# Building a Web-based Statistics Calculator and Exploring New Opportunities for Learning Bivariate Analysis 

TANG, Kwok-Chun WONG, Ka-Lok

Hong Kong Baptist University<br>The University of Hong Kong


#### Abstract

This paper first focuses on the design of a web-based statistics calculator with calculation and graphing tools accessible to primary and secondary school students and teachers. Since this project is an enhancement of an existing web-based resource package STAT.NET which was developed two years ago to facilitate the teaching and learning of statistics in primary and secondary classrooms of Hong Kong, a brief introduction to STAT.NET will be given first, which naturally constitutes the rationale of the ongoing development of the calculator. Details of various types of enhancements will then be illustrated, including: (1) input of bivariate data; (2) mean, mode, median and other statistical functions on bivariate data; (3) graphical representations of bivariate data. In addition to the technical aspects concerning with the design of a user-friendly interface, the authors will also share their experience on their own conceptual development of learning and teaching bivariate analysis throughout the design process. The reflection covers two important areas of statistical reasoning: (1) basic understanding of variables and the nature of bivariate data; and (2) interpretation of meaningful (or meaningless) numerical or graphical outputs. Finally, the implications of such reflection on teaching and learning bivariate analysis will then be critically examined.


## Background

In Hong Kong, since the issue of the five-year strategy on IT in education (Education and Manpower Bureau, 1998), the last few years have seen many IT initiatives at various levels in different educational sectors. With an investment of millions and millions of HK dollars, the government has supported multitudes of IT-related concerns ranging from schoolsubject teaching to school administration, from schoolbased curriculum to networking of teaching and learning resources. STAT.NET (accessible at http://www.hked-stat.net/), a web-based resource package for the teaching and learning of statistics in primary and secondary schools, was launched in 2000 against the backdrop of such IT surges. Comprised of six major components (viz. data handling tools, sample lesson plans and worksheets, rear life data, online resource library, question bank and online community), STAT.NET serves to (i) provide teachers with relevant online information and tools for their effective teaching of statistics; (ii) help students develop basic skills in the process of collection, selection, organization, analysis and evaluation of information and (iii) provide an opportunity for collaboration on statistical investigation between students across different levels and schools (Wong and Tang, 2000). Noteworthy is that the project not only echoes the craving need of using IT in education, but also aligns itself with major trends in both statistics education in general and the local mathematics curriculum (Wong, 2000). In particular, it encourages a more active exploratory learning approach on the part of the students and grants, as far as possible, a variety of
technological advantages to facilitate statistics education: direct access to data, flexibility in data manipulation, multiple representations of data, and connectedness to various resources outside the classroom (Shaughnessy, Garfield and Greer, 1996).

## STAT.CALC v.2.0

STAT.CALC, available at STAT.NET and with its learner interface resembling an ordinary calculator, provides an online tool for all descriptive statistics covered by the primary and secondary school mathematics syllabuses. For example, pushing a button once gives readily such measures of centrality and variation as mean, mode and standard deviation. In addition to these summary statistics, STAT.CALC, much enhanced in its latest version, can also generate tables, diagrams and graphs by organizing, sorting and classifying input data. With reference to the concept map of four stages of the process of statistical investigation developed by Friel and Joyner (1997): posing the question, collecting the data, analyzing the data and interpreting the results, STAT.CALC is a technological tool for the data analysis stage.

To further extend the potential benefits afforded by the technology, its latest version, STAT.CALC v.2.0 (accessible at http://www.hked-stat.net/common/toolbox_summary_stat.htm), has the following features:

- Statistical graphing tools (which were originally grouped in a separate toolbox) are incorporated as a companion toolset into its new user interface (Figure 1). The graphical representations buttons yield tables, graphs and diagrams quickly. The design is believed to enhance the flexibility and opportunities for multiple representations during investigation process.
- Apart from ordinary data input facility, it provides an import function to facilitate entry of massive data set captured by spreadsheets or text files. This import function grants further direct access to data from various sources.


Figure 1 User interface of STAT.CALC.

- It allows the entry of bivariate data. In the original STAT.NET toolbox, bivariate data analysis can only be carried out with spreadsheet dataset pre-linked within the resource package. Nevertheless, bivariate analysis is not uncommon in everyday situations. To say the least, we are dealing with bivariate data when we draw comparisons between data of different categories (say, genders). Thus this extended function for bivariate data makes possible further explorations of statistical data in a more flexible manner, and therefore, would bring more teaching and learning opportunities for statistical investigation.
- It provides brief explanations and tips upon delayed mouse-over of each function button. This kind of context-sensitive help provides learner with connectedness to appropriate references and supports the learner at his/her own pace.

Our participation in the design and development of STAT.NET and STAT.CALC v.2.0 tool has been giving us a genuine and concrete personal experience on the impending needs of major changes in statistics education. The inappropriate emphasis on computations, formulas and procedures in the past should shift towards statistical reasoning at various stages of the process of statistical investigation, as argued by many statistics educators (e.g. Gal and Garfield, 1997). Elsewhere, we have already explored the new teaching and learning possibilities in general with the support of STAT.NET and STAT.CALC (Wong and Tang, 2000; Wong, 2000). In this paper, our focuses will be more specific. We will examine and discuss in detail, how the new extended functions of STAT.CALC v.2.0 would give further computational support for teachers and students on the one hand, and push them to have a proper understanding of the nature of bivariate data on the other hand. The latter provides better grounds to distinguish between meaningful and meaningless numerical and graphical outputs. Finally, we will also discuss in the last section the implications for teaching and learning.

## Bivariate Analysis: Variables and Datasets

Statistics revolves around a few fundamental ideas: data, variation and chance. Larsen and Stroup (1976, p.1) may be correct in saying, "what statistics is all about can be summed up in a single word - variation." Variable is thus one of the core notions in the sense that it helps conceptualizing the idea of variation and, more importantly, helps organizing information by qualifying or quantifying the aspect that varies. In short, understanding the concept of variable is a key step to describing and/or to quantifying variability. Nevertheless this very concept is not as easy as it seems to be.

To illustrate the possible difficulties on the part of students in understanding the concept of statistical variable, let us first look at a multiple-choice question (Figure 2) set for the examination on a statistics course for the BEd programme at a university in Hong Kong. The examinees were mathematics major in the second last year of their four-year part-time studies. All of them were serving mathematics teachers and, with one single exception, teaching at junior secondary level. That is to say, they are all teaching elementary statistics.


Figure 2 An examination question set for students on a statistics course for a BEd programme.

To the authors' belief, presented here is one everyday situation commonly taken as an example for discussion in statistics classrooms. In particular, the tabulated form mimics the usual way teachers gather relevant information and present on the board. In this case, the data collected involve the distributions of two variables (namely, 'number of times going to fast-food shops in a week' and 'gender') over 25 cases. Note that the data so presented is not 'raw' but organized. (The sorted raw dataset, which is more useful for subsequent statistical investigation, is shown in Appendix.) The examination results of this question are not very satisfactory. Barely over $60 \%$ ( 16 out of 25 examinees) got it right. Almost $30 \%$ of them suggested that there were 3 variables, probably thinking of the three columns of numbers. One of them thought it consists of one variable whereas another thought there were 25. Despite the emphases on the concept of variables in the course (by the instructors and via the use of the statistical software Fathom), these results suggest an inadequate understanding of such a fundamental concept.

As a matter of fact, the examination item illustrated above originated from the setter's (i.e. the second author's) experience in developing STAT.CALC v.2.0. While the designers (including the authors) were struggling through an illustrative example similar to the one in Figure 2 above, we made a few interrelated observations:
(1) The data presented look familiar: they are put in a frequency table. But this frequency table is different from an ordinary one for univariate dataset in the way that it splits itself up into two different columns, according to the gender variable. Yet, the information presented here should still be easily comprehensible.
(2) Once we tried to start an investigation of the data using STAT.CALC v.2.0, we came to realize that we were forced (by the computer tool) to understand the data structure properly. For example, from the very start of data input, we have to ask such basic questions as "Which numbers are to be entered as 'Data'?" and 'Which numbers/information should go together (belonging to the same case/student)?" In essence, what are the variables that characterize the information gathered? In the process of data input, we may even further ask, "Are there any numbers to be entered as 'Repeat'?" which is especially useful for sorted data (see Appendix). In effect, as long as the data input needs to comply with the computer tool, it drives us towards a proper understanding of the given frequencies.
(3) We could have input the data without heeding its bivariate structure. But as soon as we wanted to generate, say, a simple or compound bar chart for the data, the prompts that followed such an option forced us to organize and, if necessary, re-structure our understanding of the data in relation to its various possible graphical representations. This experience resembles most of our experience in using statistical graphing tools (e.g. SPSS) where the step-by-step options on the one hand determine the graph and on the other hand guide us to recognize which set of numbers is to be put onto which axis, which in effect serves to structure our understanding of the dataset. We will elaborate this point in relation to bivariate data later in the next section.

In sum, by way of supporting bivariate data input, STAT.CALC v.2.0 drives teachers and students to have a better understanding of the variables involved and the bivariate structure of the given dataset. In addition to the basic understanding of bivariate raw dataset, they also need to know the intermediate steps, such as organizing, sorting and classifying raw data. These intermediate steps, though usually not explicitly shown by the tools, are crucial for displaying outputs, be they in the form of tables or graphs. Referring back to the above multiple-choice question, if the
teacher-examinees have a good understanding of the relation between the given frequency table and the hidden raw dataset together with the concealed intermediate steps of organizing, sorting and classifying, they may have no difficulties in getting the correct answer.

## Bivariate Analysis: Numerical and Graphical Outputs

## Descriptive Statistics

The new extended functions of STAT.CALC v.2.0 also urge teachers and students to be more cautious when pressing the descriptive statistics buttons to get measures of centrality or variation. Using the above examination question as an example again, after keying in the data of 25 cases (see Appendix), one can find the means of both variables, by simply pressing the "mean" button. But the mean value of X (gender in this case) should then be meaningless because of its nominal nature. Similar problem arises for other buttons for descriptive statistics. But if the "Group by X" button has been clicked before pressing the other buttons, the computer outputs become meaningful for statistical investigation. After grouping the 25 cases into two independent groups: boys and girls, their within-group centrality and variation can be examined. And more importantly, between-group differences can also be studied. In sum, for bivariate data analysis, the ability to identify the difference between nominal (or categorical), ordinal, interval and scale (or ratio) variables would be a basic requirement. Furthermore, the problem situation provides a meaningful context for the students to further their informal and intuitive understanding of within-group and between-group variations.

## Graphical Representations - Simple and Compound Bar Chart

Again, simple bar chart and pie chart become not so simple with the enabled features of handling bivariate data. Take simple bar chart as an example. There will be four different choices for learners to consider. They may have no problem when plotting bar charts by counting frequencies of each group within the X or Y variable, taking one variable into consideration only. Ambiguity comes up suddenly when considering the two variables together. With the same example of eating habit, if the learner chooses "plot Y (either sum or mean)" with "group by X", he will get a meaningful result which generates clues to the gender difference of eating habit (see Figure 3a \& 3b). (Whether the sum or mean plot is more meaningful is a context-dependent question.) But if the learner chooses "plot X (either sum or mean)" with "group by Y", he would get meaningless result which generates sum or mean of the gender code which is a nominal variable (see Figure 4 a \& 4b).


Figure 3a \& 3b: Simple bar charts showing gender difference of eating habit (Sum / Mean).


Figure $4 \mathrm{a} \& 4 \mathrm{~b}$ : Two meaningless simple bar charts with sum and mean of gender code.
The above-mentioned ambiguity is compounded if the learner has his freedom to choose plotting compound bar chart with the same set of data. In the above example, if the learner chooses "plot frequency of Y (either stacked or clustered bars)" with "group by X", he will get a meaningful result which prompts further understanding of the gender difference with eating habit details (see Figure 5 a \& 5b). If the learner chooses "plot frequency of X (either stacked or clustered bars)" with "group by Y ", he can also get another reasonable result leading to a clue to eating habit difference with gender details (see Figure 6a \& 6b). Whether these graphical representations are meaningful or not depends much on the research question posed and the context of the statistics investigation.


Figure 5a \& 5b: Compound bar chart showing gender difference with eating habit details (Stacked / Clustered).


Figure 6a \& 6b: Compound bar chart showing difference in eating habit variable with gender details (Stacked / Clustered).

By examining the above eight graphical outputs in detail, we can, on the one hand, have a fascinating picture of the efficiency and possibilities provided by STAT.CALC v.2.0 to yield various kinds of graphical outputs for one single set of bivariate data. On the other hand, we may have an
impression of the great demand of statistical knowledge and understanding on teachers and students if they want to have fruitful results during the analysis and interpretation process. Not only they need to know and appreciate the graphical details and possible use of these outputs. They are also required to have a deep understanding of the questions posed and the context in which these questions embedded. In other words, they should realize that when they are trying to represent a set of data graphically, they are at the same time making sense of the dataset and rethink about their question posed. This process itself is a way to explore, organize and present numerical or graphical output meaningfully and is thus part of the process of statistical investigation (cf. Bright and Friel, 1998).

Finally, it is worth mentioning that we have skipped the discussions on pie chart, frequency table, stem-and-leaf diagram, histogram, broken line graph and scatter diagram, etc., due to limited space. But such omission does not mean that they are less problematic or important.

## Implications on Teaching and Learning

Ben-Zvi (2000) has proposed two metaphors for studying the impact of technological tools in statistics education: the amplifier metaphor and the reorganization metaphor. He has argued that we should abandon the former and embrace the latter. The present authors agree with his suggestion. For instance, as demonstrated above in detail, STAT.CALC v.2.0 enables us to get descriptive statistics and graphical representation efficiently and accurately. In this sense, it amplifies our ability during the data analysis process. But this amplifier metaphor falls short when taking statistical reasoning into consideration. STAT.CALC v.2.0 may do very little to help students (i) acquire better understanding of variables and bivariate data; (ii) obtain better knowledge of the hidden intermediate steps worked out by computers; (iii) develop the ability to identify the difference between nominal (or categorical), ordinal, interval and scale (or ratio) variables; (iv) further an informal and intuitive understanding of within-group and between-group variations; (v) achieve a deeper understanding of the questions posed and the context in which these questions embedded during the data analysis and result interpretation stages.

By adopting the reorganization metaphor, which states that "[a]n appropriate usage of technological tools has the potential to bring about structural changes in the system of the students' cognitive and socio-cultural activities, rather than just to amplify human capacities". (Ben-Zvi, 2000, p.140), learning activities have to be transformed. New kinds of tasks which permit activities on higher cognitive levels should be supported by STAT.CALC v.2.0 and STAT.NET in order to achieve the goals of the reorganization metaphor. Due to limited space, we will use a simple proposed design of teaching and learning activities to illustrate our ideas which echo the reorganization metaphor. In the example below, points are made based on graphical representations; but we must once again make ourselves clear that construction of graphs, even their uses and interpretations are not an end in itself. As we have pointed out above, when students are trying to represent a set of data graphically, they are at the same time making sense of the dataset. This process itself is a way to explore and to organize data meaningfully and is thus part of the process of statistical investigation (cf. Bright and Friel, 1998). Moreover, we have chosen to illustrate our ideas by experiences in plotting bar charts (even with ease with the aid of technological tools) since bar charts, though being an elementary graph form in statistics, has been shown to cause significant difficulties among students (Bright and Friel, 1998; Friel, Curcio and Bright, 2001; Pereira-Mendoza and Mellor, 1991). A final particular point worth mentioning is about possible difficulties associated with bar
charts for "reduced data" as contrasted with "raw data" (Bright and Friel, 1998).

## A Simple Design of Teaching and Learning Activities as an Example

As mentioned above, STAT.CALC v. 2.0 may do very little to help students acquire basic understanding of variables and bivariate data, and/or obtain better knowledge of the hidden intermediate steps worked out by computers. But these two learning objectives can be achieved with the support of this technological tool if activities are carefully designed with reference to the reorganization metaphor.

Using the examination question in Figure 2 as an example again, we propose and outline one possible and feasible design here for a series of lessons for senior secondary students (may be around 120 minutes, depending on students' ability). In the first part of this plan, students are divided into several groups. The exploration task is then assigned to each of these groups. They are encouraged to explore and investigate the frequency table in Figure 2 freely by generating any possible, plausible or meaningful numerical and graphical outputs, with the support of Excel. After around 30 minutes, their numerical and graphical outputs will be presented, and the meanings of such outputs will then be examined and discussed by the whole class. In this first part of the plan, although they may obtain the compound bar charts similar to Figure 6a and 6b very quickly, they will probably have difficulties when further their investigation or exploration by generating other numerical or graphical outputs. For example, if anyone mistook the column 'boys' as a variable, the mean of the four corresponding numbers ( $5,3,3,2$ ) would have nothing to do with mean number of boys. When they try to generate other graphical outputs, they may be stuck by the inability of Excel in identifying the difference in nature and meaning between the first column and the other two gender frequency counts column. That is, even if we replace the first column ( $1,2,3,4$ ) by ( $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}$ ), we can have the same result! (With Excel, the first column is actually ignored!) The first part of the plan would have an ideal closure if most of the students are in big puzzlement.

In the second part of the plan, each group is going to work on the sorted raw dataset (see Appendix), with the support of STAT.CALC v.2.0 and without informing them that the frequency table given at the beginning is a reduced dataset of these raw data. Again, after around 30 minutes, their numerical and graphical outputs will be presented, and the meanings of such outputs will then be examined and discussed by the whole class. In this second part of the plan, equipped with a better tool dovetailed with the analysis procedures of raw datasets, most groups should have more fruitful and meaningful outputs (e.g. see Figures 3 to 6). Of course, some of these outputs may not be plausible or meaningful; but they should still have their impacts on students' learning. Most students would be impressed by the amount of possible numerical and graphical outputs of such an ordinary small set of data.

In the last part of the plan, students are encouraged to investigate the relation between the frequency table and the raw dataset. They are not told at the beginning that the two datasets refer to the same data. But when they try to represent the reduced and raw dataset by different numerical or graphical outputs, they have been at the same time making sense of the two datasets (Bright and Friel, 1998). Therefore, most students should have a good chance to see the simple relation quickly. The remaining time of the lesson can therefore be devoted to the discussion of the hidden intermediate steps, such as organizing, sorting and classifying, in order to have reduced data from a raw dataset. Furthermore, by examining the numerical and graphical outputs generated in the first and second part of the plan again, students are guided to appreciate that many of the reduction processes are
irreversible (e.g. see Figures 3 and 4). Finally, by contrasting the processes and products of the first two parts of the plan, basic understanding of variables and bivariate data can be achieved.

In sum, by using this brief outline of classroom activities, we plan to develop basic understanding of variables and bivariate data among students. Second, we plan to encourage students to examine the relation between reduced and raw datasets, with reference to the hidden intermediate steps. The major guiding principle of the design is shifting the activities to a higher cognitive level, which is a key orientation of the reorganization metaphor (Ben-Zvi, 2000).

## Conclusions

STAT.CALC v.2.0 is intended to be a user-friendly tool for secondary students (some senior primary pupils, perhaps). They may fill the gaps between the common, too simple and inflexible tools designed for secondary students and the sophisticated professional tools (e.g. SPSS). One good example of these gaps is the lack of specific attention to the relations between raw dataset and reduced dataset at secondary level, noticeable (to the present authors) at least in Hong Kong (Curriculum Development Council, 1999). Since STAT.CALC v.2.0 is designed to accommodate raw dataset input, either univariate or bivariate, it can generate more sophisticated results with great ease and efficiency, comparable to the basic components of data analysis of professional tools.

While exploring the impact of STAT.CALC v.2.0 in statistics education, we have embraced Ben-Zvi's (2000) reorganization metaphor. We see it not only as an efficient, reliable, accurate and user-friendly tool, but also a "legitimate extension of cognitive systems and partners in the socio-cultural arena of the statistics classroom. Using these technologies as a cognitive tool and a medium will not weaken cognition. The opposite is true. It opens up the opportunity for the development of richer, powerful, and flexible learning environment in which students are active learners of statistics. This is a demanding task for students and teachers" (Ben-Zvi, 2000, p.149). And as mentioned before, our new demands include: (i) acquire better understanding of variables and bivariate data;(ii) obtain better knowledge of the hidden intermediate steps worked out by computers; (iii) develop the ability to identify the difference between nominal (or categorical), ordinal, interval and scale (or ratio) variables; (iv) form an informal and intuitive understanding of within-group and between-group variations; and (v) achieve a deeper understanding of the questions posed and the context in which these questions embedded during the data analysis and result interpretation stages.

But teachers and students should not be left alone in the reform. Mathematics educators and curriculum developers should take advantage of the technology to reorganize the curriculum. Such reorganization should include not only a new teaching sequence of various kinds of statistical topics, but should also emphasize conceptual understanding, mathematical modeling and problem solving, real-world applications, and new methods of analyzing data. Mathematics teacher educators should join in by designing and developing appropriate forms of professional development and support systems, which can help teachers know more about the difference between traditional and technological-rich teaching strategies; prepare themselves to function in the new teaching environment. In short, the reorganization metaphor should be applied at system level (Ben-Zvi, 2000).

## Appendix: Sorted Raw Dataset on Eating Habit

X Variable: 1= boy, 2=girl (Type $=$ Nominal)
Y Variable: Number of times going to fast-food shops in a week (Type = Interval)
(25)Data item25 $=2,4 ;(24)$ Data item $24=2,4 ;(23)$ Data item $23=2,4 ;(22)$ Data item $22=2,4$;
(21)Data item21 = 2, 3; (20)Data item $20=2,3 ;(19)$ Data item $19=2,3 ;(18)$ Data item $18=2,2$;
(17)Data item17 $=2,2 ;(16)$ Data item $16=2,2 ;(15)$ Data item $15=2,1 ;(14)$ Data item $14=2,1$;
(13)Data item13 $=1,4 ;(12)$ Data item $12=1,4 ;(11)$ Data item11 $=1,3 ;(10)$ Data item10 $=1,3$;
(9)Data item $9=1,3 ;(8)$ Data item $8=1,2 ;(7)$ Data item $7=1,2 ;(6)$ Data item6 $=1,2$;
(5)Data item5 = 1, 1; (4)Data item4 = 1, 1; (3)Data item3 = 1, 1; (2)Data item2 = 1, 1;
(1)Data item1 $=1,1$

## References:

Ben-Zvi, D. (2000). Toward understanding the role of technological tools in statistical learning. Mathematical Thinking \& Learning, 2(1\&2), 127-155.
Bright, G.W. and Friel, S.N. (1998). Graphical representations: Helping students interpret data. In S.P. Lajoie (Ed.), Reflections on Statistics: Learning, Teaching, and Assessment in Grades K-12 (pp.63-88). Mahwah, NJ: Lawrence Erlbaum Associates.
Curriculum Development Council (1999). Syllabuses for Secondary Schools: Mathematics (Secondary 1-5). Hong Kong: The Education Department.
Education and Manpower Bureau (1998). Information Technology for Learning in a New Era: Five-Year Strategy 1998/99 to 2002/03. Hong Kong: Government Printer.
Friel, S.N. and Joyner, J.M. (Eds.) (1997). Teach-Stat for Teachers: Professional Development Manual Statistics. Palo Alto, CA: Dale Seymour Publications.
Friel, S.N., Curcio, F.R. and Bright, G.W. (2001). Making sense of graphs: Critical factors influencing comprehension and instructional implications. Journal for Research in Mathematics Education, 32(2), 124-158.
Gal, I. and Garfield, J.B. (Eds.) (1997). The Assessment Challenge in Statistics Education. Amsterdam: IOS Press.
Larsen, R.J. and Stroup, D.F. (1976). Statistics in the Real World: A Book of Examples. New York: Macmillan.
Pereira-Mendoza, L. and Mellor, J. (1991). Students' concepts of bar graphs - Some preliminary findings. In Vere-Jones, D. (Ed.), Proceedings of the Third International Conference on Teaching Statistics, Volume 1: School and General Issues (150-157) Voorburg, The Netherlands: International Statistical Institute.
Shaughnessy, J.M., Garfield, J. and Greer, B. (1996). Data Handling. In A.J. Bishop, K. Clements, C. Keitel, J. Kilpatrick and C. Laborde (Eds.), International Handbook of Mathematics Education: Part One (205-237). Dordrecht: Kluwer Academic Publishers.
Wong, K.L. (2000). Statistics in the 2001 Mathematics Syllabus: Statistical investigations in a technological environment. In H.K. Leung (Ed.), Proceedings of the Hong Kong Mathematics Education Conference 2000 (58-71). Hong Kong: Department of Mathematics, Hong Kong Institute of Education; Hong Kong Association for Mathematics Education.
Wong, K.L. and Tang, K.C. (2000). Weaving the web into teaching and learning of statistics: The case of primary and secondary schools of Hong Kong. Paper presented at the IMECT 2000 Conference (International Mathematics Enrichment with Communication Technology), 6-9 July, University of Cambridge, UK.
(Accessible at http://nrich.maths.org.uk/conference/reports/wongtang.html )

