COMPUTATION OF WALSH SPECTRUM BY DECISION DIAGRAM OF BOOLEAN FUNCTIONS

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Abstract

This paper proposes a computationally efficient BDD–based method for Walsh spectrum calculation, adapted for particular application where the subset of spectral coefficients is needed. This method takes the advantage of the property that for most switching functions *f* the size of BDD is usually quite a bit smaller than the size of the Multi-Terminal Binary Decision Diagram (MTBDD) for the Walsh spectrum. The selected Walsh coefficients can be computed by processing different nodes in the BDD for *f* and the pair of calculated coefficients is stored in two fields assigned to the root node. Complexity of the algorithm to calculate a pair of Walsh coefficients is proportional to the size of the BDD for *f*.

1. Introduction

The orthogonal transforms such as Rademacher, Walsh, Radhemacher–Walsh, Walsh–Paley, Reed–Muller etc. are used in spectral techniques for different applications in digital circuits especially in digital signal processing, design, synthesis, testing etc [7, 8, 9, 10, 11, 12]. The main limiting factor for application of spectral techniques is their computational complexities, either in direct methods or fast methods like FFT algorithms, due to exponential increase of input variables 2^n [9, 11]. Storage and time requirements are also major disadvantages of spectral techniques. However the use of Binary Decision Diagram (BDD) in presentation and manipulation of Boolean function has drawn considerable interest due to their computational simplicity, reduced storage requirement and ability to implement operations on function as graph algorithms [2, 3, 4, 5]. But the common disadvantage of the available techniques with BDD is that they always calculate the entire spectrum of switching function. This can be considered as important disadvantage, because in many logic design and fault detection using spectral methods only some selected coefficients are needed [1].

This paper describes a computationally efficient BDD–based method for Walsh spectrum calculation, adapted for particular application where the subset of spectral coefficients is needed. We calculate the pairs of Walsh coefficients using a 2×2^n window by exploiting the periodicity of Walsh function. We split the window into a set of (2×2) window and distributed them over the MTBDT for the Walsh transform by simultaneously matching the recurrence in both Walsh matrices and in a decision tree. The both recursive structure originate in the decomposition of the domain group of order 2^n into the direct product of cyclic subgroup of order 2. Due to the restriction to 2×2 sub-windows the related calculations match the basic Walsh transform matrix $W(1)$ and we just have to take into account the periodic change of sign in the Kronecker product of $W(1)$ by itself corresponding to -1 in $W(1)$. It is also possible to calculate the set of Walsh coefficients by our method for any switching function.

2. Background Concept

The Walsh function of order n is defined as [9]:

$$wal\,(j,k) = (-1)^{\sum_{i=1}^{n} k_i j_i} \tag{1}$$

Where $k, j \in \{0, 1, \ldots\ldots\ldots\ldots\ldots, 2^n{-}1\}$ and $k = \sum_{l=0}^{n-1} k_l \cdot 2^l$, $j = \sum_{l=0}^{n-1} j_l \cdot 2^l$

In many application this Walsh function used in different ordering but they produce the same set of Walsh coefficients [8]. Since the natural ordering of the Walsh function is easy to represent in matrix form hence we are using it in this paper. Walsh function of natural order can be written in the matrix form as follow:

$$W_n = \begin{bmatrix} W_{n-1} & W_{n-1} \\ W_{n-1} & -W_{n-1} \end{bmatrix} \text{ for } n = 1, 2, \ldots\ldots.. \tag{2}$$

and by definition $T_0 \underline{\underline{\Delta}} 1$.

Let $f(X)$ be a Boolean function of n variables, $X = \{x_1, x_2, \ldots\ldots\ldots, x_n\}$, $x_i \in \{0, 1\}$ and $i = 1, 2, \ldots\ldots\ldots, n$. Then all 2^n Walsh coefficients of the function f can be obtained by using the following relation.

$$W_n\, F = R \tag{3}$$

Where R represents the complete set of Walsh spectrum. For computing Boolean domain by Walsh spectrum of the given Boolean function use reverse of the equation (1).

$$F = [W_n]^{-1} R = \frac{1}{k}[W_n] R \text{ where } k = 2^n \tag{4}$$

The order of the coefficients is determined by the number of x_i variables in the corresponding *XOR* function. Since the order of the transformation matrix is $2^n \times 2^n$, hence all 2^n spectral coefficients requires $2^n \times 2^n$ multiplication and $\{2^n \times 2^n - 1\}$.

Examaple 1

For a three variable switching function $f(x_1, x_2, x_3) = [0, 0, 0, 1, 0, 1, 1, 1]$ the Walsh spectrum can be calculated using equation (2) and (3)

$$R = \{+4, -2, -2, 0, -2, 0, 0, +2\}$$

3. Decision Diagrams

The matrices and vectors which are the integral part of spectral techniques are represented by the different type of decision diagrams which are used as the data structure to perform all the manipulations and calculations. The Walsh spectrum is an integer-valued function and can be represented by the Multi-terminal Binary Decision Diagram (MTBDD) [6]. The MTBDD is derived by the reduction of the corresponding Multi-terminal Binary Decision Tree (MTBDT). Since the switching functions are a subset of integer valued functions if logic values 0 and 1 are interpreted as integers 0 and 1. Applying this particular application Binary Decision Diagram (BDD) become MTBDD. Figures 1 and 2 MTBDT (f) for example 1 and MTBDT (R).

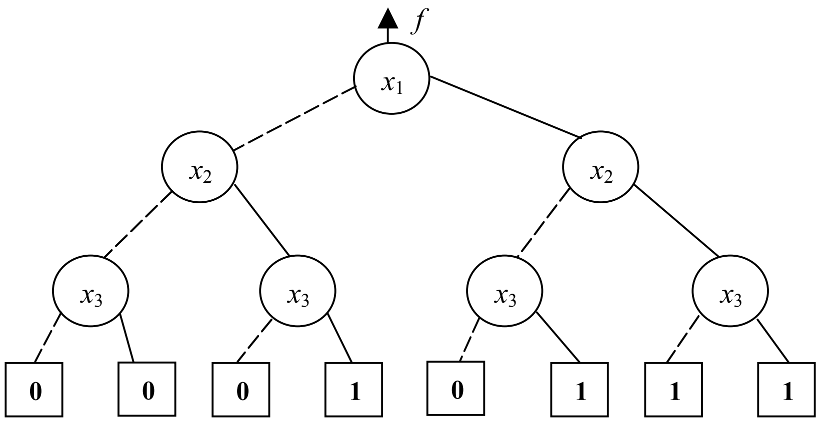

Figure 1: MTBDT for truth vector (f)

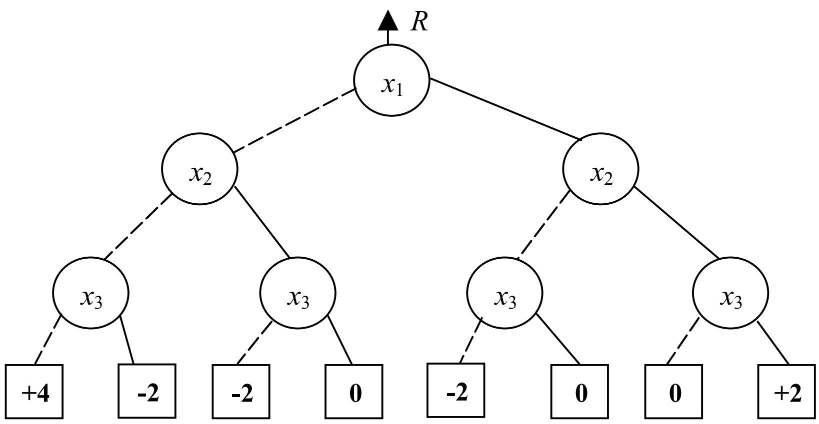

Figure 2: MTBDT for Walsh Spectrum (R)

Here the solid and lashed lines indicate the logic 1 and 0 respectively, which defines the path. In an MTBDT (f), a level consists of nodes to which the same variable in $f(x_1, x_2, \ldots\ldots\ldots\ldots., x_n)$ is assigned. We assume that levels in an MTBDT are denoted by indices of the variables assigned. Thus the levels are denoted by 1 to n, where n is the number of variables, with the root node at the level denoted by 1. The same convention can be applied by to MTBDDs.

4. Walsh Coefficient Calculation

First of all we assign to each node v two fields denoted by plus (v) and minus (v) in the MTBDT of given function (f) [1]. After that we calculate the values of these fields as follows:
Assume that we want to calculate a pair of Walsh coefficients W_d and $W_{d+2^{n-1}}$, $d \in \{0, 1, \ldots\ldots\ldots, 2^n-1\}$. We determine the binary equivalent $d = (d_1, \ldots\ldots\ldots\ldots, d_n)$ through the relation $d = \sum_{i=0}^{n-2} 2^i x^i$, $d_i \in \{0, 1\}$. Then we introduce a parameter vector P by deleting the first bit in the binary representation for d. Thus $P = (d_2, \ldots\ldots\ldots\ldots\ldots, d_n)$. The j^{th} element of P determines the way of calculation of fields *plus* $f(v)$ and *minus* $f(v)$ assigned to the nodes at the level j in the MTBBD (f) given as following:

(i) If v is a terminal node showing the constant c, then

$$plus\ f(v) = minus\ f(v) = c \tag{5}$$

(ii) If v is a non-terminal node with the successors $v.low$ and $v.high$ then

$$plus\ f(v) = plus\ f(v.low) + plus\ f(v.high) \tag{6}$$

$$minus\ f(v) = plus\ f(v.low) - plus\ f(v.high) \tag{7}$$

for $p_j = 0$, and

$$plus\ f(v) = minus\ f(v.low) + minus\ f(v.high) \tag{8}$$

$$minus\ f(v) = minus\ f(v.low) - minus\ f(v.high) \tag{9}$$

for $p_j = 1$

Clearly, if $j = n$ and the outgoing edge of a nodes v point to the constant nodes showing the values c_q and c_{q+1}, $q \in \{0, 1, \ldots\ldots\ldots, 2^n-1\}$ then *plus* $f(v) = c_q + c_{q+1}$ and *minus* $f(v) = c_q - c_{q+1}$. In figure (3) and (4) we show the application of (5), (6), (7), (8) and (9).

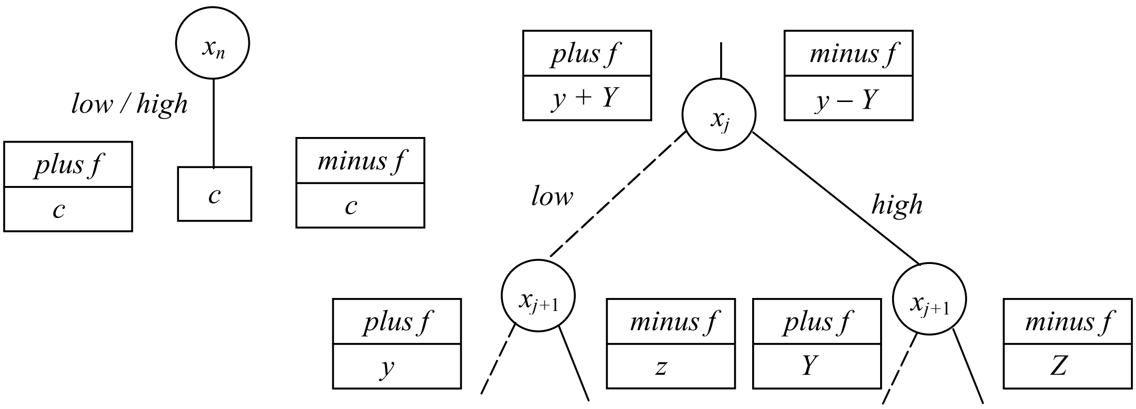

Figure 3: MTBDT (f) for $p = 0$

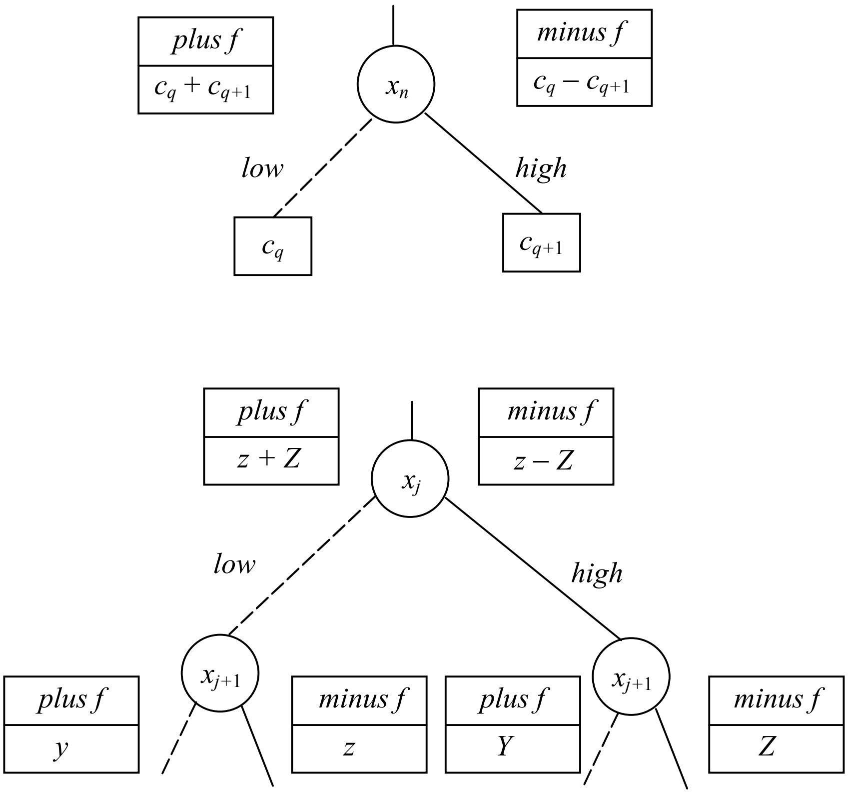

Figure 4: MTBDT (f) for $p = 1$

A procedure for the calculation of the pair of the Walsh coefficients W_d, $W_{d+2^{(n-1)}}$ can be formulated as follows:

(i) Express the decimal index d of the required Walsh coefficients W_d by a binary sequence $d = (d_1, d_2, \ldots\ldots\ldots., d_n)$, where n is the number of variables.

(ii) Generate a parameter vector P as:

$$P = (d_2, d_3, \ldots\ldots\ldots., d_n)$$

(iii) Traverse MTBDT (f) level by level starting from level n. Calculate the values of fields *plus f* and *minus f* for all non-terminal nodes.

(iv) The pair of Walsh coefficients (W_d, $W_{d+2^{(n-1)}}$) in the natural ordering is defined as

$$W_d = plus\ f\,(root)$$

$$W_{d+2^{(n-1)}} = minus\ f\,(root)$$

Where root is the root node in the MTBDT (f).

Example 2

The calculation of pair of Walsh coefficients W_3 and W_7 for any three variable function is shown in Fig. (5).

Since the $d = (0\ 1\ 1)$ for W_3 and $d = (1\ 1\ 1)$ for W_7, for both coefficient $P = (11)$, which shows that they can be calculated simultaneously. The values of fields *plus f*(v) and minus *f*(v) are shown on the left and the right side of the node. If the procedure explained in Fig. (5) is applied to example 1 we get $W_3 = 0$ and $W_7 = +2$.

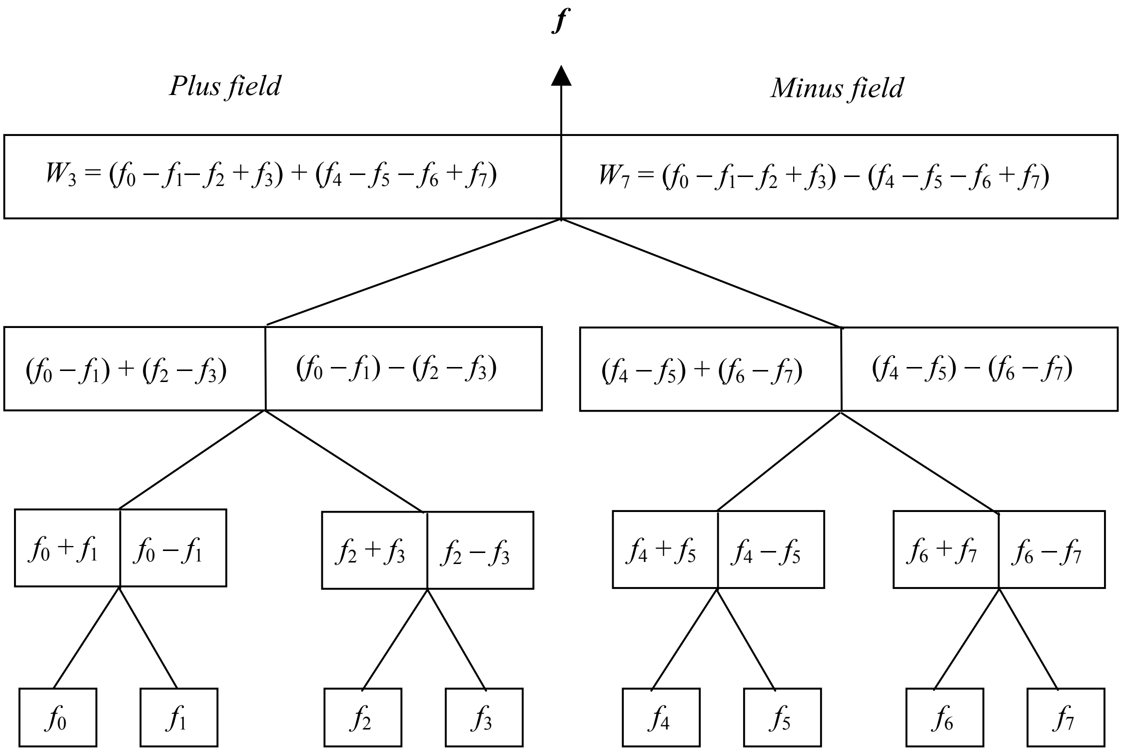

Figure 5: Calculation of W_3 and W_7

We can calculate the subset of Walsh spectrum with r coefficients by running this procedure a few times for different values of elements of the parameter vector P. The choice between the fields *plus f* or *minus f* of the successors which will be used in the calculation of a Walsh coefficients depends on the index of the spectral coefficients which is going to be calculated.

5. Conclusion

The proposed method is more suitable for the calculation of a subset of Walsh coefficients. Unlike the existing DD based (decision diagram) method the algorithm for implementation of the proposed method does not produce MTBDT for spectral coefficients. Instead we assign to each node in the MTBDT (f) two fields denoted as *plus* and *minus* fields. These fields are used to store the results of intermediate calculation. In this way the algorithm efficiently exploits the property for a great majority of switching function. The software for this algorithm is developed in C.

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