LEARNING LINEAR ALGEBRA USING TI-92

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Abstract

The school of mathematical sciences of Universiti Sains Malaysia has for the first time offered a course on integrating hand-held technology in the academic year 2001/2002. The course is intended for the pre-service teachers and for other students in mathematics. The course is module into four categories namely calculus, statistics, differential equations and linear algebra. Work done in this paper was a course content of linear algebra. In this paper we demonstrate the use of graphing calculator TI-92 in solving linear algebra problems. The objective of the course is to enhance the students' understanding in linear algebra concepts, in the sense that they would be able to see the concepts better through visualization. The intension also focus on providing the students an efficient method of showing the connections between the algebraic, numerical and graphic approaches, making it easy for the students to be active learners as well as generating an efficient method for solving problems modeled after 'real-life' solutions. Topics involving matrix multiplication and problems related to linear equations were introduced in this course.

1. Introduction

Graphing calculator has many interesting features apart from its portability and easy to use, it helps in visualizing underlying concepts that might not be visible explicitly. Learning mathematics has not been always easy especially when it comes to graphs plotting and data handling. The calculator has lighten up the work in mathematical calculation by short-cutting tedious steps such as, calling the same data over and over again without having repeatedly writing it down. The result is significant with large quantity of data. Integrating the technology of graphing calculator into the Linear algebra course has changed the perception of a group of a final year students in the Universiti Sains Malaysia towards Linear algebra. This paper describes how the students learn linear algebra in a different way such that it gives the students sense of awareness, excitement and involvement. Through the classroom activities they have been given opportunities to create their own set of algorithm or module for them to generate their mind to further enhance their conceptual knowledge. In this course we have introduced one specific type of graphing calculator, which is the TI-92 model, the latest version of graphing calculator, which is more sophisticated than the previous

model. The students taking the course, has been subjected to conceptual basics before hand, which means, they had taken an ordinary linear algebra course in the past. In this paper we give a brief exposition of students learning activities in a classroom, surveys and analysis of the surveys. The paper is organized as follows. First, the explorations and students learning activities are presented. Section 3 provides survey summary and analysis. Finally, section 4 conclude.

2. Explorations and Learning activities

This is a laboratory-based course, which runs for 12 weeks in a semester with 4 weeks for calculus(as it includes the introduction to the tool) and 3 weeks each for linear algebra, calculus, statistics and differential equation. In each week 4 hours are allocated for graphing calculator laboratory. There are two levels of maturity in this linear algebra course. First level begins with the preliminaries and illustration on basic examples such as matrix edition, matrix operations and other graphing calculator functions related to linear algebra.

In the second level of maturity we revisited topics in level one, this time addressing real-world applications. At this level we have chosen four different topics for the TI-92 explorations. The topics are identified as below:

1) Chemical balancing
2) Eigen value
3) Cryptography
4) Least square

In the explorations, interrelations between theoretical and practical concepts are discussed. This maturing procedure continues until several real-world levels of practicality have been incorporated. At this stage, by the help of the calculator they used all their creativity to manipulate the data to suit their interests. This level of maturity process allows students to learn how to refine a simple model so it will reflect a closer approach to reality. When this happened in such a very short moment, the students began to learn to appreciate the use of graphing calculator in their learning process towards better understanding in conceptual basis. In this paper, we show explorations on least square and cryptography. The other two explorations can be found in [2].

2.1 Least square

Least squares fitting is when you take a set of data values, ordered pairs, through which you wish to determine a curve of best fit. That is, we wish to find a curve, a type of equation, that we feel best models the data set. A good guess could be made based upon a scatter plot of the data. This means that we will need to determine the coefficients of the terms of the equations(s) for our model data, this we can do by turning our problem into a linear algebra problem. We will determine a matrix of numbers based on our known data values and solve the resulting linear algebra for the unknown coefficients of the model equation. For the model of an nth ordered polynomial, it can be represented as $f(x) = a_n x_n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$. The n pairs of data, (x_1, y_1), (x_2, y_2), ... , (x_n, y_n) must satisfy the equation and the unknown coefficients a_0, a_1, ... , a_n, are to be determined from the data. The matrix of values, A, constructed from the values of the data ordered pairs depicted in figure 2.0

$$A = \begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^n \\ 1 & x_2 & x_2^2 & \cdots & x_2^n \\ 1 & x_3 & x_3^2 & \cdots & x_3^n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_m & x_m^2 & \cdots & x_m^n \end{bmatrix}$$

Figure   2.0

The linear algebra problem we wish to solve, but cannot be solved, is to determine U from $AU = Y$. This can be done by determining W from $(A'A)W = A'Y$, an approximate problem with the solution vector W containing approximate values of the unknown coefficients of our model equation. This approximate problem makes use of what statisticians and mathematicians call the generalized inverse to find the best approximation, W, to U. In this exploration we use fish data shown in figure 2.1. Given the following fish data, determine a linear, quadratic, and cubic best-fit curve. First, we need to enter the problem data into a named data set area and complete a scatter plot of the data.

Table 2.1
Mean lengths (cm) of bullheads in a stream(monthly basis)

J	J	A	S	O	N	D	J	F	M	A	M
1.5	2.8	3.4	3.9	4.2	4.4	4.4	4.5	4.5	4.5	4.6	4.9
5.2	5.9	6.4	6.5	6.7	7.8	7	7.2	7.2	7.2	7.3	7.5

There are a few steps to be carried out before we get to the solutions and the interpretation of the data. First we need to enter the data and create a scatter plot. To begin with, the *Data/Matrix editor* is used to enter the values. *Type: Data* is chosen and the variable name is *fish*. Once in the data editor, we enter the months in numerical values along column 1 (c1) and the corresponding fish length along column 2 (c2). We are now ready to plot the data, this must be done from the Data/Matrix Editor screen for data plots. We press F2 and scroll to a plot number that is unused, press F1, select *Plot Type...Scatter, Mark...Box*, c1 in the x-box, press down arrow, type c2 in the y-box, select NO for Use..., press ENTER, press [♦] GRAPH. For this problem, the matrices are the following (figure 2.2).

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & \vdots \\ 1 & 23 \\ 1 & 24 \end{bmatrix}, \; Y = \begin{bmatrix} 1.5 \\ 2.8 \\ \vdots \\ 7.3 \\ 7.5 \end{bmatrix}$$

Figure 2.2

The vector $W = [a_0 \; a_1]'$ is the matrix of approximate unknown coefficients (the notation $'$ is used herein to indicate the transpose of a matrix). Next is to enter the matrices A and Y into the calculator. We enter the coefficient matrix A and Y vector using direct method: The *Data/Matrix*

editor is again used to enter the values of the matrices and this time we use *Type: Matrix* and give the variable name *af1*. Column 1 (c1) consist of 1's throughout 24 cells and Column 1 (c1) will consist of the second column in A. For the Y vector we do the same (entering the data values for Y (fish lengths)) and give the variable name *yf* . Alternatively(the indirect method) we can use the list in the columns of the "fish" data we had and use to form our matrix. However, from our experience, the direct method above is less time-consuming. The generalized matrix problem, $(A'A)W = A'Y$, may be solved in the TI-92 in the form $W=(A'A)A'Y$ where we used the following command to compute the solution (coefficients) using the generalized inverse. We type then type

$$(af1^T * af1)^{-1} * af1^T * yf , \text{ENTER} \qquad (\text{eq*}).$$

The TI-92 returns the solution vector W=[2.54529   .228043]′, the approximate solution. The elements of the vector W are the coefficients of the interpolating least squares equation. The linear least squares equation that models our data is:

$$2.54529 + 0.228043x = y \qquad (\text{eq1})$$

Plot a scatter plot of the data and the linear least squares function on the same plot. Enter the equation above in the equation editor (Y =) and plot it along with the original fish data. We then press [♦] GRAPH; the scatter plot and the linear model should be plotted together. This is shown in figure 2.3. To determine a quadratic best-fit function for the data, the matrix operator will be the following matrix A.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ \vdots & \vdots & \vdots \\ 1 & 23 & 529 \\ 1 & 24 & 576 \end{bmatrix}$$

Again, we have Y = [1.5 2.8 … 7.3 7.5]′ and the vector W = $[a_0 \ a_1 \ a_2]'$ is the matrix of (approximate) unknown coefficients. We again solve the problem $(A'A)W = A'Y$ in a similar fashion as in the linear version. This leads to the solution, [2.06349   .339228   -.004447]′. Substituting these coefficients leads us to the best- fit quadratic (interpolating) polynomial below (eq2). The plotting is shown in figure 2.4.

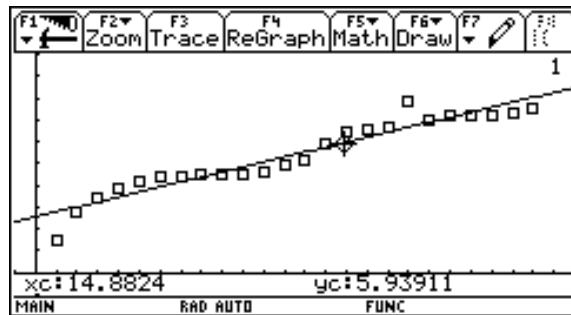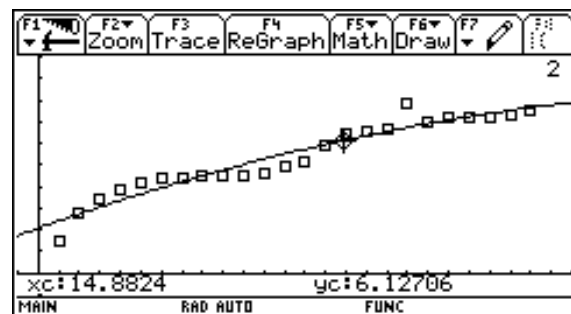$$2.06349 + .339228 - .004447 \, x^2 = y \qquad (\text{eq2})$$

Figure 2.3

Figure 2.4

Finally, we use model functions (eq1) and (eq2) to make guesses of fish length (y values) at when the experiment started, x = 0, month 25 (x=25, one month after the last data value), one year after the last data value x = 36, and at month 100.

Linear model, with x = [0 25 36 100), yls 1 gives
      y = [2.54529 3.79953 10.7548 25.3496]
Quadratic model, x = [0 25 36 100], yls 2 gives
      y = [2.06349 3.79472 8.51239 -8.48371]

The TI-92 commands we use to complete the above computations are:

[♦] HOME, type *y1({0,25,36,100})*, ENTER, type *y2({0,25, 36, 100})*, ENTER.

Of the two sets of values, we feel that those emanating from the quadratic model will indeed make us happy (though for x = 0, y = 2.375 is a much worse guess than y = 1.79471 as the fish at the end of month 1 was 1.4 cm long). As for long time behaviour, we conclude that the quadratic model seems to do much better in month 36, thus Linear Algebra saves the day again.

2.2 Cryptography

   One way to disguise a message is to replace each letter in the alphabet by some other letter and to use this substitution throughout this message. For instance, if we replace each letter by the next one

in alphabetical order and replace Z by A, the message "THIS IS A SECRET MESSAGE" becomes "UIJT JT B TFDSFU NFTTBHF". However, substitution codes such as this are easy to break because of the differing frequencies with letters appear in English sentences.

We can disguise a message more effectively by converting each letter to a number and then multiplying the numbers by a matrix to scramble the letters more thoroughly. In this exploration, conversion is done by converting 26 letters of the alphabets A-Z and the space, period, and question mark into 29 numbers 0, 1, 2, ..., 28 respectively. We use matrix A below as our enciphering matrix.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$$

Our enciphering scheme converts messages to numbers and group the numbers by twos and multiply each pair of numbers by the matrix A. We begin with enciphering the message "THIS IS A SECRET MESSAGE". The matrix M below is the result after converting alphabets to numbers, and C is the enciphered message. The command **mod(C,29)** replaces each number in C by its remainder after division by 29. Below exhibit the exploration with the TI-92.

$$[1 \quad 2 \; ; \quad 3 \quad 5] \rightarrow A$$

$$\begin{bmatrix} 19 & 8 & 26 & 18 & 0 & 18 & 2 & 4 & 26 & 4 & 18 & 6 \\ 7 & 18 & 8 & 26 & 26 & 4 & 17 & 19 & 12 & 18 & 0 & 4 \end{bmatrix} \rightarrow M$$

$$A*M \rightarrow C$$

$$= \begin{bmatrix} 33 & 44 & 42 & 70 & 52 & 26 & 36 & 42 & 50 & 40 & 18 & 14 \\ 92 & 114 & 118 & 184 & 130 & 74 & 91 & 107 & 138 & 102 & 54 & 38 \end{bmatrix}$$

$$\text{mod}(C,29) \rightarrow C$$

$$= \begin{bmatrix} 4 & 15 & 13 & 12 & 23 & 26 & 7 & 13 & 21 & 11 & 18 & 14 \\ 5 & 27 & 2 & 10 & 14 & 16 & 4 & 20 & 22 & 15 & 25 & 9 \end{bmatrix}$$

How can we decipher C ?. By applying A^{-1} to it, of course! This has to be done somewhat carefully, because we want to use only the numbers 0, 1, 2,... ,28. thus, since A^{-1} has some entries not in the range

$$A^{-1} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$$

we will use in its place a matrix B whose entries are all in this range and which have the same remainders after division by 29. Finally, the answer is

$$B = \begin{bmatrix} 24 & 2 \\ 3 & 28 \end{bmatrix}.$$

We then multiplied this matrix B with C to get the original messages back and this is done by a touch of a button.

3. Survey summary and analysis

We have carried out a survey on this course. The survey are divided into two parts, Part A and Part B. Part A contains questions on the students' understanding and feeling about Linear Algebra before they were introduced to the technology and Part B measures how much their perceptions have changed (assuming they have changed somewhat) after discovering the graphing calculator's usefulness in Linear Algebra problems. Twenty-four students responded to the survey. The results of Part A are shown in Table 3(a) below.

Table 3(a)
Percentage respondents for Part A

Item	1 (%)	2 (%)	3 (%)	4 (%)
VLow	4.2	4.2	16.7	0
Low	12.5	12.5	16.7	12.5
High	41.7	37.5	23.3	25
VHigh	41.7	45.8	33.3	62.5

There are four questions contained in Part A. The first question asked was on their level of understanding of the theories and concepts of Linear Algebra. The second question was on their level of interests in Linear Algebra, the third question asked was on their level of appreciation towards the application of Linear Algebra in the real world and lastly they were asked to rate their feeling of whether learning or applying Linear Algebra could be fun. The students were required to answer in a scaling format, which were rated from **very low** to **very high** based on the scale of **1 – 10**. Results from Part B, is shown in figure 3(b) below.

Table 3(b)
Percentage respondents for Part B

Item	5 (%)	6 (%)	7 (%)
A/SA	95.8	83.3	87.5
Not sure	0	4.2	4.2
D/SD	4.2	12.5	8.3

Similar to Part A, 4 questions were included in Part B. The first question was on how the students' understanding and appreciation of Linear Algebra have changed. The second asked for how much their understanding have enhanced after learning to use the graphing calculator. The third question asked whether their experiences with the graphing calculator in Linear Algebra have given them any insight into the value or limitations of integrating technology into the classroom.

Analysing the students' responds for Part A and Part B, we see that prior to taking this course 16.7% of the students had very little interest in Linear Algebra and 33.4% had almost no appreciation towards its application in the real world. However, 95.8% felt that their level of

understanding/appreciation towards Linear Algebra have improved after three weeks of lab explorations and 83.3% said that the experiences using this technology have enhanced their understanding in concepts of Linear Algebra.

The final question in Part B required the students to write about new experiences that they have gained after attending the Linear Algebra course. 91% of the students wrote that they were amazed by the technology of the graphing calculator especially in the problems that require tedious calculations which could invite mistakes. In the 'Secret Codes and Matrix Multiplication' exploration, 45.8% of the students felt that it was enjoyable to be able to use the graphic calculator in coding and decoding secret messages and 27.3% of these wished that they were given more time to explore the topic as this was done in only an hour of lab time. In addition, 37.5% think that having this tool to help in 'Balancing Chemical Equations' improved their understanding in this topic of Chemistry which they have learnt previously and 16.7% also appreciate the application of Linear Algebra in the "Least Square Method". In general, the students feel that with this technology, they need not bother themselves with messy calculations and they were able to concentrate more on the applications. As the three weeks progress, they were able to see how Linear Algebra can be applied to solving real world problems and not just a subject where you reduce matrices and solving equations. However, 9% voiced their concern that original concepts may be left out if excessive calculations are done to solve Linear Algebra problems. Therefore, they suggested that the students are equipped with the basic concepts prior to taking the course.

4. Conclusions

   The graphing calculator definitely has enhanced the learning process as well as improving the teaching methodology. This scenario can be observed through the explorations that we have done earlier, where modeling of the data is done in such a short time and tedious calculation skipped. We can also render our choice on the data, by a touch of a button. As a result, the students will have ample time to play around with the data, and at the same time without worrying about the calculations mistake, they can now interpret data more efficiently. Hence, graphing calculator is the right tool, not just to linear algebra alone but to other mathematics subjects in general. Of course, we do not object on any opinion or say, that there will be some original concepts being left out if the calculator is used excessively. Due to this concern, we suggest that students must be equipped with conceptual basics prior to attending the course.

References

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