Pattern Spectrum Procedures to study the reconstruction accuracy

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Abstract

This paper presents a procedure to reconstruct the image from different levels of dilated shapes. The dilated portions are identified with the use of different mathematical morphological structuring element (SE) and compared with the original binary and reconstructed by using shapeness index. The range of this index is 0 to 1. If the shapeness index is 1, then the two images are in the same pattern with exact geometric similarity. If it is 0, then the two images are geometrically and topologically dissimilar. The reconstruction of the image is done by the following procedure. Successive dilations are performed on the image with markers, each dilation being followed by an intersection with the original image until convergence. This method decomposes an object into a number of simple components based on homothetics of a set of SE. Mathematical morphological transformations that are based on set theoretic principles are employed to decompose a binary shape by means of various SE. This procedure is based on various steps such as morphological reconstruction, skeletonization, shape decomposition and pattern spectrum. These procedures are used in an integrated manner to study certain aspects of binary image with an aim to derive morphological rules from the topological structure of a binary shape. To derive these rules several binary shapes are simulated and the procedures based on mathematical morphology have been systematically implemented to verify the accuracy of morphological rules and in the reconstructed shapes. To deal with these aspects of studying the reconstruction accuracy a procedure, based on pattern spectrum is adopted. Using this procedure shapeness indices of original and reconstructed shapes are computed and compared. It verifies the similarities of original image and reconstructed image by using pattern spectrum method. Details of the study of prototype nature are discussed

Introduction

Mathematical morphology [5] is a set algebra used to process and analyze data based on geometric shapes. It examines the geometrical structure of an image by probing it with small patterns, called structuring element (SE) by varying size and shape. Shape description is a very important issue in pictorial pattern analysis and recognition. Therefore, many theories exist that attempt to explain different aspects of the problem. Mathematical morphological reconstruction is part of a set of image operators often referred to as geodesic. Reconstruction is one classical way in mathematical morphology to achieve robustness by conditioning transformations to a reference set or image, and therefore controlling the spatial extensions of the morphological transformations. In morphology, a quantitative measurement for the size distribution of objects in an image is given by pattern spectrum. This size distribution in the form of pattern spectrum may be used for object recognition. This paper presents a procedure to reconstruct the image from different levels of dilated shapes and to compare with the original image. The pattern spectrum procedure has some invariant properties over Fourier spectrum [3]. The ability to reconstruct the original image from the descriptor, as accurately as possible and the shape descriptor is unaltered by a rotation of the given image about an axis. Through pattern spectrum procedure [2], we derive shapeness index (I) for the binary images.

Morphological Transformations

The discrete binary image, X is defined as a finite subset of Euclidean two dimensional space Z^2. Let B denote a structuring element which is a subset in Z^2 with a simple geometrical shape and size. Morphological dilations and erosions are defined as set transformations [1]. The morphological operators can be visualized as working with two images. The image being processed is referred to as the image and other image being a structuring element. Each structuring template [Fig 1] has a designed shape that can be thought of as a probe of the image. The translation of set B by the point x in Euclidean space is defined by $B+x =\{b+x:b\in B\}$, where the plus sign signifies vector addition. The reflection of set B is obtained by rotating B 180^0 with respect to origin (i.e) $-B=\{-b:b\in B\}$. The four basic morphological transformations are dilation, erosion ,cascade of erosion-dilation and closing [Eqn. 1].

Erosion :*XQ(-B) ={x : B+x Ì X}*
Dilation: *XÅ(-B) ={x : B+xÇX¹Æ}*
Opening: *XoB =[XQ(-B)] ÅB*
Closing: *X·B =[XÅ (-B)] QB* *(1)*

In the multi-scale approach, the size of the SE will be increased form iteration to iteration.

$$nB = \underbrace{B \oplus B \oplus B\oplus \ldots \oplus B}_{n\ times}$$

where *n* is the discrete size parameter. These transformations are systematically used to reconstruct and to find the shapeness index.

Pattern spectrum

The pattern spectrum [2,6,8] of size n by a structuring element is defined as the pixel wise difference between the target image morphologically opened by a homothetic set of structuring element of size n and that opened by structuring element of size n+1. The morphological analog of frequency in conventional signal processing, is size. Hence any geometric spectrum, obtained morphologically, is a measure of size-content (of the structuring element) in the given image. Thus such a geometric spectrum of an image is plotted against the size (of the structuring element) axis. The n^{th} entry in the pattern spectrum [Eqn.2] is defined as:

$PS(n) = Area\{Xo\ nB \setminus Xo(n+1)B\},\ n = 0,1,\ldots,N_{max-1}.$
$PS(n) = 0\ ,\ n ³ N_{max}$ *(2)*

where X is the image B is the structuring element, $S \setminus Q =\{x\in S: x\notin Q\}$, N_{max} is the minimum size of the SE B, such that the erosion of image X with N_{max} B results in the null set. Thus the n^{th} entry in the pattern spectrum is the cardinality of the set difference between the opening of the image X by the

structuring element B of size n and (n+1).It is a shape-size descriptor, which can detect critical scales in an image object and quantify various aspects of its shape-size content. Since opening removes the portion smaller than the SE, the difference of the images opened by the SE of size n and size n+1 contains the portion whose size is exactly n. The number of pixels in the set obtained by subtracting the opened objects from the original one gives the area of those objects that cannot contain the SE. Thus, iterative application of the morphological opening and the measurement of the residues, while increasing the size of the SE gives the size distribution of the objects contained in the given image.
The problem of shape representation [9] and shape-size description is very important in computer vision and image processing. Toward this goal, we investigated a shape-size descriptor, called pattern spectrum, which can detect critical scales in an image object and quantify various aspects of its shape-size content. By scale we define here the smallest size of a shape pattern (generated by a prototype pattern of unit size) that can fit inside the image. The pattern spectrum conveys different types of information about shape (binary image). First the boundary roughness of X relative to B manifests itself as contributions in the lower size part of the pattern spectrum. Second, the existence of long capes or bulky protruding parts in X that consist of patterns sB shows up as isolated impulses in the pattern spectrum around positive r= s. where B is SE, r is scale. Finally, the negative-sizes illustrate the existence of prominent intruding holes in X. In this paper we studied the some binary shapes such as square, rectangle, circle, hexagon, octagon, triangle ,fractal and irregular shapes.

Reconstruction of binary image

Morphological reconstruction is a process of taking a partial connected component and recreating the entire connected component, based on the intensities in the input image X. We can reconstruct the connected components by selectively dilating these components (one pixel at a time) with some SE. Successive dilations are preformed on the image with markers, each dilation being followed by an intersection with the original image until convergence or idempotence. Reconstruction retrieves [Fig3,5] the original shape of the retained particles after an erosion which eliminates small objects.

Morphological reconstruction of the shape can be done by means of several predefined morphological rules by changing their characteristics. It involves two steps, first we have to derive the skeleton (networks) of the shape. Skeleton [10,11] is a line thinned caricature to summarize the shape, size, orientation and connectivity of the shape; will be precisely decomposed from its shape by the morphological rule that is designed precisely. In the next phase, these skeleton subsets, decomposed [7] from the shape, need to be dilated by way of trying to reconstruct the shape outline by means of certain predefined morphological rule. These decomposed subsets need to be dilated by an explicit number of iterations to reconstruct the basin, by the predefined morphological rule. The image reconstruction can be built by an infinite sequence of dilation and intersection, until the result reaches stability. The dilation and intersection is called conditional dilation.

Study of Reconstruction Accuracy

A procedure to reconstruct the image from different levels of dilated shapes is investigated [3,4]. The dilated portions are identified with the use of mathematical morphological SE with different scale. Our method decomposes an object into a number of simple components based on homothetics of a set of SE. Mathematical morphological transformations are employed to decompose a binary shape by means of various SE. These procedures have been used in an integrated manner to study certain aspects of binary image. These aspects include the derivation of morphological rules from the topological structure of a binary shape, and vice versa. To derive these rules several binary shapes are simulated and the procedure based on mathematical morphology have been systematically implemented to verify the accuracy of morphological rules and in the reconstructed shapes [Eqn3].

$X=U_{n=1}^{N} [S_n(X)\ Å\ nB]$

$$\text{where } s_n(X) =((XQnB) \backslash (XQnB)oB) \quad (3)$$

To deal with the aspect of studying the reconstruction accuracy a procedure based on pattern spectrum is adopted. Using this procedure shapeness indices of original and reconstructed shapes have been computed and compared. It verifies the similarities of original image and reconstructed image by using pattern spectrum method. Pattern spectrum shows that a structuring element (B) represents the position of a target area(X) as distribution of the shape of structuring element or scale when the scale and the shape of structuring element are determined. The scale shows the size of structuring element. Pattern spectrum $PS_x(r,B)$ [Eqn.4] is given by the following equation, where A(X) means that the area of X and r is the scale. (r is greater than or equal 1)

$$PS_x(r,B) = A((X\ o\ B_o) - (X\ o\ B_r)) \quad (4)$$

$A(X\ o\ B_r)$ is a measure of the pattern content of X relative to the pattern rB. By varying both r(scale) and the shape of B(structuring element) we obtain a shape-size spectrum of X, which is the full pattern spectrum of X relative to all the patterns that can fit inside X. The higher a spectrum at the maximum r, the more alike an area X is as a structuring element B. Shapeness is a likeness between X and B. B-shapeness $S_x(B)$ [Eqn. 5] is as follows, where r_{max} means that maximum of scale r.

$$S_X(B) = PS_X(r_{max-1},B) / A(X)$$

$$S_X(B) = (PS_X(r_{max},B) + PS_X(r_{max}-1,B))/ A(X) \quad (5)$$

If X is completely similar to B, $S_X(B)$ equal to 1.The second equation is used when the spectrum at r_{max-1} is not maximum. The procedure is explained in flowchart[Fig 9].

Case study

In this paper we studied different shapes such as triangle, square, rectangle, octagon, hexagon, circle, and irregular shapes for reconstruction and accuracy of reconstruction[Fig. 3]. We plotted all these shapes in one image and carried out the reconstruction procedures. For each shape there is a different pattern spectrum value through different SE like square, octagon, Rhombus. At the n-1 scale it gives the maximum area. For SE it takes different scale required for convergence the same image in pattern spectrum[Fig 4,6,7,8]. These values are given (Table 1-4). Flowchart [Fig. 2] shows various steps involved in the adopted procedure. Fractal[Fig 3] shape is considered with different SE. It gives 80 to 90% accuracy of reconstruction. Square shape with square SE gives 100% accuracy. But with different SE it gives 10 to 40% of accuracy. Similarly the rectangle shape with square SE gives 100% accuracy but with other SE it gives 10 to 30% of accuracy.

Conclusion

By scale we define here the smallest size of a shape pattern [Fig 9a 9b,9c]generated by a prototype pattern of unit size) that can fit inside the image. Scale in this approach has been quantified by linearly convolving the image with a Guassian function of standard deviation σ or local weighted averages of spatial span $\sigma \geq 0$; the real number σ is the scale parameter. In this paper we view size distributions via the concept of a pattern spectrum. We can also extend the size distributions and pattern spectrum to continuous-space graytone images and arbitrary multilevel signals, as well as to discrete-space binary and graytone image by introducing a discrete-size family of patterns. From the experimental results, it was shown that the shapeness index I was available information for individual recognition and it could be classified with 100% accuracy. To improve the classification accuracy, we will consider a new structuring element like ellipse or asymmetrical shape.

Table 1: Pattern spectrum of Fractal(256x256)

Scale	1	2	3	4	5	6	7	8	9	10	15	20	25	38	n
PS(16,Octagon)	15	20	34	36	40	42	46	49	61	83	85				0
PS(26,Square)	0	14	17	26	33	34	36	37	39	41	60	82	89		0
PS(39,Rhombus)	4	11	17	18	21	24	35	35	36	37	42	49	82	87	0

Table 2: Pattern spectrum of Square(100x100)

Scale	1	2	3	4	5	6	7	8	9	10	15	16	20	25	30	32	n
PS(17,Octagon)	4	12	24	40	60	84	112	144	180	220	480	544					0
PS(33,Square)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4950	0
PS(33,Rhombus)	4	12	24	40	60	84	112	144	180	220	480	544	840	1300	1860	2112	0

Table 3: Pattern spectrum of Rectangle(140x140)

Scale	1	2	3	4	5	6	7	8	9	10	15	20	21	25	30	32	n
PS(16,Octagon)	4	12	24	40	60	84	112	144	180	220	480						0
PS(32,Square)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7875	0
PS(32,Rhombus)	4	12	24	40	60	84	112	144	180	220	480	840	924	1300	1860	1984	0

Table 4: Shapeness index I of binary image with different SE.

Structuring element \Shapes	Square	Rectangle	Triangle	Octagon	Hexagon	Circle	Irregular Shape	Fractal
SE(Square)	1.0	1.0	0.5	0.6	0.6	0.4	0.6	0.9
SE(Octagon)	0.1	0.1	0.4	1.0	0.4	0.9	0.8	0.8
SE(Rhombus)	0.4	0.3	0.5	0.5	0.7	0.4	0.7	0.9

	1	1	1	
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
	1	1	1	

Octagon

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

Square

		1		
	1	1	1	
1	1	1	1	1
	1	1	1	
		1		

Rhombus

Fig 1. Binary patterns in Z^2 (SE)

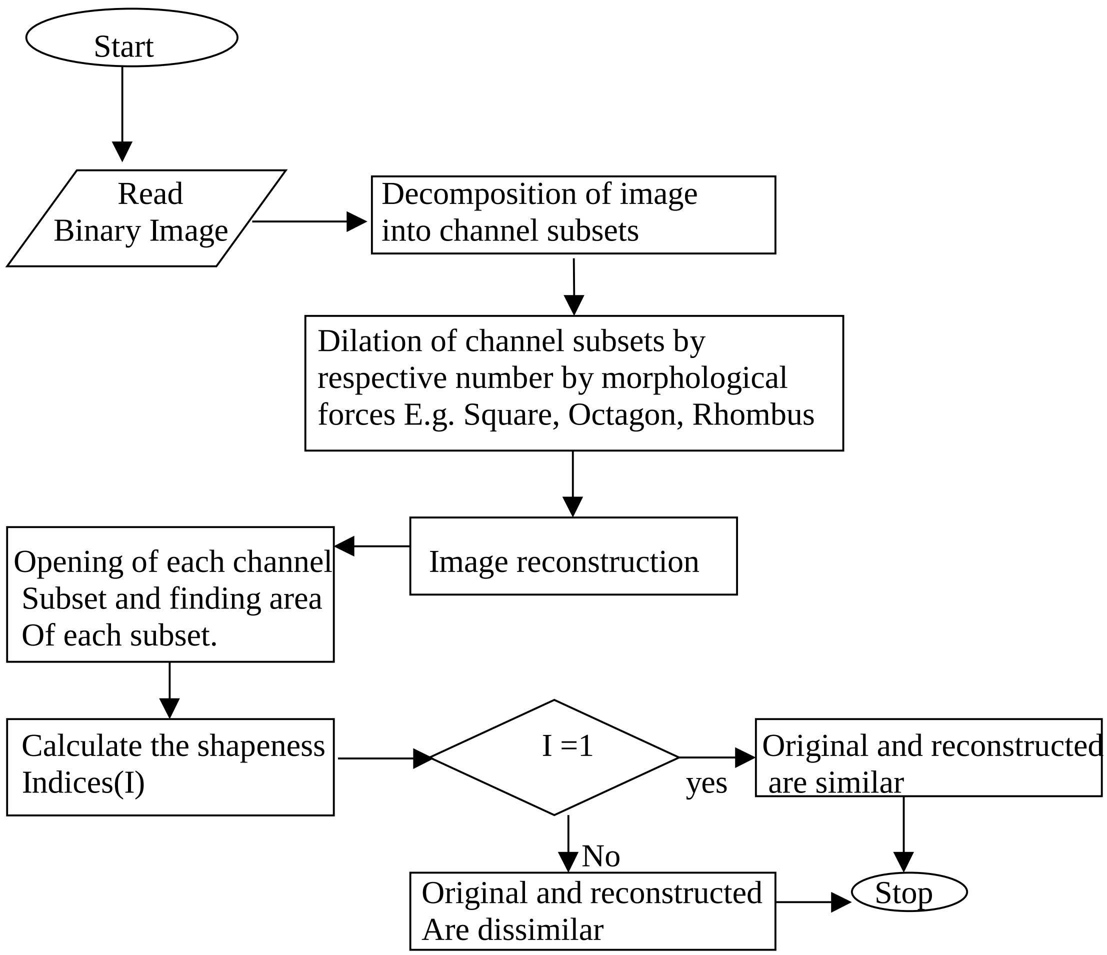

Fig.2 The methodology for pattern spectrum to study of accuracy

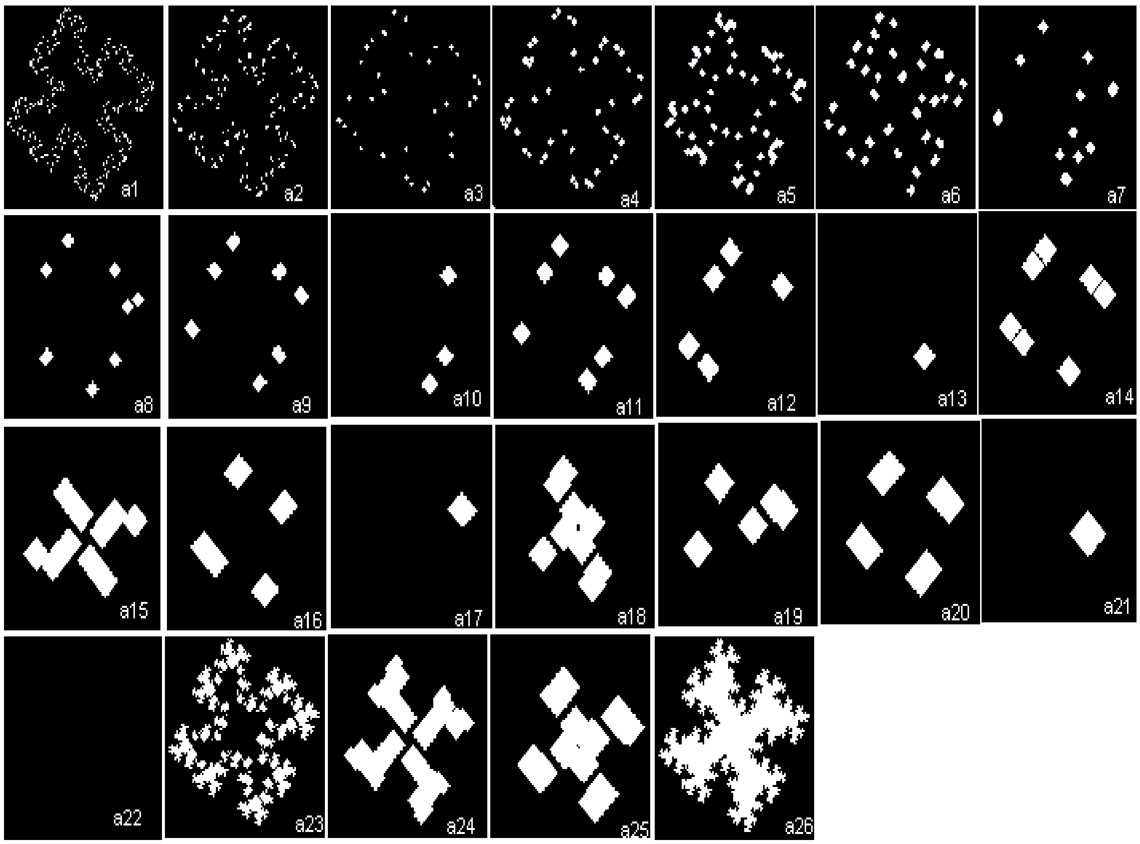

Fig 3 Reconstructing Fractal using Rhombus as SE. a1-a22: different scale subsets of dilation of original image. a23: union of a1-a10 a24: Union of a11-a17, a25: union of a18-a22, a26: union of a23-a25.

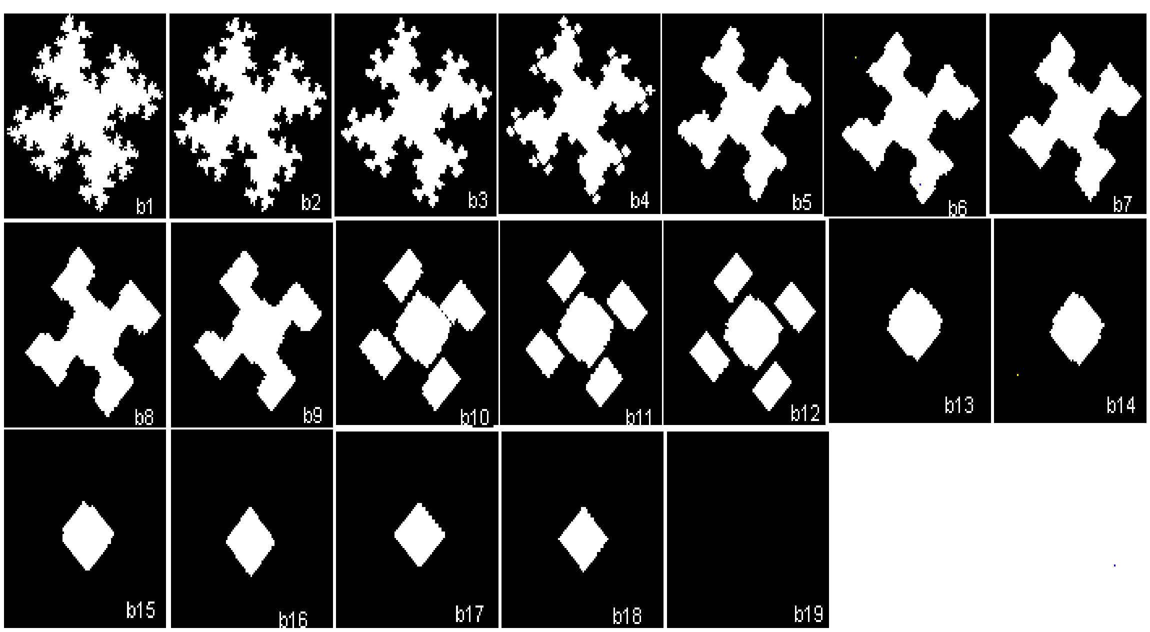

Fig 4 Pattern Spectrum of Fractal image in different scales with Rhombus as SE

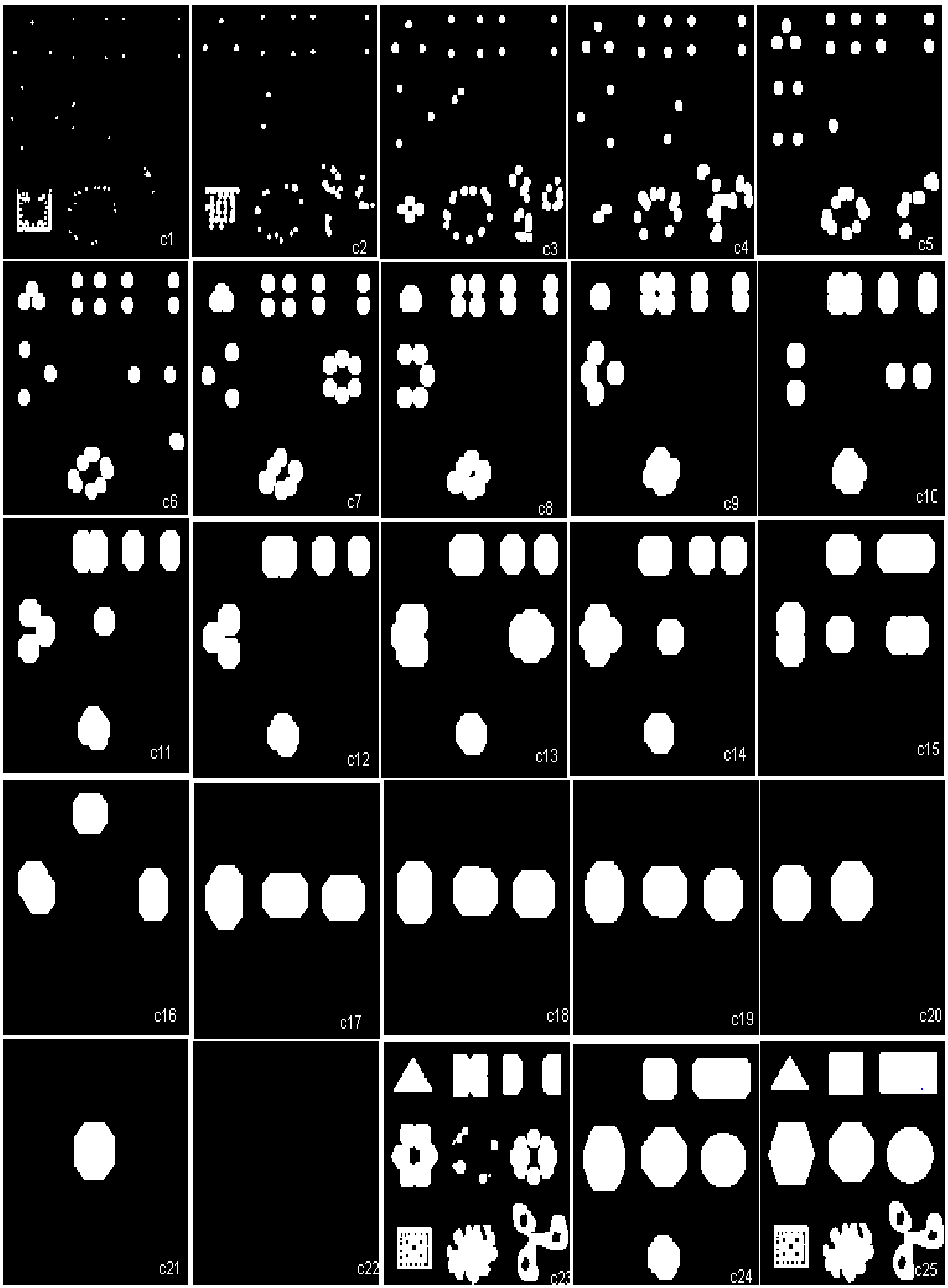

Fig 5 Reconstruction of combination of different shapes in different scale with Octagon as SE.
c1-c22 :different scale subsets of dilation of original images, c23:union of c1-10, c24:union of c11-c22, c25:union of c23-c24.

Fig 6 :Pattern spectrum of Square with square as SE. d1: image with one openings, d2: image with n-1 openings, d3: image with n openings.

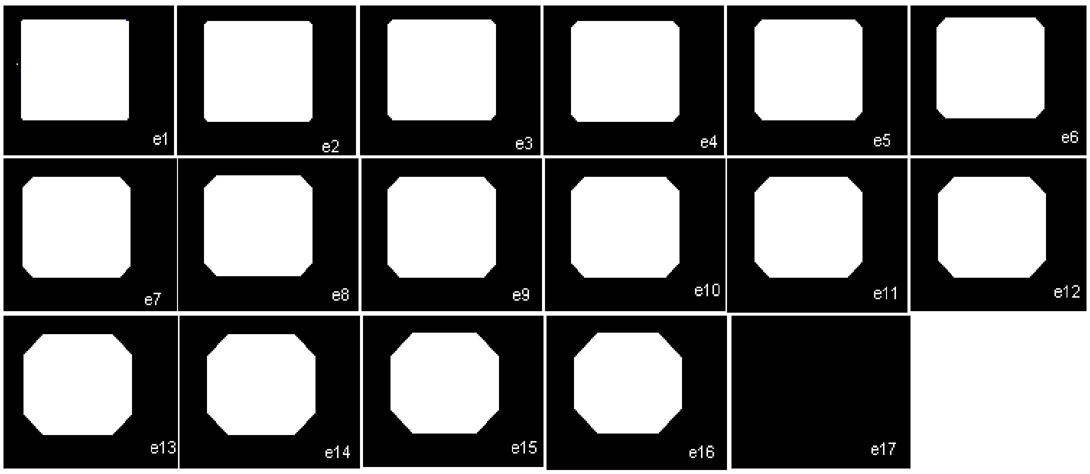

Fig 7 Pattern spectrum of Square with Octagon as SE. e1: image with one openings, e2-d15 different scale openings, e16: image with n-1 openings, e17: image with nth openings.

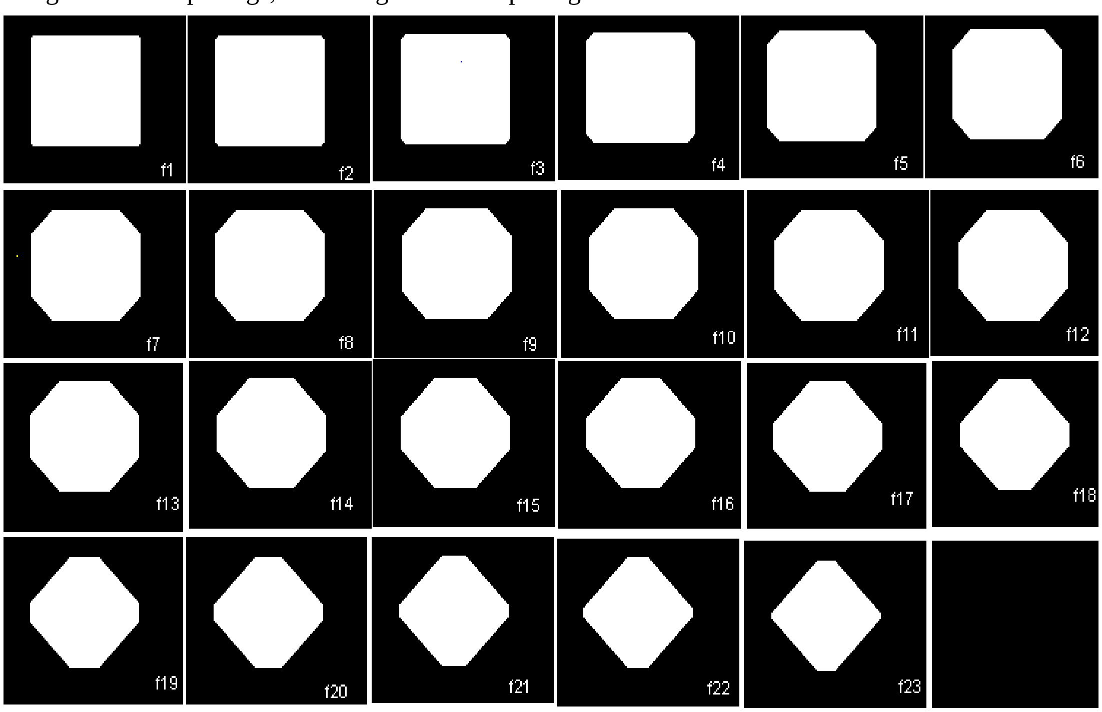

Fig 8: Pattern spectrum of Square with Rhombus as SE. f1: image with one openings, f2-f22 different scale openings, f23: image with n-1 openings, f24: image with nth openings.

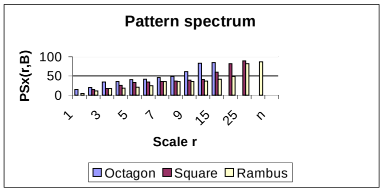

Fig 9a :Pattern spectrum of Fractal

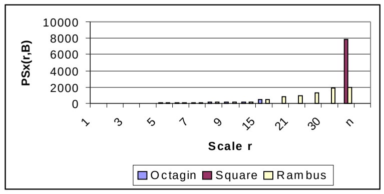

Fig 9b: Pattern Spectrum of Square

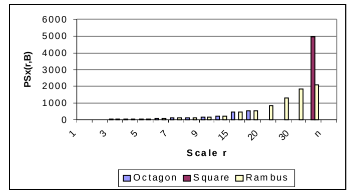

Fig 9c :Pattern Spectrum of Rectangle

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