Debate and Argument Centred on the Projected Display of a Graphics Calculator: Examples of Instrumented Activity

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Abstract

This paper considers 'instrumented activity' in relation to the introduction of vector calculus in a Year 12 class. The first lesson on the vector topic is described where the display from a student's graphics calculator was relayed onto the whiteboard via an overhead projector and other students used their own calculators. An episode of whole-class discussion from the second lesson is presented during which the teacher connected his own calculator to the overhead panel. Students referred to the display as they suggested a variety of approaches for solving quadratic equations and argued about the efficiency of calculator and calculus methods. The paper identifies appropriation by students of their graphics calculator can enhance learning.

Introduction

Graphics calculators have been required equipment in Western Australian Tertiary Entrance Examinations (TEE) in mathematics since 1998. The introduction of the calculators to the examination was accompanied by widespread adoption of them for mathematics in schools, leading to changes in classroom practice. This paper explores practices associated with the use of an overhead projector to relay the display from a graphics calculator onto a whiteboard. The context of the *instrumented activity* (Guin & Trouche, 1999) was the introduction of vector calculus in a Year 12 class, where the focus was analysing quadratic functions and solving quadratic equations. *Instrumented activity* means tool use is integrated into activity.

I discuss the first two lessons on the vector calculus topic. In the first lesson, the teacher outlined how vector and calculus concepts could be combined in the interpretation of parametric graphs. A student supported the discussion by producing graphs on her graphics calculator, for display on the whiteboard. The focus in the second lesson was projectile motion. An episode from the lesson is reported which illustrates strategies for calculator use encouraged by the teacher, limitations of calculator use, and students' ability to argue! The teacher produced graphs on his own calculator for display via the overhead projector. He had wide experience in teaching with technology, all students had owned graphics calculators for nearly two years and some of them were highly competent users.

The paper is part of a study of teaching/learning in the class over twenty-one lessons. The teacher's calculator or students' calculators were routinely connected to the overhead projection panel. This practice facilitated the development of new concepts (Forster & Taylor, 2002) and allowed the teacher to encourage efficient problem solving methods and to conduct revision (Forster, Taylor & Davis, 2002a, 2002b). This paper, like the ones just cited, contributes to the small body of literature that directly addresses use of an overhead projector panel with hand-held calculators (e.g., Doerr & Zangor, 2000; Guin & Trouche, 1999).

Instrumented Activity

Noss, Healy and Hoyles (1997) portray mathematics learning as a process where students forge links between their actions and the numeric, geometric and symbolic representations that they develop. Noss et al. hold that the mediums through which students express their mathematics (e.g., a computer) profoundly structure the mathematising process, and a problem is students do not link mathematical understanding developed with a tool to understanding developed without it. Guin and Trouche (1999) define the problem in relation to the use of graphic and symbolic calculators. First, students do not allow for limitations of the graphical display. Second, students accept uncritically that calculator outputs are valid when the tool cannot show the canonical properties. For instance, students copy graphs from the screen to show curves stopping on asymptotes; and accept the outputs for limits to infinity when the storage capacity of the calculator has been exceeded during calculation. Drivjers (2000) describes related problems including students' inability to (a) 'help a calculator' when it does not produce results that are known to exist, (b) reconcile calculator symbolic outputs and traditional (pencil and paper) notation, and (c) cope with decimal outputs when exact values are needed. The internal limitations of the calculator, as well as syntax requirements and the non-transparent organisation of processing capabilities can be barriers to utilising the tool and can mediate negatively students' learning. Furthermore, students need to be aware of the calculation power of the tool in order to benefit from its efficiencies.

In general, tools become instruments when users appropriate them for the tasks that they undertake. Guin and Trouche (1999) give an example of a teacher appropriating an overhead projector and calculator projection panel, where he arranged for students to take turns sitting at the front of the room for entire lessons and link their calculators to the panel. Students in this role were observed to guide, assist and mediate class discussion. The set up "favoured classroom debates . . . and was essential to counterbalancing the rather individualistic relationships students tend to have with a small [calculator] screen" (p. 209). Use of the overhead influenced students' actions and thus influenced their learning.

Doerr and Zangor (2000) describe another configuration of the practice with the overhead projector: a teacher encouraged students to come to the front of the room during class discussion, to operate a calculator already attached to the panel. Furthermore, students often initiated the action. Like Guin and Trouche (1999), Doerr and Zangor observed that the projected display supported communication between students, whereas personal use of a calculator by students, even during group work, led to individualised learning.

Goos, Galbraith, Renshaw and Geiger (2000) report other formats for utilisation of the calculator-overhead combination in the classroom. For example, with an investigation of function transformations, the teacher assigned different functions to small groups of students and, after exploration, the groups explained and defended their conjectures using the projection panel as a means of communication. At other times, the teacher demonstrated calculator operations via the panel, and students displayed their partial solutions and completed them with class input.

In the Year 12 class that I observed, use of the calculator-overhead set up was flexible so that sometimes a student's calculator was linked to the panel and at other times the teacher's calculator. The panel had a long lead which could reach all students in their usual seats. When students were 'plugged in', as the teacher expressed it, the display variously supported teacher-led revision or instruction (Forster, Taylor & Davis, 2002a, 2002b) or students took the initiative and shared solutions that they had devised. Sometimes the teacher adopted an open, discursive style of questioning (Forster & Taylor, 2002) and at other times he was more didactic. Both styles are evident in the episode below.

Data Collection

I attended twenty-one 50-minute lessons in the Year 12 class, at an all-girls' college. I took the role of an observer-participant, observing whole-class work and acting as an assistant teacher during seatwork. I set a video-recorder to record continuously and included the display from the overhead panel in the field of view. In addition, I audio-recorded the conversations of all 13 students in the class. During seatwork, I made brief field notes of how students' operated their calculators. They all owned HP38G limited symbolic calculators. I transcribed the audio-recordings of all episodes of whole-class work and these included one-to-one interactions between students. The transcripts were the main source of data for this paper.

Instrumented Activity in Practice

Part 1: Preliminary work

In the first lesson on vector calculus in the Year 12 class, the teacher introduced the topic by asking students to recall vector relationships for rectilinear motion from their Year 11 work. Then he drew the graph of a straight line on the whiteboard, on **i**, **j** axes, and asked: "How can we bring in the calculus"? Students guessed velocity and acceleration would be obtained using differentiation. Next, the class revised how to plot the position vector $\mathbf{r}(t) = 2t\mathbf{i} + (t-1)\mathbf{j}$ on their graphics calculators (see Figure 1) and discussed how to choose the *x*, *y* and *t* ranges. Students favoured resetting the calculator to the standard scale and using the autoscale facility, which adjusts the *x* and *y* ranges to match the extremes produced in the *t* range (see Figures 1b-d). In addition, the teacher described how to calculate the *x* and *y* scales using the components and t = 0 and t = 12, to yield the domain of [0, 24] and range of [-1, 11] on the graph.



Figure 1. Screen displays for graphing $\mathbf{r}(t) = 2t \mathbf{i} + (t-1)\mathbf{j}$

During the revision a students' calculator was linked to the overhead projection panel. The rest of the class used their own calculators and could check their results against the projected display. After obtaining the graph, the teacher asked the student who was 'plugged in', "Can you just plot that again for us please" and asked the class "Can you look up here please. Watch it as it's plotting. What can you say about it slowing down or speeding up or anything else as it goes across"? The rate at which the graph is plotted from left to right reflects the speed of a particle with position vector $\mathbf{r}(t)$. Several students answered that the speed was constant.

Then, position vectors $\mathbf{r}(t) = 2t\mathbf{i} + t^2\mathbf{j}$ and $\mathbf{r}(t) = \sin t\mathbf{i} + \cos t\mathbf{j}$, for parabolic and circular motion which were new Year 12 content, were plotted. The teacher asked the class to observe and comment on the parabolic motion. When students offered it was "going down" and "getting quicker", the teacher described how calculus would be used: "and the part of the calculus in this is to work out when it [the plotting] speeds up, why it speeds up and all those things". With the circle, the teacher explained how to adjust the scales to avoid distortion (so the *x* range: *y* range = 2:1, which is the ratio of the dimensions of the calculator screen). He also asked for the range of *t* that would produce a single cycle (i.e., 2π); and asked the class to access the table of values and deduce the Cartesian relationship between the x and y coordinates. Next the revision shifted to the definition and meaning of magnitude and argument, and conventional and calculator syntax for them; and finished with the vector equation of a straight line. Following the whole-class revision, students started a textbook exercise that entailed pencil and paper methods, with opportunities for calculator use. Part of the exercise was set for homework.

Hence, the teacher integrated analytical approaches and a visual approach on the calculator to introduce the Year 12 vector calculus topic, and so modelled the integration that is recommended (e.g., Guin & Trouche, 1999). In particular, while he agreed to use the autoscale facility on the calculator for setting the scales for the straight line, he also encouraged students to determine the scales using mathematics. Setting the scales is known to be a major source of error and difficulty (Ward, 1997) so warrants the attention that it was given.

The functions for the **i** and **j** components were well chosen in that they illustrated each type of path (parabolic etc.) that students would eventually encounter. Moreover, parametric graphing on a calculator is 'realistic' in that the rate at which a graph is plotted reflects speed. Hence the graphs potentially are valuable for understanding the mathematics of motion. 'Realism' is increased when scales of a graph are set to preserve and not distort spatial relationships. Even so, students do not always see (speed etc.) relationships that are obvious to the teacher so the broad overview was important in that it introduced properties that would later be explored in detail.

Handwritten and calculator syntax were articulated (for vectors, magnitude etc.), which is significant in view that translation between them can be problematic (Drivjers, 2000). The teacher's practice was to write all expressions in conventional syntax next to the calculator display on the board, so equivalence was exhibited, for example between the vector form of the line $\mathbf{r}(t) = 2t\mathbf{i} + (t-1)\mathbf{j}$ and the corresponding parametric equations X1(T) = 2*T and Y1(T) = T-1. This was one way instrumentation of whole-class discussion with the overhead projector functioned to support students' use of their calculators.

The table of values linked to the graph, another key calculator feature, was visited as well. Hence, a calculator capability and the way to access it were discussed explicitly—and both these aspects are important in establishing efficient calculator use.

The student whose calculator was lined to the panel sometimes pre-empted the teacher's instruction and sometimes acted in response to his suggestions. In the process, she (the student) might have advanced her own expertise with the calculator. The whole-class work was highly interactive: it evidenced the communal activity centred on the projected calculator display that is noted in the literature (e.g., Doerr & Zangor, 2000). In addition, students who did not contribute to class discussion were not just listening. They actively used their own calculators. In addition, some discussed the work with each other and many made notes.

Part 2: Requirement for a sketch

In the second lesson on the vector topic the teacher set up the overhead projector with his own calculator. Soon after the start of the lesson, he presented the situation of a stone being thrown from the window of building. He specified the position vector: $\mathbf{r}(t) = 2t\mathbf{i} + (10 + 12t - 4.9t^2)\mathbf{j}$ and asked:

- T Its 4.9 *t* squared. Now, can anyone suggest why might be 4.9 *t* squared?
- Sts Gravity
- T A bit of gravity acting on it. So, part (a) . . . Plot the motion of the stone, please. Plot, sketch the graph, to show the motion of the stone.

After looking to see students' progress, the teacher said to the class:

T Can I suggest you plot the graph for about 6 seconds, and steps of about 0.1 usually works (see Figure 2b).

The teacher continued to walk around the class and a student asked him one-to-one:

- St 1. Can't we just go for like,
- T However many seconds it takes to hit the ground.
- St 1 Yes. Its much shorter.

Later, the teacher asked:

- T Once again, set your *t* range and then what?
- St 2 Autoscale.
- T Put it into autoscale (see Figure 2c). Just a rough sketch is all I want.



Figure 2. Screen displays for $\mathbf{r}(t) = 2t \mathbf{i} + (10 + 12t - 4.9t^2) \mathbf{j}$

Again a visual approach was used and was possible because graphics calculators were available. The teaching approach was to ask students to attempt the graphing and, when some appeared to need support, the teacher suggested how to set the scales. The features on two students' hand-drawn graphs (based on sketches in my field notes and on conversation with the students) are shown in Figure 3.



Figure 3. The features shown on two students' hand drawn graphs of $\mathbf{r}(t) = 2t \mathbf{i} + (10 + 12t - 4.9t^2) \mathbf{j}$

The graph in Figure 3a was drawn by the student who asked about the range for t (St 1 above). She used the POLYROOT function to solve for t when y = 0 (see Figure 4). POLYROOT produces the roots of polynomial equations. She entered 0 to 3.106 for the t range in her calculator, then, plotted the graph using autoscale: the x and y scales were automatically set so the turning point and the endpoint at t = 3.106 were just visible. She copied the graph, read the y intercept on the screen and calculated the x intercept (2 x 3.106) and added these to the hand-drawn graph. Her actions seem to reflect Guin and Trouche's (1999) definition of a *resourceful work method*, which entails the use of a range of techniques and sources of information: sometimes observations prevail, at other times theory is called on. Importantly, she chose the t range carefully, using the root of the function for the j component, thus incorporating analytical understanding, but passed calculation to the calculator. Hence, she took the given context into account to produce a realistic graph.

HOME HOME
POLYROOT([-4.9,12,10]) [657049889243,3.106…
STOP

Figure 4. Solving $10 + 12t - 4.9t^2 = 0$ to yield the *x* intercept of the graph in Figure 3.

The student who drew the graph in Figure 3b used the 0-6 range for t that the teacher suggested. She moved the cursor on the calculator graph to obtain an approximation for the xintercept: t, x and y values are visible on the calculator screen (see Figure 2d). Thus, she did not incorporate properties of the physical situation in her solution or determine the range of values for t prior to applying the autoscale routine. Nor did she bring to bear symbolic or calculator methods to improve the estimated x intercept, or copy the part of the graph that matched the situation. It seems she did not *predict* or *anticipate* any aspect of the graph; or *check* her solution for *consistency* with the given context, which are important aspects of calculator use. Her actions seem to reflect a mechanical work method (Guin & Trouche, 1999), characterised by reliance on the calculator, with no reconciliation to make the calculator model match reality. In fact, the student regularly took first place in tests in the class, was strong in algebra, and on occasion I observed her using her calculator judiciously. These are characteristics of a rational work method, as defined by Guin and Trouche, where the student relies heavily on analytic methods; and of a theoretical work method, which entails an even heavier emphasis on analytic approaches. Maybe the visual, calculator-based approach that was promoted in this episode did not appeal to her; but nevertheless, she did follow the teacher's instruction to produce 'just a rough sketch'.

Issues for pedagogy are (a) how to cater for students' preferences, where approaches are to include pencil and paper only tasks, and tasks where calculators can be chosen but are not specified and (b) how to provoke a minds-on approach, whatever environment (calculator or pen and paper) is intended. Asking students to attempt a graph allows opportunity for creativity. Asking them also to bring in whatever mathematical knowledge they have so as to draw an accurate graph might prompt some to advance beyond passive acceptance of calculator outputs. As well, for mathematics to be meaningful, instructions should be given to take the 'real-life' contexts of questions into account. In regard to context and the first lesson, the calculator graphs were dynamic and resembled reality, but they were only representations of the real thing. With tasks of that type, students should be encouraged to relate the characteristics of the graph to their real experiences--of seeing projectiles and sensing changes in speed, for instance, in going around a bend in a car. Hence, a role for the teacher in fostering instrumented activity is to encourage students to interpret tool outputs in light of analytic understanding and of the context. Figure 3a indicates the outcomes when a student has this orientation.

Part 3: Distinguishing processes on the calculator

The teacher plotted the graph on his own calculator and projected it onto the whiteboard. At this stage, he asked for the meaning of the $\mathbf{r}(t)$ components in terms of the given context. Next, he asked how the maximum height and the time when the stone hit the ground could be found, and students answered "the maximum value for y" and "when y equals zero". Then, a student asked:

St 3 But you don't get, like, the extremum and the roots [meaning you cannot read accurately the coordinates of turning points and roots on the parametric graphing screen on the calculator].

- St 4 Yes, you can. Not on that [indicating the display on the whiteboard], you have to go into Function [where functions are plotted] and use the different parts of the equation.
- St 3 But it isn't a function graph.
- St 4 No, no, no.
- St 5 She said you go into Function.
- T You could go into Function, and which part do we enter?
- St 4 That one [pointing to the **j** component of the $\mathbf{r}(t)$ expression written on the board], but you use *x*'s and *y*'s [see Figure 5]
- St 3 Oh.



Figure 5. Graphing $y = 10 + 12t - 4.9t^2$ as $y = 10 + 12x - 4.9x^2$ and obtaining the root.

Thus, another calculator method was found for determining the y intercept. The display of the parametric graph from the teacher's calculator and his board work acted as coherence objects. They scaffolded the conversation. The instrumentation facilitated the way Student 3's query was answered. The method involved graphing the **j** component as a function: t was replaced with x and students needed to remember the horizontal coordinate on the graph represents time, which is potentially confusing.

Part 4: A plethora of approaches

The exercise continued:

- T Part (b) is when does the stone hit the ground and how far from the building is it at this time [writing the key phrases on the board]? So, it hits the ground when y = 0. Do we have any plans to get the intercept? What are we going to do? We could solve it manually using the quadratic equation. [The class laughs].
- T Kate [St 1], which method are you going to use?
- St 1 I used POLYROOT [see Figure 4].
- T Anyone have any other methods they used on the calculator?
- St 5 Solve [see Figure 6].
- St 6 Draw it [see Figure 2? and Figure 5].



Figure 6. Displays for solving $y = 10 + 12t - 4.9t^2$ with y = 0 using Solve.

- T You could go to Solve but what's the problem with using Solve sometimes, whoever said that?
- St 5 It only gives you one [solution] at a time.
- T It only gives you one at a time so you have to?
- St 1 But if you have already looked at the [parametric] graph you already know whereabouts it is [so you can enter the approximate value for T prior to solving for T, see Figure 6b].

- T Yes, if you have already looked at the graph. That's so important. This looks like it's about three seconds [referring to the calculator display, see Figure 2b].
- St 1 Just magically.
- T Just magically.
- St 1 Its 3.106 seconds.
- St 3 Did you get that off there [indicating the display]?
- St 1 No, I used POLYROOT earlier.
- T With POLYROOT we get the answer straight away. It's quicker. What do we get with POLYROOT?
- St 1 3.106
- T Then, distance equals 6.212 metres [writing t = 3.106 and substituting into x = 2t on the board].

With long-term calculator use, students typically develop preferred methods of solution that utilise only a selection of the commands that they have met (Guin & Trouche, 1999). However, this does not mean the most efficient and effective methods are chosen. Hence, important aspects of whole class discussion in a calculator environment are discussing a range of methods and the limitations and strengths of them. Here the teacher pointed to the limitation of the Solve facility, Student 5 defined the problem, then, Student 1 and the teacher established how to circumvent the problem. Estimating the t value before utilising Solve is an instance of setting up the calculator to produce appropriate answers, an aspect of calculator use in which Drivjers (2000) found students are deficient.

Next, Student 1 challenged the Solve approach and restated her preference for POLYROOT. The result was the teacher valued the POLYROOT approach over Solve with "It's quicker". Indeed, the numerical POLYROOT approach requires only a single screen so is quicker than using Solve or using the graph of Figure 5d. Numerical methods on the calculator seem to be given little attention in the literature, even though they can be efficient. However, an advantage of using a graph together with the automated ROOT finding facility is the cursor jumps onto the root, which allows students to check visually that the value is appropriate (see Figure 5d).

Importantly, students in the Year 12 class frequently asked substantive questions about mathematics and challenged the teacher and each other (see Forster, Taylor & Davis, 2002). Reasons for the high participation were the teacher valued students' answers by centring discussion on them, which is illustrated above, and sometimes he commended students. Thus, many solution approaches became available, not only the methods of the teacher and a few students.

Last, I mention the traditional approach using the quadratic equation was also mentioned, in a tone that put it out of the question. I return to this stance in the next section and highlight it as problematic.

Part 5: Introducing the calculus and arguments for efficiency

- T Okay, part (c). What is the maximum height of the stone [writing the question on the whiteboard]? Okay, how do we find the maximum height of the stone?
- St 7 Find the maximum value of *y*.
- T Okay part (c) is find the maximum value of y [writing it on the whiteboard as he speaks]. How are we going to find the maximum value of y?
- St 7 Plot it.
- T What do you mean plot it, Abi [St 7]?
- St 7 Graph it.

- T Whereabouts though?
- St 7 In Function [see Figure 5, which shows the cursor on the maximum point].
- T Okay. Listen, Remember there are sometimes more efficient ways. Can anyone think of a quicker way of doing this?
- St 2 It's the turning point.
- T Like it's a quadratic. Can't we just differentiate it and make it equal to zero?
- St 8 But its easier to plot it.
- T Easier to plot it?
- St 8 Yes, you just type it in.
- T Shall we have a race?
- St 8 I've done it [laughs].
- T Surely [writing as he speaks] if I put 'dy/dt = 12 9.8 t'. Solve that equals zero. 't = 12/9.8'. Surely that's quicker than plotting it?
- St 8 Yes, but you have got more chance of making a mistake there.
- There is widespread chatter amongst the class about the issue.
- T Surely that's quicker Sally said wouldn't it have been quicker if we had plotted this thing [pointing to the **j** component of $\mathbf{r}(t)$] in the first place, she would already have had the graph ... So, Sally is probably quite correct to say if we had plotted this function in the first place as an *x*, *y* function, it's the quickest way to get extremum [or maximum point].
- St 3 But that's still not the quickest way [indicating his differentiation method on the board], even though you haven't plotted it.
- St 1 No. Because you have to put it back. It makes it less accurate and it takes it longer.
- T So now, y equals
- St 8 See now, you are doing it [the substitution] manually.
- T Okay, I have to concede defeat, partially.

So, calculus was introduced. I included the piece to entertain the reader and there is no room for extended comment. However, the motivational aspects of lively debate need to be acknowledged, although the calculator approaches were a distraction and asking students to use calculus might have served the vector calculus topic better. A second aspect is students *privileged* (Kendal & Stacey, 2001) calculator approaches, as the teacher and students did alike in Parts 1-4. Guin and Trouche (1999) found, in tension with this, that a general attitude of *using analytic methods* and *supporting them with calculation on the calculator* leads to more efficient integration of calculator use in activity in the long term.

Conclusion

Instrumented activity in the two lessons extended to students using their own graphics calculators, and the teacher and students taking advantage of the overhead-projector / calculator set-up (and, at a more fundamental level, included appropriation of writing implements, paper, the whiteboard, etc.). The projected display supported students' individual calculator use and scaffolded shared activity: it was at the centre of discussion on setting up and using the calculators, was the means for the teacher to model desirable calculator practices and identify calculator capabilities, and the students and teacher referred to the display when discussing mathematics relationships.

An emergent issue in the analysis of the two lessons was the privileging of graphics calculator approaches. First, the visual approach with parametric graphs seemed to serve the introduction of vector motion well. Second, an implication of the last excerpt is that sometimes students' suggestions for calculator methods warrant being strongly disputed so that the class is not distracted from making mathematical progress. Moreover, on the basis of the variation in quality of two students' calculator assisted solutions, recommendations are to *direct* students to bring in mathematical knowledge and to take into account given contexts when problem-solving with the calculators.

In conclusion, I suggest other aspects of the action which supported students' learning well were the teacher made explicit and had students identify mathematical relationships that were relevant to the calculator graphs and other outputs: it cannot be assumed that 'new' relationships are recognised in the electronic medium and that calculator solution processes are known. Furthermore, debate and argument on the efficiency of calculator procedures was usually commendable in that it encouraged optimal use of the technology; and the juxtaposition of conventional syntax and calculator syntax on the whiteboard potentially assisted equivalence recognition.

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