	-2 (1.1)	95 (0.6)	4 (1.3)
Romania r	1 (0.3)	-12 (0.9) 🔻	5 (0.4)	-3 (0.8) 🔻	93 (0.5)	15 (1.3) 🔺
Russian Federation r	1 (0.2)	-1 (0.4) 🔻	3 (0.4)	-2 (0.7)	97 (0.4)	3 (0.9) 🔺
Singapore	11 (0.8)	9 (1.0) 🔺	43 (2.5)	35 (2.8)	46 (2.7)	-44 (3.1) 🔻
Slovenia	5 (0.6)	1 (0.7)	15 (1.2)	7 (1.3) 🔺	81 (1.4)	-9 (1.6) 🔻
Thailand <sup>†</sup>	5 (0.6)	2 (0.8)	10 (0.6)	5 (0.9)	85 (1.0)	-6 (1.4) 🔻
International Avg. §	5 (0.2)	0 (0.2)	16 (0.4)	4 (0.5) 🔺	79 (0.4)	-4 (0.6) 🔻

() Standard errors appear in parentheses.

These trends are a little bit different from the case of the computer. As shown in Table 2, there is a small but statistically significant shift in frequency of computer use between 1995 and 1999: from the 'never' 1995 to 'once in a while'. In international average, 4% of students moved from 'never' to 'once in a while' in using computers in mathematics classes during the period. Significantly more students used computers in mathematics classes 'almost always' or 'once in a while' in 1999 than in 1995 in six countries: Canada, Hong Kong, Korea, Singapore, Slovenia, and Thailand.

### 3. Discussion about the Results

### •Lower Level Calculator Use in some Asian Countries

The TIMSS-R reported the relatively low level calculator use in some Asian countries. It may be explained from three perspectives: financial limitation, cultural characteristics, and evaluation system.

Firstly, it may be due to the countries' financial limitations for calculators. The necessary condition to use calculators in classes is the provision of technological facilities. In Iran, Romania, and Thailand almost one-third  $(32\sim38\%)$  of the students had calculators at home in 1995(KICE, 1997, p.161 Table) and these countries were very passive in calculator use (KICE, 1997). Therefore the financial limitations can be a reason for low level calculator use. Most Korean teachers using (graphic) calculators in their mathematics classes get those equipments via rental system and now schools begin to have their own graphic calculators for a class.

Secondly, it may be explained by their understanding of nature of mathematics and conservative cultural characteristics rather than the economic situation. The main problem seems to be the teachers' understanding of the nature of mathematics and mathematics activities: 'pure mental activity without any instrumental help is essential in mathematics activities'. Many Korean secondary mathematics teachers worried that technologies, especially calculators might make students' computing skills decrease and even students using graphic calculators to learn quadratic functions also worried about the possible deduction of their calculating abilities (Chang, 2000).

Thirdly, the curriculum, including the evaluation system that does not affect on integrated approach to using calculators, may be the most effective factor that prevents calculator adoption in classes. In Korea, no calculators are allowed in mathematics exams and students have to solve problems without calculators in classroom exams and the college entrance exams. The answers of items of the tests are not so complicated, and paper and pencils suffice. Therefore students do not have to use calculators intricately to solve mathematics problems and using calculators seems to be time consuming for students and teachers under such evaluation environments (Bitter & Hatfield, 1994).

### • Trends in use of calculators

The reason why the international trends shifted toward the less frequent calculator use and what makes the differences in Belgium need to be carefully analyzed. The quality of mathematics activities dealt with in the mathematics classes and the policy of MOE related to calculator use might be the factors of such trends.

The trends of decreasing in calculator use in mathematics classroom make us consider the didactical aspects of calculator use. Oldknow and Taylor(2000) identify the roles of the ICT as follows: (1)problem generator; (2)result checker; (3)context provider; (4)tools for demonstration/ communication; or (4)tools for problem solving. Most activities with ICT in most mathematics textbooks require ICT as a tool for simple operations. Using ICT simply as a result checker may limit and hinder their full capacities and this might cause the trends of decreasing use of calculators mentioned above

The curriculum contents and evaluation systems might be crucial factors. In Korea, the current 7th national mathematics curriculum document strongly recommends the use of ICT for instruction (MOE, 1998), but the contents and the curriculum sequences are almost same as those of the  $6^{th}$  curriculum. Students are allowed to use calculators to check their answers, carry out routine

computation, and solve complicated problems but not during in exams. Teachers prefer to solve problems with paper and pencils rather than with calculators. Calculators are not required to solve problems in mathematics textbook. This situation makes no change in mathematics lessons. Recently Ellington (2003) conducted a meta-analysis investigating 54 research studies to determine the effects of calculators on achievements and attitudes in pre college mathematics classes. She reports the positive effects of calculators on problem solving skills when calculators were an integral part of testing and instruction.

In summary, according to TIMSS and TIMSS-R, calculator use in some Asian countries is extremely lower than the international average. Though the results of meta research (Ellington, 2003; Ruthven, 1996) report the positive effects of calculators on mathematics achievements, the trend toward less frequent use of calculators in mathematics classes between 1995(TIMSS) and 1999(TIMSS-R) is revealed.

Inappropriate use of calculators may cause such trends of less frequent calculator use and therefore the content and quality of mathematics activities should be considered to maximize the ability of calculators to enhance mathematics learning. Some sample activities assumed to maximize the capacity of technologies will be presented in the following chapter.

#### **Calculator-based Secondary Mathematics Activities with CASIO CFX9850** Π

Using calculators in mathematics classrooms requires more time and financial supports. Therefore the efficiency should be counted at the pedagogical perspectives when we use them in mathematics classes. Calculators should be used as tools to enhance mathematics learning. The roles of calculators as tools can be identified in different ways: (1) to deepen and enrich the traditional curriculum; (2) to provide alternative approach to traditional problems; and (3)to attack new types of problems. Some calculator-based sample activities with the graphic calculator CASIO CFX9850 will be suggested. The traditional curriculum means the 7<sup>th</sup> Korean mathematics curriculum hereafter.

### 1. Enhancing the Traditional Curriculum

• Enhancing traditional learning by visualization

As shown in Figure 1,  $\frac{\sin x}{x}$  is greater than  $\cos x$  and less than 1. The visual presentation of the graphs of y=1,  $y = \frac{\sin x}{x}$ , and  $y = \cos x$  provides the intuitive evidence that  $\frac{\sin x}{x}$  approaches 1 as x approaches 0 based on the Sandwich Theorem.



Figure 2. Visual presentation of Sandwich Theorem

### • Deepening traditional learning by visualization

Investigating the roles of coefficients in quadratic functions with graphic calculators leads students to interesting explorations. For example, the role of B in  $y = x^2 + Bx + 3$  is much different from the role of A in  $y = Ax^2 + 2x + 3$ . According to the role of the value B in  $y = x^2 + Bx + 3$ , the following questions can be asked: Are there any fixed points common to all graphs? If yes, what are they and why does every graph go through the point(s)?; Does the value of B change the shape of the curve? Why and Why not?; Does B change the line of symmetry of the graph?; or What is the locus of the vertex? At the [Dyna] mode, students observe the fixed point (0, 3) and begin to figure out the reason of their observations.



**Figure 3. Graphs of**  $y = x^2 + Bx + 3$ 

The last question may lead students to another situation of quadratic functions. Since the coordinates of the vertex in this case are  $\left(-\frac{A}{2}, -\frac{A^2}{4}+3\right)$ , the locus of the vertex is the parabola  $y = -x^2 + 3$ . The family of graphs and the locus of their vertices are shown in Figure 4. Students may use graphic calculators to find out the clue of the answer or to check their answers before or after answering the questions respectively.



Figure 4. Locus of the vertex

### 1. Providing Alternative Approaches to the Traditional Curriculum

### • Alternative approaches to traditional curriculum contents

Most Korean  $10^{\text{th}}$  graders' mathematics textbooks deal with graphs of ax + by + c + k(dx + ey + f) = 0 very technically. For example, students are asked to show that "x + y - 6 + k(2x - y) = 0 is a line through the intersection of the two lines, x + y - 6 = 0 and 2x - y = 0" (Shin & Choi, 2002, p.32). In the textbook, students are led to change the given equation into the form of Ax + By + C = 0, i.e. (1 + 2k)x + (1 - k)y - 6 = 0, investigate whether this equation can be a line by checking the values of k and find the coordinates of the intersection of the two lines x + y - 6 = 0 and 2x - y = 0, and so on. There is nothing to attract students to this problem in this manner and no one is motivated to check whether the

graphs represent a family of lines. Students seem to feel it merely as a weird problem that bothers them.



An alternative approach to this problem can be presented by starting with the investigation of dynamic graphs of  $y = \frac{kx + 2x - 3}{k - 1}$  through graphic calculators (Figure 5).

They may lead them to a much more active involvement in the activity. Students investigate the graphs' common characteristics and may be asked the reason why their findings are right. After careful observations, they begin to think about the reason why their observations are correct. Their efforts to find proof can be highly motivated than before.

· Alternative approaches: Modeling before computational skills



Yoonie wants to make a topless rectangular box by cutting off the rectangular shaped hardboard. The width and length of the hardboard are 120cm and 90cm each. What is the value of x, the dimension of the cut-off squares, to maximize the volume of the box?

If the side of the square is x and the volume of the box is y, y = (120 - 2x)(90 - 2x)xand 0 < x < 45. It is easy to write the function to solve the problem. According to the recent Koran mathematics curriculum, 7<sup>th</sup> graders can make the function but students under 11<sup>th</sup> grade cannot find the x, because students are supposed to solve these maxima problems of non quadratic functions with paper and pencil. Under this restriction, this problem can only be solved by differentiation which is the 11<sup>th</sup> graders' activity.

Graphic calculators can change the situation. The following Figures 6 are screens of CFX9850 Plus to solve the problem. Students input the function, define the dimensions of the visual screen, and graph the function. Using [G-Solve] function, the maximum value is easily detected. It is another good activity to determine the appropriate ranges of the visual screen.



**Figure 6.** The maxima of the cubic function y = (120 - 2x)(90 - 2x)x

### 3. Attacking new types of problems

### • Attacking iteration problems of historical mathematics explorations

Finding an approximated value of irrational numbers has been a big project to human beings in history. We can find the approximation of an irrational number, say  $\sqrt{5}$ , as follows:

Suppose that an arbitrary rational number  $x_0$  is the initial estimation of  $\sqrt{5}$ .

Since  $x_0 \neq \sqrt{5}$ ,  $x_0 > \sqrt{5}$  or  $x_0 < \sqrt{5}$ . Assume that  $x_0 > \sqrt{5}$  .....(1)

Let multiply the both side of the inequality (1) by  $\sqrt{5}$  and then divided by  $x_0$ .

$$\sqrt{5}x_0 > 5$$
 and  $\sqrt{5} > \frac{5}{x_0}$  .....(2)

From the inequalities (1) and (2),

$$\frac{5}{x_0} < \sqrt{5} < x_0$$

Similarly, if  $x_0 < \sqrt{5}$ , then  $x_0 < \sqrt{5} < \frac{5}{x_0}$ .

Hence  $\sqrt{5}$  is between  $\frac{5}{x_0}$  and  $x_0$ .

Let the average of the two extreme values be  $x_1$ , the next estimation of  $\sqrt{5}$ .

i.e. 
$$x_1 = \frac{1}{2}(x_0 + \frac{5}{x_0})$$
.

Do the same procedure to find  $X_2$ , the next estimation of  $\sqrt{5}$ .

i.e. let 
$$x_2 = \frac{1}{2}(x_1 + \frac{5}{x_1})$$
.

Continually, we can find more precise approximation values  $x_3$ ,  $x_4$ ,  $x_5$ , ....

Graphic calculators make this procedure very easy and simple. The followings are Casio CFX-9850G PLUS screens for this activity: Enter any number (e.g. 4) at "RUN" menu, press "Exe", and it will return a number. Input "(Ans + 5 ÷ Ans) ÷ 2" and press "Exe", and it will return 2.625 (Figure 1-2 next 2 lines). Press "Enter" key repeated, and it returns 2.264880952, 2.236251251, 2.236067985, and so on. Finally we can get the value 2.236067977 as an approximation of  $\sqrt{5}$  (Figure 7).

### **Figure 7.** Estimation of $\sqrt{5}$

This activity can be followed by a question leading to meaningful mathematical investigations: How can you confirm that the last number is a precise approximation of  $\sqrt{5}$ ?

If the sequence of  $\{x_n\}$  converges to some value, let the limit be A. Then since  $x_n = x_{n+1} = A$ for an infinitely large number *n*,  $x_{n+1} = \frac{1}{2}(x_n + \frac{5}{x_n})$  becomes  $A = \frac{1}{2}(A + \frac{5}{A})$ . A is the positive root of  $x = \frac{1}{2}(x + \frac{5}{x})$ , i.e.  $x^2 = 5$ .

This method to find the approximated values by repeated computation is known as the Babylonian method. It is interesting that the Babylonian method is exactly the same as the Newton Method for the roots of quadratic equation  $x^2=5$ . According to Newton Method, for  $x_n$ , the initial approximation of the root of f(x)=0, the next estimation  $x_{1+1}$  is  $x_n + \frac{f(x_n)}{f'(x_n)}$ .

Since  $f(x) = x^2 - 5$ ,  $x_{n+1} = x_n + \frac{x_0^2 - 5}{2x_n}$  is equivalent to the equation above,  $x_{n+1} = \frac{1}{2}(x_n + \frac{5}{x_n})$ .

• Exploring the Euler number e

Problem: Ms. Math is working at a hotdog store and looking for another job. Now she earns \$2.8 per hour. This morning, in the newspaper, she found an advertisement for a job position with working conditions the exactly same as her current one except the salary. According to the advertisement, the daily payment will be increased. The salary system is as follows:

<b>BEST SALARY SYSTEM!!!</b> Everyday we will pay you more!!!	4th day $(1+\frac{1}{4})^4 = 2.44 $ %/h			
1 st day $(1+\frac{1}{-})^1 = 2$ \$/h				
2nd day $(1+\frac{1}{2})^2 = 2.25 $ %/h	n-th day $(1+\frac{1}{n})^n$ \$/h n			
3rd day $(1+\frac{1}{3})^3 = 2.37$ \$/h	INCOME WILL BE INCREASED FOREVER Everyday!			
	Ace Hamburger Store Call at 012-345-6789			

The following questions can be asked: According to the payment system, is the advertisement of paying more money everyday true?; Ms. Math is going to work 3 more years. For more income, does she have to quit her recent job for the new one? What is your advisement for Ms. Math? To answer these questions, students can make a table of  $y=(1+\frac{1}{x})^x$  at [TABLE] menu and graph it for positive integer x. This problem will lead students to the monotonic increasing function with upper bound and finally to the limit, Euler number "e". The current Korean mathematics textbooks introduces "e" technically as a limit of  $(1+\frac{1}{x})^x$  as x approaches to 0 by presenting the ready made table of values. Graphic calculators "enable to curricula to move beyond mechanics to experience with ideas" (NRC, 1990, p.18) of *e* in much more abundant context.

### **IV.** Conclusion

The technology principle assumes that technology can help mathematics learning, but using technology does not automatically guarantee the quality of mathematics education. The recent trends shifted toward the less frequent calculator use in mathematics classes shown in TIMSS and TIMSS-R, let us consider the didactical aspects of activities with calculator. Calculators "are able to" deepen and enrich students' learning and let them focus more on modeling rather than computational skills. The curriculum sequences can be changed with graphic calculators: The optimizing problems can be presented to 8th or 9th graders before calculus courses.

Several activities with graphic calculators for secondary school mathematics classes are presented in this paper. Calculators are used as a tool for mathematical explorations and problem solving in all suggested activities.

There are several tasks to be done for preparing students for modern ICT society. Firstly, activities maximizing the capacity of technologies need to be developed with didactical concerns. "Calculators should not be used merely to deliver the same mathematics in new ways" (Bright, 1994, 27). Secondly, the curriculum needs to be integrated with ICT. Curriculum sequences and contents should be reorganized according to the needs of the technological societies. Thirdly, the evaluation should be aligned with the curriculum (Romberg, 1994; NCTM, 2000). If the curriculum documents allow and recommend use technologies in classes, students should be allowed to use them in test also. Fourth, in- and pre-service mathematics teachers have to be the real users of calculators and should be significantly supported for their professional experiences to learn how and when to use them. Mathematics teachers habituated to a paper-pencil curriculum only do prevent students to extend their learning with technologies. Teachers who do not know the appropriate use may hinder the calculator use in mathematics education.

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