Enhancing Students' Understanding in Secondary Mathematics through the Use of Graphing Calculators

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Abstract

The graphing calculator is one form of handheld technology. It was specifically designed and used as a tool in effective teaching and active learning in secondary mathematics. Graphing calculators empower students to use their ability to access, visualise, and create graphical representation, which will enable students to develop their mathematical thinking skills and concepts. By using graphing calculator students learn through exploring, investigating and discovering. Research carried out through SEAMEO RECSAM (Southeast Asian Minnistry of Education Organization, Regional Centre for Sicence And Mathematics) has shown that if graphing calculators are appropriately employed they can enhance students in developing their relational understanding in mathematics.

Mathematics and Students' Understanding

Over the past few decades many mathematics teachers faced with the perennial problem of teaching students who do not understand mathematics, they have tried to change the way they teach mathematics. These teachers have begun to wonder how they should teach those students in their classes who always seem to struggle to understand the mathematics presented in ordinary lessons? They have pondered over questions like: What exactly don't they understand, and why? What do they understand—and in what sense do they understand? What do I need to do to assist more students to understand?

According to Sierpinska (1994), understanding is the mental experience of a subject by which he/she relates an object to another object. Personal understanding of a concept is grasping or acquiring the meaning of the object. Understanding is a matter of being able to do a variety of thought-provoking things with a topic, such as explaining, finding evidence and examples, generalising, applying, analogising and representing the topic in new ways. (Blythe, 1998; Wiske, 1998).

Skemp (1978) describe two different meanings generally associated with "understanding" in mathematics. They are *relational understanding* and *instrumental understanding*. He explains the first one as "knowing both what to do and why" where as the instrumental understanding he described as, " rules without reasons." According to Skemp (1978), instrumental knowledge of mathematics is knowledge of a set of "fixed plans" for performing mathematical tasks. The characteristic of these plans is that they prescribe a step-by-step procedure to be followed in performing a given task, with each step determining the next. The kind of learning that leads to

instrumental knowledge of mathematics consists of the learning of an increased number of fixed plans, by which students can find their way from starting points to required finishing points. In contrast, relational knowledge of mathematics is characterized by the possession of conceptual structures that enable the learner to construct several plans for performing a given task (Skemp,1978, p.14).

Algebra is an important subject in mathematics. Students have to learn and understand the language, including the symbolisms, and grasp basic algebra, algebraic concepts such as polynomial and simplification, before they can expect to have any chance of success in studying pre-calculus, linear algebra and other more advanced areas of mathematics. Yet, algebra is one of the subjects that most students learn without understanding. The reason might be that with algebra, there has traditionally been an overwhelming emphasis on the manipulation of symbols, and because of that it has been too abstract for many students. Teachers expect their students to spend large amounts of time attempting to paper-and-pencil algorithms associated with elementary algebra. Students are required to memorise algorithms, but in the absence of any form of understanding, they simply do not learn them, or if they do, they quickly forget them.

Empowerment Through Tools: Graphing Calculator for Learning Mathematics

The graphing calculator is a relatively new form of technology, one which was specifically developed as a tool to assist the teaching and learning mathematics. Graphing calculators are small, pocket-sized computers with built-in graphing capabilities. They have the ability to draw and analyse graphs, carry out complex computations, perform matrix algebra, calculus and carry out statistical analyses (Waits & Demana, 2000).

However, the teachers have to achieve a sensible balance in their students' use of traditional paper-and-pencil techniques and operations through using graphing calculators. Teachers who have gained experience in the use of graphing calculators have discovered ways by which paper-and-pencil techniques and graphing calculators can complement each other in the mathematics classroom. Waits and Demana (1998) suggest that one method teachers can use to achieve a good balance is to have students employ the following strategies:

- Solve problems using paper-and-pencil and *support* the result using a graphing calculator;
- Solve problems using a graphing calculator, and then *confirm* the result using paper-and-pencil algebraic manipulation method;
- Use manipulative and paper-and-pencil techniques during initial concept development, and then use graphing calculators in the extension and generalising phases; and
- Use graphing calculators to investigate and explore the various connections among different representations of a problem situation.

SEAMEO RECSAM and Graphing Calculator

SEAMEO RECSAM offered full scholarships for 6-week training course on *SM-0613: Using Information Technology in Teaching Secondary Mathematics Through Cooperative Learning Model* from 1 July until 10 August 2002. The Ministry of Education of Brunei Darussalam, Indonesia, Lao PDR, Malaysia, Myanmar, Philippines, Singapore, Thailand and Vietnam selected their mathematics teachers/participants to attend this course. The participants were exposed to the knowledge on the use of graphing calculator as a tool in teaching and learning mathematics. The graphing calculator used in SM-0613 were TI-83 Plus and Casio CFX-9850GB Plus.

Polynomial: Cubic and Quartic Functions

Five participants who attended course SM-0613 developed the mathematics lesson plan on Polynomial: Cubic and Quartic functions incorporating with graphing calculator. The lesson plan was well planned and carefully designed based on the suggestion of Waits and Demana (1998). The trial-out lesson was held on 2 August 2002 from 9:00 to 11:30 am. with 21 Form 5 (Grade 11) students of Chung Ling Secondary School, Penang Malaysia. These students had basic knowledge only on cubic function and they did not learn quartic function yet. All of the students never used the graphing calculator before. The Malaysian participant who played a role as classroom teacher taught them how to use graphing calculator and quartic function through out the trial-out lesson. The trialed out reports are as follows:

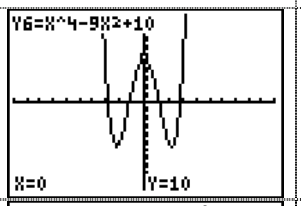
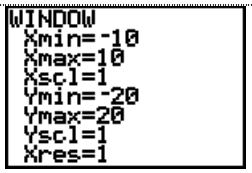
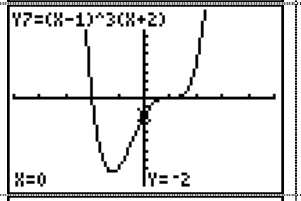
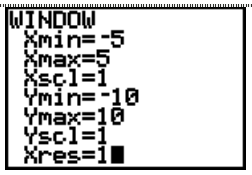
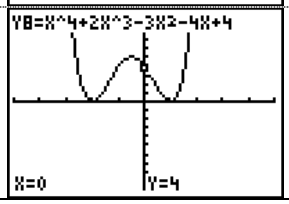
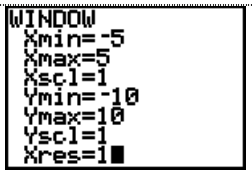
1) Results and Finding of the Trial-Out Lesson

The students worked together in pairs; each pair of students was given only one worksheet and one graphing calculator. The students worked on Worksheet No.1 and No 2, then they played game before the lesson was over. The Worksheet No.1 and No.2 are in the appendix. The examples of the provided answer sheet of Worksheet No.1and No.2 are shown in Figure 1 and Figure 2 below.

The results and finding shown that at first, the students felt difficult to predict and sketch the graph of quartic functions. Through the use of graphing calculator the students were able to conjecture the graph and explore the essential facts that were being asked in the worksheets. They used graphing calculator to draw graph, to confirm and check the solutions of the given cubic and quartic functions.

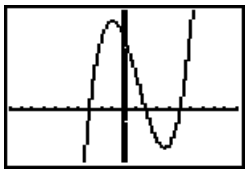
The students were able to use WINDOW and GRAPH Menu of graphing calculator in exploring and investigating the behaviour of cubic and quartic functions. They could illustrate the graphs and recognize the graph of cubic and quartic functions that satisfies the functions given. The students felt at ease when they used graphing calculators to explore the domain and range of cubic and quartic functions. The graphing calculators enhanced students to learn the Polynomial: Cubic and quartic function with understanding. They get immediate feedback and they learn through exploring and discovering.

The example of the provided answers to question II

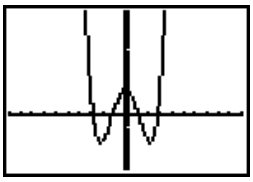
Equation	Graph	WINDOW	Degree	No. of Times Graph Touches X-AXIS	No. of Real Roots
6) $y = x^4-9x^2+10$			4	4	4
7) $y = (x-1)^3(x+2)$			4	2	4
8) $y=x^4+2x^3-3x^2-4x+4$			4	2	2

The example of the answers to question III.

III. a



Cubic Function
Graph Curves from Quadrant 3 to Quadrant 1



Quartic Function
Graph Curves from Quadrant 2 to Quadrant 1

b. Most number of times graph touches x-axis: cubic function **3** times
quartic function **4** times

It can be *less than 3 for a cubic function or less than 4 for a quartic function.*

c. The *number of real roots is also the number of times the graph of the function touches the x-axis.*

d. The number of real roots is *equal to or less than the degree of the function.*

e. The graphs $y = x^3$ and $y= -(x^3)$ have the same but one is the reflection of the other with respect to the y-axis.
The same condition holds true for the graphs of $y = x^3 -4x^2 -11x+30$ and $y = -(x^3-4x^2 -11x+30)$

f. Same observation holds true for quartic functions. However, the graph of the functions reflects each other with respect to x-axis.

Figure 1: The example of the provided answer sheet of Worksheet No.1

The Figure 2 below displays the answers to Worksheet no.2

Answer	Equation	Windows	
		Domain	Range
H	1) $y = -x^4$	$-5 \le x \le 5$	$-3 \le y \le 0$
C	2) $y = x^3 - 4x^2 + x + 6$	$-10 \le x \le 10$	$-10 \le y \le 10$
B	3) $y = -x^3 - 2x^2 + x + 2$	$-10 \le x \le 10$	$-10 \le y \le 10$
I	4) $y = 0.005x^3 + 60$	$-10 \le x \le 10$	$50 \le y \le 70$
G	5) $y = x^4 - x^3 - 43x^2 + x + 42$	$-10 \le x \le 10$	$-550 \le y \le 40$
A	6) $y = -x^4 - 2x^3 + 3x^2 + 4x - 4$	$-5 \le x \le 5$	$-10 \le y \le 3$

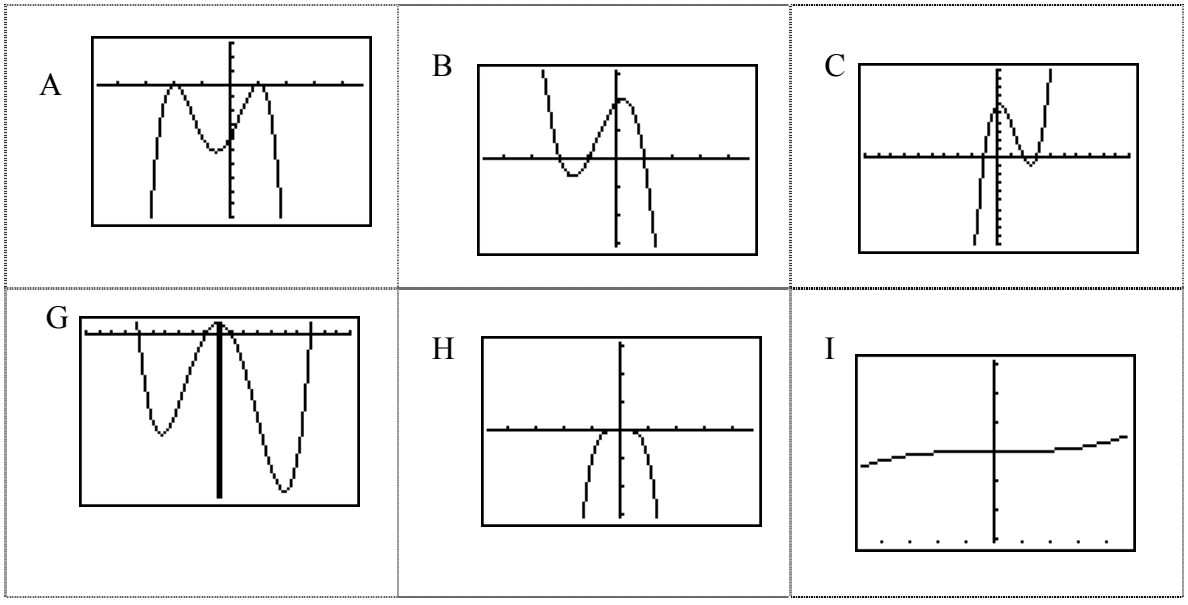
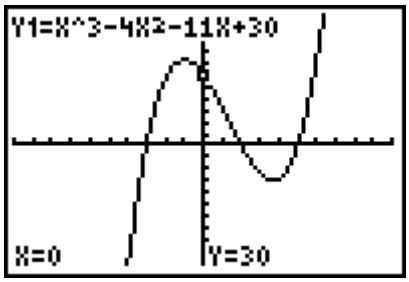
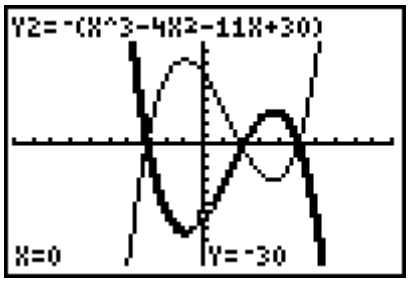


Figure 2: The example of the provided answer sheet of Worksheet No.2

2) The Students' Feedback

From the students' feedback after they worked on Worksheet No.1; they were able to find out the answer more than we expected. There was one instance where an answer key in the Worksheet No.1 was wrong. The results and finding are described as in the following.

The suggestion solution of Worksheet No.1 **from the students.**	The solution answer key of Worksheet No.1 **provided** in the trial-out.
3) e. The graphs $y = x^3$ and $y = -(x^3)$ have the same but one is the reflection of the other with respect to the Y- axis. For the graphs of $y = x^3 - 4x^2 - 11x + 30$ and $y = -(x^3 - 4x^2 - 11x + 30)$, One is the reflection of the other with respect to X-**axis.**  	3) e. The graphs $y = x^3$ and $y = -(x^3)$ have the same but one is the reflection of the other with respect to the Y**- axis**. The *same condition holds true* for the graphs of $y = x^3 - 4x^2 - 11x + 30$ and $y = -(x^3 - 4x^2 - 11x + 30)$

The students learned the Polynomial: Cubic and Quartic function with *relational understanding.* This was, because the students were able to explain what to do, and to tell *why* to their peers and they could share to whole class.

Through the use of graphing calculator, it enhanced students to develop their relational understanding in mathematics.

3) The Students' Perception on the Use of graphing Calculator

The semi-structured interviews with the students were conducted after the trial-out. The following excerpts are derived from audio taped and the transcription.

- Using graphing calculator it is more accurate and very powerful;
- I think using graphing calculator it save a lot of time, faster and convenience;

- I think graphing calculator has to use after the teacher taught the basic concepts, how to draw, calculate *x* and *y* value to get coordinate and wrote in the table form;
- I never use graphing calculator before, this is the first time I see something like this;
- It is quite difficult at the beginning, it has many button to press;
- The mathematics class today is fun, interesting I learn new skills and know new thing;
- I will recommend my teacher to use graphing calculator in mathematics class;
- Using graphing calculator is very useful, I learn better and faster; and
- Graphing calculator can make our learning in mathematics much more easier and interesting.

4) The Teacher' Perception

After the trial-out lesson, the informal interview with the teacher from Chung Ling school was conducted. She revealed that the trial-out lesson was very interesting. The students enjoyed the class; it was different from her normal teaching of mathematics. The use of graphing calculator as a tool enhanced the students to investigate and observe better than the normal way in using paper and pencil. It saves a lot of time in calculating the tedious polynomial functions. She shall recommend the school to use graphing calculator in mathematics classes.

Conclusion

If mathematics educators are to help their students think mathematically there is a need to find ways to improve the students' understanding of connections between various mathematics facts and procedures. There are now grounds to believe that the use of graphing calculators can help students who previously could not cope with algebra, to learn algebra. It is also appears to be likely, that all students, even the very capable ones, will benefit by using graphing calculators in investigating and exploring in mathematics class.

The teachers in Southeast Asia are using graphing calculators in motivating students to learn and to improve their understanding in mathematics lessons. Research carried out through SEAMEO RECSAM has shown that if it is appropriately employed, instead of students sitting back passively listening to the teacher, and then attempting to memorise "facts and procedures for examinations," they can be motivated to work actively and cooperatively with their peers through the use of graphing calculator. From 1998 until the present, SEAMEO RECSAM plays an important role in conducting workshops, training courses and researches on graphing calculator for mathematics teachers in ten countries in Southeast Asia.

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Appendix

Worksheet No. 1

I. Predict how the graph of each of the following functions look likes by sketching. **DO NOT USE** the graphing calculator.

a) $y = x^3$	b) $y = x^3 - 3x + 2$
c) $y = x^4$	d) $y = x^4 - 9x^2 + 10$

II. Using TI-83 Plus graphing calculator to graph the functions below and fill in the following table:

Function	Graph	Degree of the Polyno-mial	No. of times graph touches x-axis	No. of real roots
1) $y = x^3$				
2) $y = x^3 - x$				
3) $y=x^3-3x+2$				
4) $y = x^3- 4x^2-11x+30$				
5) $y = x^4$				
6) $y = x^4-9x^2+10$				
7) $y = (x-1)^3(x+2)$				
8) $y = x^4+2x^3 -3x^2-4x +4$				

III. Can you now answer the following questions?

a. What is the most number of times the graph touches the x-axis for a cubic function? _____ quartic function?_____
Can it be less? _____

b. What can you say about the number of real roots of the function and the number of times its graph touches the x-axis?

c. Therefore, how are the real roots of the function related to its degree?

e. Now, using your TI 83 Plus calculator, graph $y = -(x^3)$ and $y = -(x^3- 4x^2 -11x +30)$. Compare it with the graphs of $y = x^3$ and $y = x^3- 4x^2 -11x +30$ that you have drawn before. How do the graphs differ?

f. Will your observation hold true for quartic functions?

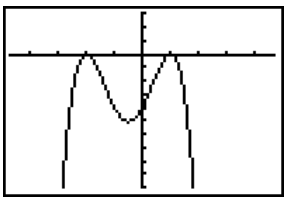
g. How did you find the activity on graphing polynomial functions using the TI-83 Plus graphing calculator?

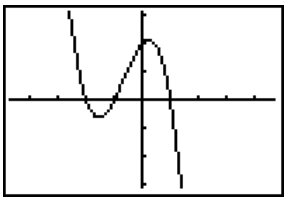
this  **or** this  ?

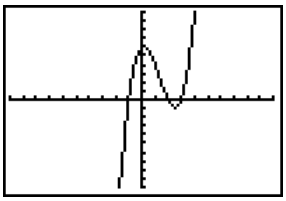
Worksheet No.2

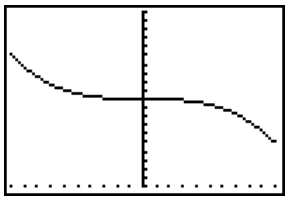
Match each of the following polynomial functions with the correct graph in the following sheet. Write the letter of the correct answer in the space provided. Use your TI-83 plus Graphing Calculator to sketch the graph and determine the domain and range of the function.

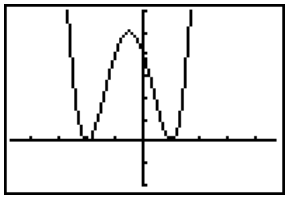
Answer	Function	WINDOW	
		Domain	Range
	1) $y = -x^4$		
	2) $y = x^3 - 4x^2 + x + 6$		
	3) $y = -x^3 - 2x^2 + x + 2$		
	4) $y = 0.005x^3 + 60$		
	5) $y = x^4 - x^3 - 43x^2 + x + 42$		
	6) $y = -x^4 - 2x^3 + 3x^2 + 4x - 4$		

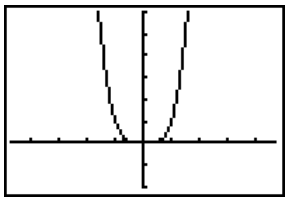
A 

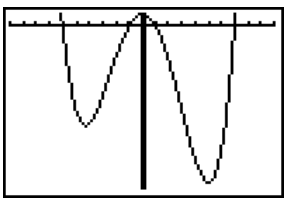
B 

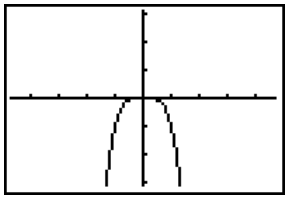
C 

D 

E 

F 

G 

H 

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