Equational Reasoning in Programming

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Abstract
Equality plays an important role in our life, and we practise equational reasoning everyday. We can take advantage of our ability of reasoning with equalities and make explicit the equational reasoning in programming and symbolic computation. Based on this observation we developed an equational programming system called CFLP (Constraint Functional Logic Programming system). We present various examples to show the importance of equations in programming.

1. Introduction

Modern life demands a certain level of mathematical maturity. One of the most important is the ability to reason with equalities. We do sophisticated reasoning with equalities in arithmetic in everyday life, although we are often unaware of it.

Let us take a concrete example. Suppose we buy an item priced at 975 yen. Since in Japan consumer tax is 5 %, we have to pay over 1000 yen. We calculate the exact amount of money that we have to pay, while at the same time we fumble in the pocket and try to find appropriate coins and notes so as to avoid filling up the pocket with many coins of change. We may decide to give a note of 5000 yen and three 10 yen coins. How do we decide?

The involved reasoning is complex; it is not mere simplification of numerical expressions. It involves equational reasoning. In the above example we need at least 6 steps of equational reasoning even if integer arithmetic is assumed. Most people can somehow perform this kind of mathematics. Indeed they master it at a fairly early age. They can comfortably handle reflexivity, symmetry and transitivity of equality relation defined over numbers.

In this paper we will show that reasoning with equality over various domains of objects is also important, and easy to practise if we are provided with appropriate tools for reasoning. One particular example that we are interested in here, and is relevant to mathematics education, is programming. All the programming examples including these texts are in *Mathematica* Notebook.