DESIGN NONLINEAR SYSTEM WITH SLIDING MODE CONTROL

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Abstract

This paper presents result of a research and development of nonlinear system synthesis using sliding mode method to perfect the current sliding mode technique. Actually, the problem is the adjustment certain input control of nonlinear plant so the variable state system reaches the equilibrium point. It was done by choosing a manifol or certain area that once state system enter this area or manifol it will slide to equilibrium point.

By defining an area of certain surface, many researchers succeed in using this sliding mode method to determine adjustment certain input control for both linear and nonlinear systems. The main disadvantage of controller of this method is the chattering effect caused by forcement of system to enter the area or sliding surface.

This research is to develop a controller synthesis method of new sliding mode in order to produce or to determine adjustment input control by deleting or smoothing chattering effect without decreasing the performance.

Replacing the sign function by tan inverse function may deleting or smoothing chattering effect. It is done by ordering the system parameters by considering stability aspects and uncertainty of the system.

Based on the result of the simulation pendulum control system, manipulator robot is driven by DC motor in relation to tracking and stabilization and tan inverse function produce a better response of closed loop system.

1.Introduction

The focus attention of researchers on modern control theory is that there is not a general method that can be used in analyzing and synthesizing nonlinear control system, because nonlinear differential system is basically difficult to be solved. Only a few and simple nonlinear differential equation can be exactly solved. Moreover many nonlinear differential equations are only possible to be solved by approaching and its solution limited on certain condition. To solve the difficulty, the similar differential equations are grouped, then developed the appropriate analysis method for each group. Therefore, many techniques can be resulted in analyzing nonlinear system[1, 3, 6]

All the techniques depend on the unlinearity and order of the analysis system. Quasi-Linear System is a nonlinear system with small unlinearity degree, this system can be analyzed by linearization approach.

Describing Function Method is one choice used widely to analyze the Quasi-Linear System, the linearization approach can be used in all system order.

The nonlinear system can also be approximated by several linear area (Piece-wiece linear approach) allowing linearization segment by segment (piece linearization) without considering the degree of unlinearity and also used for all order system.

Phase Plane Method is one of the nonlinear analysis systems that is used only for nonlinear system of the order 2. By using two dimensions graphic method, the nonlinear method gives the information of stability and time respons.[1,3,4,6]

Lyapunov method is used in analyzing nonlinear system with high degree unlinearity. This method is used to analyze the stability of nonlinear system, however, it will find many difficulties especially in finding out the appropriate Lyapunov function. [2, 3, 4, 5.]

Synthesis of nonlinear control usually follows this procedure:

- Linearization of nonlinear system, that is approaching the characteristics of nonlinear system by linear system based on cutting Taylor series to the power around the certain operation point. Conceptually, it is done by counting matrix value Jacobi (first order) of the nonlinear function of the system characteristics around the equilibrium point and identifying them as characteristic matrix of linear system as limit of nonlinear system.
- Control system with configuration and general method to designing linear control system, such as LQR (linear Quadratic Regulator), LQG (Linear Quadratic Gaussian), pole placing, quadratic stabilization [12] to synthesize control system of linear time invariant

The main disadvantages of this approach are:

- Controller is not synthesized to improve the real nonlinear system performance. It only improves the nonlinear system as a result of approximation of nonlinear system itself.
- If the system dynamics move away the operation point, system can easily be uncontrolled, because control system will only operate at the operation point area.

Techniques or sliding mode method is developed from Variable Structure Control (VSC) to solve the disadvantages of this designing nonlinear control system. Sliding mode method is the method to adjust feedback by previously defining a surface or area so the system that is controlled will be forced to that area and then the operation point slides to the equilibrium point.

This method was introduced by Aizerman and Gantmacher [36], Emelyanov [36], Filippov[27], Utkin[30]. The combination of sliding controller and observer in linear case developed by Bondarev36, Hendrick and Ragavan[28] for nonlinear case.

The advantages of sliding mode method are:

- ✓ It can be used to synthesis nonlinear system with high order.
- ✓ It can be used for MIMO (Multi Input, Multi Output) System.
- ✓ The resulted control system will robust or resist to disturbance.
- ✓ It can be used for system with uncertainty so it directly solves inaccuracy.
- ✓ This method has been applied in many cases such as in underwater vehicle [39], motor vehicles and industries.

This research oriented to enhance and develop sliding mode method for nonlinear system single-input single-output or multi-input multi-output by considering uncertainty degree.

The focus of this paper is sliding mode method that based on these principles:

1. Notational simplification
2. High order differential equation represented by differential equation order 1.
3. Protecting the stability and solving the tracking problem
4. Solving inaccuracy
5. Generalization of Lyapunov Method
6. Construction of dynamic equivalent form for the nonlinear system with Filippov discontinue.

The result of this method can be applied on a real system including simple system of pendulum controller, robot manipulator motion equation that is driven by DC machine and link robot manipulator.

Nonlinear system discussed in this paper is only about time continue system stated by state equation. Explanation is limited to affine nonlinear system. It means that input affects linearly system movement. Method of this research is limited to the system by taking surface linear sliding. This limitation is not decreasing the generalization of using of this method.

2.Sliding Mode Method

The main focus of sliding mode method is how to adjust certain input control of nonlinear plant that causes variable state system to reach the equilibrium point. To do this we choose a manifol or certain area so once state system enters the area or manifol, it will slides to equilibrium point.

Many researchers have done many efforts to know linear or nonlinear system where sliding mode method possibly used. The appropriate requirements that have been derived from [11, 32, 39]. Utkin [39] has derived the requirement so the sliding mode can be used in nonlinear system. Controller for nonlinear plant synthesized based on this sliding mode technique can be implemented in various creatures for many systems, such as manipulator robot, satellite, CSTR (*Continuously Stirred Tank Reactor*). Generally, it can be said that most of continuous system that has n derivative can use sliding mode method.

The sliding mode technique developed in this research is different from previous sliding mode method. The previous sliding mode method produces chattering effect caused by discontinuous differential equation, caused by the sign function. This problem has completely derived from [16, 32, 39]. Sliding mode method in this research uses other function, that is tan inverse function that can delete or smooth chattering effect in nonlinear of linear system. This research is oriented on enhancement of the current sliding mode method by using tan inverse function that has a better configuration and it can be realized and guarantee the system performance.

3.Plant

Consider the single-input dynamic system :

$$x^{(n)} = f(\bar{x}) + b(\bar{x})u \qquad (1)$$

where the scalar is the output of interst $\bar{x} = [\dot{x}, \ddot{x}, ..., x^{(n-1)}]^T$, the scalar u is the control input $f(\bar{x})$ is not exactly known, but the extent of impresion on $f(\bar{x})$ is upper bounded by a known continous function of $\bar{x}$, the control gain $g(\bar{x})$ is not exactly known.The control problem is leading $\bar{x}$ in certain time into the desired condition $\bar{x}_d = [\dot{x}_d, \ddot{x}_d, ...x_d{}^{(n-1)}]^T$ with system model depends on f $f(\bar{x})$ dan $b(\bar{x})u$. By defining vector tracking error:

$$\tilde{x} = \bar{x} - \bar{x}_d \qquad (2)$$

By taking $\lim_{t\to\infty} \tilde{x}(t) = 0$ maka $\bar{x}(t) = \bar{x}_d(t)$ In this research was taken two cases: stability and tracking.

4.Sliding surface

Sliding surface was used in this research:

$$S(\tilde{x},t) = \left(\tfrac{d}{dt} + \lambda\right)^{(n-1)} \tilde{x} \qquad (3)$$

Where $S(\tilde{x},t) \in R^{(n)}$ is a time-varying sliding surface
λ is stricly positive constant, stated number related to the transient response system, sliding condition system reaches at:

$$\frac{1}{2}\frac{d}{dt}S^2 \le -\eta\,|S| \qquad (4)$$

5.Equivalent control

In order S to reach zero, it should be forced $\dot{V}(S) \le -\eta|S|$ controller should be taken to make the condition above reached in a certain time, where $V(S) = 0 \Leftrightarrow S = 0,$. This condition called sliding condition. Equivalent control is certain u value that taken and caused the system reached sliding condition, while u the control feed-back:

$$u_{total} = u_{ekivalen} - k\,\mathrm{sgn}(S(\tilde{x},t)) \qquad (5)$$

while controller value resulted from this research:

$$u_{total} = u_{ekivalen} - k \arctan A(S(\tilde{x},t)) \qquad (6)$$

where A is positive constant

6.Selection k and lambda value

k value was selected for this method divided into 2 parts:

a. For the system without uncertainty
k selection for this system was seen at the resulted simulation graph until the system limit or plant that was not produced chattering effect.
k selection depends on the system that will controlled, the big value of k tends to increase the appearance of this effect.

b. For the system with uncertainty at f and b

For the system with uncertainty at f, the k value was chosen:

$$k = F + \eta \qquad (7)$$

where F is a limit of uncertainty at f, and η is a limit at Lyapunov function.
For the system with uncertainty at f and b was chosen the k value:

$$k \ge \hat{b}bF + \eta\hat{b}b^{-1} + \left|\hat{b}b - 1\right|\left|\hat{f} - \ddot{x}_d + \lambda\dot{\tilde{x}}\right| \qquad (8)$$

For the MIMO system, the k value chosen was:

$$(I - D_{ii})k_i + \sum_{j\neq i} D_{ij}k_i = F_i + \sum_{j=i}^{n} D_{ij}\left| x_{ri}^{(n_i-1)}\,\hat{f}_j \right| + n_i$$

Selection of k value above for the system with higher order was done same as the system without uncertainty, and the value has contribution to transient response system.

7.Simulation and Application

To study various aspects from the analytic concept of this research was done by synthesis controller for the dynamic pendulum nonlinear, then synthesis the controller of the manipulator robot system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ \sin(x_1)+x_3 \\ x_2+x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \qquad (9)$$

for both stabilizaton case and tracking by considering the uncertainty of the system.

8.Observation and Comparation

All the results of this research simulation consider the uncertainty effect of the system. From the result obtained, there is sign function on the controller causing chattering effect on the resulted response system for the certain k values. For example k = 5 for the controller system of the dynamic pendulum, k = 20 for the controller system of robot driven by DC motor. At this k value, system to control enteres the sliding surface and causes chattering effect. This condition is also operating for tracking problem. The following figures are the examples of the stabilization problem on the controller of pendulum dynamic.

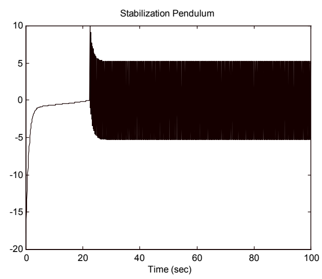

Figure 1.

Figure 1 showed the chattering effect on the controller system of the pendulum dynamic without high enough degree of uncertainty. By using other function suggested in this paper, the result shows at Figure 2.

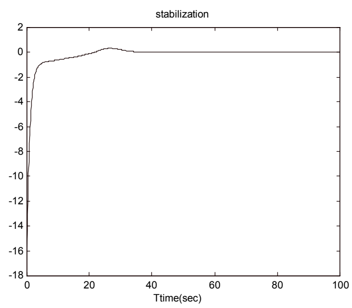

Figure 2.

For the tracking problem shows at Figure 3.

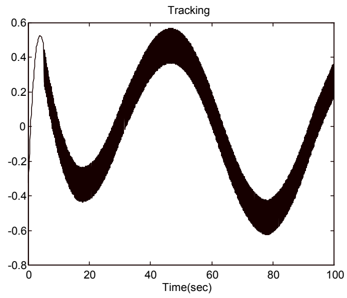

Figure 3.

Figure 3 shows the chattering effect was high enough on the controller of the dynamic system of the pendulum without uncertainty for tracking problem. By using of the function suggested in this paper, the result shows at Figure 4.

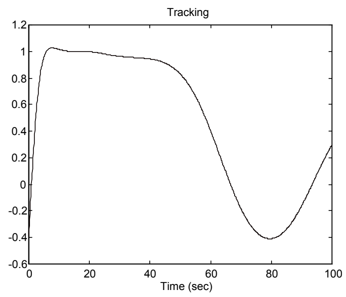

Figure 4.

The results of the research has been done on the others function, the researcher choose tan inverse function as an approach function from the sign function with changes the amplitude, produce the better result of the system response. It can deleting or smoothing the chattering effect.

From the result of this research is also obtained that the selection of amplitude value of the arc tan function can also deleting the effect.

The resulted system response by using switching control arc tan tends to be faster. It can be seen at Figure 5.

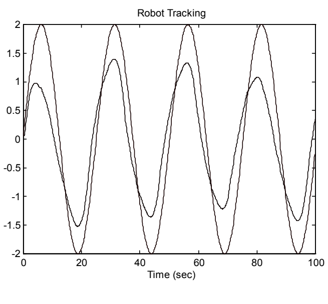

Figure 5.

The graph above shows the response x-1 on the simple robot system with tracking by using sign function, while if use a function $2/\pi$ arc tan showed in Figure 6.

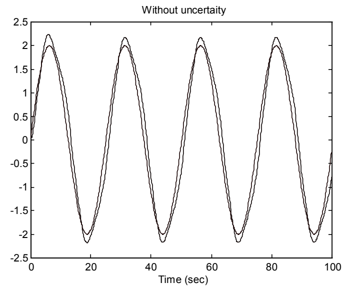

Figure 6.

This figure shows faster, so the resulted error $x-x_d$ is smaller.

9.Current and Future Research

Using the function of inverse tan shows that chattering effect can be smoothened and deleted by taking declivity curve inverse tan. Observation and accurate comparation [38] through digital simulation and taking of certain function as replacing the function of switching controller (the sign) gives the better result.

Two examples that have been given [38]: first the system order 2 with single input; second system order 3 with single input. By using both examples, the synthesis controller has been realizable configuration by considering uncertainty of the system.

The new finding of this research is the development and improvement of previous sliding mode method. It was developed by generalization Lyapunov theory. The taking of tan inverse function proposed in this research could solve the disadvantages of the previous sliding mode method. So this paper gives contribution to the nonlinear control system especially sliding mode method.

The difficulties of this research were to determine k and value causes the sliding condition.

Optimization is the performance control system that should be considered in this sliding mode method. The using of general performance applied in optimal control system, such as fuel or time can be considered as one performance that should be reached in synthesis. Application of the principle of dynamic programming from Bellman or Pontryagin helps in seeking new method.

The development of sliding mode method for the certain system that represent the control system such as Hamiltonian or Langrangian system, probably attain more interesting result.

Solution of the differential equation of the dynamic geometry Filipov can only be approached, however it will not give a special solution for the differential equation. There is no general method to solve a set of differential equation like this.[15, 27]

Therefore the approximation technique of differential equation that is optimized based on the performance of closed loop system is an interesting subject to study.

The new things we are researching and for future research:

1. Using inverse tan function as a replacing operation control of "switching".
2. Using optimization principle in determining tan parameters causes the system to reach sliding condition.
3. Using new stability concept such as Ultimately Unformally Bounded (UUB) from Corless and Gutman, so the stabilization or resulted tracking can be proved.
4. Elaborating Lyapunov geometry approach that was developed by La Salle and Slotine to derive controller validity.

5. To prove the solution of differential equation of sign function by using the differential geometry approach.
6. Comparing the solution of differential equation of sign function from current sliding mode method with solution differential equation of tan inverse function that will be developed analytically or geometrically.
7. To determine the optimal k value with geometry and analytic approach method. So it is not necessary to use trial and error method.
8. Applying the new sliding mode method to the real controller system.

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