

CAS in the Classroom: A Status Report

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Abstract

Computer algebra systems have been around for nearly twenty years now and have gradually made their way into tertiary and secondary mathematics classrooms. Their inclusion has accelerated in the last ten years through the advent of CAS graphics calculators. The potential impact on the curriculum, on teaching and learning and on assessment has been the subject of numerous studies and has received a considerable amount of attention in the literature. To start with there were “show and tell” articles that gave examples of how particular topics can be treated using a CAS. More recently topics of empirical inquiry have been pedagogy associated with the use of the CAS, assessment and key skills. In this paper we present an overview with regard to these topics and detail both the issues and proposed courses of action.

Introduction

In this paper we present a review of the literature on the use of Computer Algebra Systems (CAS) in mathematics education. This use has expanded in recent years. One reason for this expansion is the impetus of the calculus reform movement whose proponents advocate a multiple representation approach to teaching and learning of calculus at all levels, exemplified in the textbook by Hughes-Hallet et al (2002) and as the SONG (Symbolic, Oral, Numeric and Graphical) approach by Gretton and Challis (2000). Another reason is that technical advances in miniaturisation have allowed the inclusion of CAS on calculators such as the high-end models TI-92, HP49G, and Casio FX-2. Use of CAS in the tertiary sector appears to be widespread, but by no means uniform. Most reports in the literature centre on the introductory undergraduate and service mathematics curriculum, while the use in higher-level mathematics subjects is not made as explicit. Some instances of the latter are discussed by Kulich (2000), in the setting of abstract algebra. At this stage the secondary education agencies in a number of countries are actively engaged in the investigation of the inclusion of CAS calculators in the high school curriculum. There is a wide range of views as to where the use of CAS is headed and how, if at all, it should be regulated. Advocates of the use of technology argue for its inclusion in all aspects of mathematics learning while others are concerned about a perceived loss of basic skills through the inclusion and would be inclined to censor the use of the technology.

In the sections below we consider some of the dominant themes in the literature. These are the implication of the CAS for pedagogy, the question of how to incorporate CAS in assessment and the definition of key skills students are to possess when the use of CAS is assumed. We do not claim to be exhaustive in our review and acknowledge that in general there are multiple instances given in the literature of the points raised. However, for economy of space usually only one reference is cited.

Implications of CAS for pedagogy

In the sections below we review the constraints CAS raises for learning, the resources it offers, and students’ and teachers’ responses to the inclusion of CAS.

Constraints to learning and problem-solving

Guin and Trouche (1999) identify three types of constraint that a CAS raises for learning: *internal, organisation and command constraints*. Internal constraints are those linked to the internal representation of mathematical objects and their processing. Examples include different representations for an algebraic expression depending on the mode of calculation (Bloom, Forster and Mueller, 2001), limitations of graphical displays stemming from the discretisation of the screen (Pierce and Stacey, 2001) and inconsistent production of exact results (Guin and Trouche, 1999). Organisation constraints are linked to the manner in which commands and facilities are accessed. Command constraints include knowing the commands to select and the specification of syntax for them. Galbraith and Pemberton (2000) observed that these two constraints gave rise to more student queries in CAS laboratory sessions at the undergraduate level than other aspects of CAS use.

While according to Pierce and Stacey (2001) anomalies can provoke discussion between students, Guin and Trouche (1999) dispute that students naturally conjecture what is happening on the screen and suggest that explicit intervention at a conscious level is required. One reason for explicit intervention is students might lack the mathematical background needed for informed decisions. So Guin and Trouche, along with others (for example, Lagrange, 1999), recommend addressing strategies for comparison of algebraic expressions, the interpretative demands brought about by discretisation of the graph, distinguishing efficient from inefficient calculator procedures and when calculation using pen and paper might be warranted. Kutzler (2000) notes that it is not unusual for students to use technology when pen and paper approaches would be more efficient.

Another type of difficulty in a CAS environment is that syntax and commands sometimes require expanded views of the conventional meanings for them. For example, a symbol (for example, 'y') that might conventionally be thought of as a 'variable', can call up a unique value in the calculator because it was stored in the y memory (Heck, 2001). This is particularly true of CAS calculators, which usually still incorporate aspects of traditional scientific calculators. However, once a command is applied to an expression, the notion of 'variable' gains a particular meaning. For instance, the derivative command implies expressions are functions of the variable with respect to which one is differentiating.

The CAS 'Solve' command requires an understanding that this command is an algebraic command and so its application to an expression may result in a numeric value but can also be used to isolate a variable in terms of other variables and parameters. Drivjers and van Herwaarden (2000) describe the difficulties students experienced in accommodating this idea. These include confusion with specifying the variable for which to solve. Student difficulties with the CAS in part revealed but did not cause weak understanding of the nature of parameters. They highlight that conceptual understanding needs to be addressed. Other recommendations were that technical aspects of solving on the calculator need explicit attention in class and that effective use of the tool, in general, requires that students know the expected form of the output.

Going beyond consideration of single applications, Guin and Trouche (1999) and Lagrange (1999) suggest that some of the calculator approaches recommended in the literature could mediate against conceptual development in a domain. Examples are that inferring limits from graphs on the calculator and using the limit command to yield limit values might create obstacles to expert conception of limits; and deducing rules, for example for derivatives, from numerical outputs of CAS commands encourages rule based understanding and not conceptual understanding. Imperatives for research include the range of pen and paper and technology approaches to introduce a given concept, for productive learning (for example, Kendal and Stacey, 2001); and how to scaffold students' use of CAS, particularly for weaker students (for example, Galbraith and Pemberton, 2002).

Resources the CAS calculators afford for action

A potentially valuable aspect of a CAS for learning and problem-solving is the ability to quickly produce multiple examples to support inquiry and conjecture. The accuracy of the output is an added advantage (provided data are entered correctly and commands are used appropriately). Examples include combining symbolic and graphical approaches on the calculator to explore the notion of derivative (Lagrange, 1999) and using graphs to explore Riemann integrability (Kawski, 1997). On the basis of teaching experience, Kawski notes how inquiry with visual approaches can lay the foundation for the development of analytic methods and Kidron (2002) observes that visual approaches allow the exploration of intuitive ideas in a way that algebraic methods do not. In the affective domain, Pierce and Stacey (2001) observed that the emotional neutrality of technology was beneficial for inquiry approaches because students could explore without fear of embarrassment.

In learning (as distinct from problem-solving), Kutzler (2000) observed using a CAS allowed students to concentrate on new techniques while passing assumed ones to the technology. A CAS can support learning by allowing students to concentrate just on one task such as the performance of equivalence transformations without worrying about the need for simplification and, once competence is developed, pen and paper computation can include other processes.

Kutzler (2000) suggests also that a CAS can be an excellent compensation tool that allows less gifted students to deal with advanced topics. This role of the CAS is also discussed by Geiger, Galbraith, Goos and Renshaw (2002) who give the example of a student using the algebraic capabilities of the CAS to proceed to other aspects of a solution. However, such use of a CAS might amount to blackbox use that many argue against (for example, Bloom and Bloom, 2000): the absence of mathematical understanding necessitates blind acceptance of the output, without the critical interpretation that is recommended. Leinbach (2001), on the basis of classroom experience, argues that advantage to the weaker student might only be limited to the use the CAS as “an expanded answer book for accomplishing mechanical and manipulative tasks” (p. 131) without advancement in understanding.

At least for the more able students, it is widely reported that the CAS allows more complex and more realistic problems to be tackled. Leinbach (2001) gives the example of students recreating an historical development of an algorithm for finding the roots of a cubic equation. Kidron (2002) also describes students recreating history, this time Euler’s algebraic method for expanding functions as power series, where both the graphical and symbolic capabilities of a CAS were used. Here students’ appeared to gain confidence when they could visually confirm algebraic results.

Additional features of CAS that are potentially beneficial for learning are programming and animation capabilities. The possibility of programming to extend the capabilities of a CAS allows customisation for special applications. Alexopoulos and Abraham (2001) argue that customisation of CAS languages that are relatively ‘natural’, is an ideal way of introducing students to programming. Blyth and Naim (2001) describe the use of animated diagrams in Maple for problem solving and although they see it is easier to use only algebraic methods, they recommend the animations because of their visual appeal to students.

Student profiles

A variety of student approaches to CAS use are presented in the literature. Guin and Trouche (1999) and Goos, Galbraith, Renshaw and Geiger (2000) describe low level use where trial and error procedures are common, and outputs are accepted without verification. At a second level, students rely heavily on the technology for calculation. Reasoning is based on the accumulation of consistent machine results and some checks may be instituted. At a third *resourceful* level (Guin and

Trouche), or working in *partnership* with technology (Goos et al), multiple information sources are explored and students balance the authority of mathematics and the technology-outputs. Goos et al. define a fourth, high level of use as comprising *an extension of self*: CAS becomes an integral part of activity and the student's mathematical performance is extended. Galbraith and Pemberton (2002) also describe that use of a CAS can allow students to extend their mathematical capabilities. In their view this relies on sound mathematical understanding, and the extension occurs through using the CAS as a tool for computation, rather than for learning. Leinbach (2001) expresses similar views to Galbraith and Pemberton.

Guin and Trouche (1999) define two other student approaches to CAS use: a *rational* work method, characterised by reduced use of the technology and more emphasis on paper and pencil approaches, and a *theoretical* work method characterized by heavy and systematic use of theory, which was also applied to guide technology use. Moreover, they observed that in the long-term students who adopted a rational work method used CAS calculators more efficiently than those who strongly favoured calculator use and that the ways in which students accommodate the use of CAS calculators seems to proceed from the ways they use graphics calculators. According to Guin and Trouche (1999) accommodating CAS syntax and commands occurs in two phases: discovery of various commands, their effects and organisation and then students focus their attention on a limited number of commands and coordinate their use with other information sources.

Overall, CAS use by students (as distinct from teacher demonstration) is seen to lead to more student inquiry and discussion between students, and hence more control by students over their own learning (for example, Kidron, 2002; Pierce and Stacey, 2001).

Teacher responses to CAS technologies

Three themes dominate in the literature on teachers' responses to introducing CAS for student use. These are time and other pressures, teacher privileging and emergent classroom practices. Lumb, Monaghan and Mulligan (2000) identify issues that include gaining access to computer rooms, time to get to know where CAS might be useful, time for planning lessons and time for writing worksheets. As well, students need time to learn to use the system to the level of competence where they spontaneously choose to use it to solve problems. Vlachos and Kehagias (2000) and Blyth and Nairn (2001) are others who make similar comments. Monaghan (2002) suggests that to relieve the pressure on teachers, electronic templates of activities could be made available, and teachers could be modify them to suit their style and needs.

Kendal and Stacey (2001) and Drivjers and van Herwaarden (2000) observe that the methods and representations evident in students' individual work are the methods and representations the teacher presents or privileges. However, the power and scope of the suggested approaches might be limited and can be explained by teachers' beliefs about learning. Kendal and Stacey (2001) give the example of a teacher who promoted the CAS for exploring the symbolic-graphical link because in his view it aided students' conceptual understanding, but discouraged it for symbolic computation as he believed that it did not assist understanding.

Emergent practices include a move away from expository teaching and towards student inquiry (for example, Pierce and Stacey, 2001) and the variety of different ways teachers use the overhead panel attached to their own or a student's calculator (Goos et al., 2000). The set up with the overhead seems to encourage class discussion and enables the teacher to be aware of students' uses of the tool so that problems can be addressed.

CAS, assessment and the ‘key skills’ agenda

In the literature we reviewed there is general agreement that use of CAS in teaching and learning must be accompanied by an inclusion of the CAS in assessment in order to maximise learning outcomes and to encourage acceptance of the CAS by students. At the same time, and partly because of the need for valid assessment, the question needs to be considered as to what are the key skills students need to demonstrate in the CAS age. In the UK the *Key Skills (Core Skills; Generic Skills)* are those identified as underpinning good practice in the labour market and workplace, now and in the future. They are to be distinguished from *Basic Skills* but are deemed to build on Basic Skills. In Australia these are known as *Key Competencies*. Galbraith and Haines (2001) identify the Key Skills relevant to CAS as Information Technology, Application of Number and Problem-Solving whereas the Key Competencies relevant to CAS are listed as using mathematical ideas and techniques, using technology and solving problems. At the heart of the question of what are key skills is the more fundamental question as to what constitutes ‘doing mathematics’.

For Gretton and Challis (2000) the central questions in relation to skills are what exactly is doing mathematics and what is the objective of teaching mathematics. For them mathematics is not concerned solely with carrying out mechanical algorithms but includes the communication of ideas and the solution of real problems. They categorise the ‘key and transferable skills’ as communication, solving problems, working in groups, using IT, improving one’s own learning and application of number. They also suggest that, with the ready availability and affordability of the CAS, the perception of doing mathematics can move from symbolic manipulation to the communication of ideas, not only with people but also with machines. Peschek and Schneider (2001) also oppose the view that mathematics equals calculation. They advocate setting up a theoretical frame of reference within which to assess the experiences and considerations of using a CAS. They take the stand that which basic knowledge and which concrete basic skills should be taught at school has to be set by society and experts in a process of negotiation.

Leinbach Pountney and Etchells (2002) espouse similar views. For them ‘doing mathematics’ is mainly concerned with reasoning and problem solving and calculations are only a means to an end. In keeping with this view they distinguish three classes of mathematical skills where the skills commonly associated with calculations are part of the most basic group. This group, Group A, concerns technical skills, memorization of facts and the use of templates, Group B involves the information transfer to new situations while Group C requires understanding including justification and formal proof. They illustrate their scheme with examples from elementary calculus and conclude that the tasks essential to mathematical analysis and reasoning are still performed by the student, with the CAS being used only as a tool. In a related paper Pountney, Leinbach and Etchells (2002) raise the question of appropriate use of CAS in assessment. Using the scheme in Leinbach et al (2002) they define the development of Group C skills as the goal of mathematics education and argue that any appropriate assessment tool has to include the testing of items that belong to the Group C category. They also contend that the introduction of CAS has trivialised some assessment tasks and that the availability of a cheap CAS option cannot be ignored but must be taken advantage of to avoid instruction in mathematics being viewed as redundant. Thus they argue for a change in assessment to include CAS and discuss how assessment items ought to be constructed so that they are neither trivial nor so hard as to exclude almost everyone but the most skilled problem solvers. To ensure that an average student is able to pass an examination, questions need to be constructed in such a way that the basic skills testing precedes the testing of group C skills. Assessment items constructed in this way should lead to students of all abilities gaining maximum benefit from the use of CAS at their level. They give examples of assessment items and sample solutions demonstrating differing levels of achievement to illustrate their arguments. In their

experience assessment items thus constructed do not have any impact on the number of students who pass. However in their view the inclusion of CAS enables one “to teach the mathematics that we profess to be our goal”.

Irrespective of whether the main concern is secondary or tertiary education almost all authors argue that the inclusion of the CAS enables one to test higher order skills. Brown (2001a) expects the CAS calculator to have a greater impact on mathematics assessment than graphics calculators because many of the standard processes in the mathematics classroom are automated in this new setting and so the question of how and what gets assessed will require serious consideration. Leigh-Lancaster and Stephens (2001) examine the role of state examination boards with regard to the CAS inclusion. They are concerned with the organisational response to the introduction of CAS in secondary education systems. They distinguish three possible responses: no change, partially differentiated examination questions or the piloting of an innovative curriculum and assessment package. The last approach is currently being tested in some upper secondary classes of the Australian state of Victoria. Here the introduction of the CAS into the curriculum was accompanied by redesigned examination questions.

The issue of the impact a CAS has on examination questions is discussed mainly in the secondary school context. It has given rise to several classification schemes in much the same way as the inclusion of graphics calculators in assessment. Examples of such schemes are those discussed in Monaghan (2000) for the A-level exams of the UK and Kokol-Voljc (2000) in a more general setting. Monaghan summarises the early stages in considering the implications of the use of CAS calculators for traditional examinations. He describes a classification into questions where the CAS calculator has no impact and questions where it might give the candidate some advantage over a non-CAS user. He notes that the degree to which an area of the UK A-level syllabus would be affected by the use of a CAS calculator would vary from topic to topic and reports on attempts to construct examination papers that embrace CAS use. He observes that it is harder to construct examinations that make full use of CAS than those that bypass it and discusses some of the techniques used in the CAS-active sample paper. These include more structured questions, more use of parameters, more basic principles and more contextual questions. One of the problems he raises is the potential danger that CAS-enabled examinations become too hard.

Kokol-Voljc (2000) classifies questions as *CAS-insensitive questions*, *questions changing with technology*, *questions devalued with CAS* and *questions testing basic abilities and skills*. She observes that the main effect of the CAS inclusion for questions changing with technology is a shift in emphasis from “technical/ mechanical/routine” work to “mathematical/semantic/conceptual/applications work”. Questions devalued with CAS are those that test essentially algorithmic skills, such as the evaluation of limits, or the calculation of derivatives from rules. Once a CAS is available, the value of such questions in providing feedback to students and teachers on the performance of operations is replaced by feedback on the “technical ability to use a CAS”. The same applies to questions in her fourth category. An alternative classification she discusses is one based on the role the CAS use plays in the solution of a problem: *primary CAS-use*, *secondary CAS use*, *routine CAS use*, *advanced CAS use*, or *no CAS use*. She maintains that the “major educational goals of mathematics assessment are the testing of students’ achievement in understanding of mathematical concepts and their use in modelling and that there is confluence between these goals and areas in which CAS is most useful in teaching and learning”. She argues for careful planning of questions to ensure that the achievement of these goals is tested and implies that the proposed classification schemes are useful to facilitate this.

What then happens to the manipulation skills that students were to demonstrate in assessment? These are often tested in the questions that fall into the second and fourth category of

Kokol-Voljc's classification. While one might argue that requiring a demonstration of these skills alone in an examination question is of little value other than to show technical competence, that might not be linked with mathematical understanding, questions of this type can be accommodated in a two-tier examination. Herget, Heugl, Kutzler and Lehmann (2000) also note the distinction between 'perform an operation' and 'choose a strategy' and advocate a two-tier examination, one part of which would be 'calculator-free' with no calculator of any kind permitted. Kutzler (2001) expands on this notion and sees such exams as a well-balanced compromise meeting both the desires of technology supporters and the reservations of those who are concerned about the use of technology in the classroom. Kissane (2000) notes that such examinations potentially defeat the purpose of allowing CAS systems but regards it as a potentially useful interim measure and it is the approach taken in Denmark is (see Brown, 2001b). There, since 2000, two written exam papers are required for the Upper Secondary School Leaving Examination. One paper allows the use of CAS calculators and the other allows no calculator at all. A two-tier approach is also discussed by Forbes (2001) with a split of approximately 1:2 for the non-calculator versus the calculator paper. He acknowledges that a split into two types of papers is far from ideal and that in the new circumstances alternative assessment procedures, for example oral examinations, might be appropriate. In relation to pen and paper methods, Lee and Sabarudin (2001) who discuss the use of the CAS in their calculus units in Singapore allow students to pass some of the heavy manipulation to the CAS but only if they can still solve simple algebraic problems by hand and can show understanding by demonstrating intermediate steps.

Torres-Skoumal (2001) argues that the introduction of CAS necessitates a re-evaluation of the goals of mathematics education and that this needs to be reflected in the types of assessment. She believes that the use of group work can be of benefit for learning in the presence of technology and gives assessment criteria for group work that address the following areas: content, communication, presentation, use of technology, results, conclusions and extensions. In addition she discusses the potential benefit of the CAS for self-assessment. The examples she gives focus largely on algebraic equivalence and the need to explain apparently different results obtained from paper and pencil manipulation and CAS-commands.

On the other hand, Balderas-Puga (2001) regards the use of CAS as a perfectly good way to ensure more complete testing of the syllabus in conjunction with giving students access to a greater variety of examples.

Concluding Discussion

We have seen that the inclusion of CAS in mathematics teaching raises many issues. With regard to pedagogy the foremost of these is the need to teach students how to use the CAS and how to critically evaluate any associated output. In particular, students need to learn syntax and commands and the meaning of the commands that might be wider than conventionally assigned meaning, as well as the organisation of the CAS. Another important question is when to use CAS. This judgement needs to be the subject of instruction as it is not necessarily apparent to students whether or not the CAS will be of benefit (see for example Malabar and Pountney, 2000). Pierce and Stacey (2002) also note that the value of the CAS depends on how effectively it is actually used by the student. In the long-term, effective and efficient use of a CAS by students needs to be underpinned and guided by sound knowledge of mathematical relationships and basic mathematics processing structures. The development of appropriate teaching materials is time consuming for the teacher. In addition, major considerations are the contribution of individual tasks to conceptual development in a domain, and how best to scaffold weaker students use of the technology.

Assessment using CAS will require change in the assessment practices. This is apparent when considering current CAS-free assessment items and the classifications discussed earlier on have made this explicit. The redesign of assessment procedures draws one to the discussion of how to assess the use of CAS, what use of CAS to assess and what manual manipulation skills should be retained and tested. A recurrent theme in the literature in this context is the two-tier examination mode. However, we question the value of such an approach. Apart from noting the obvious difficulty of getting any broad agreement on what might go into a calculator-free exam, we do not share Kutzler's (2001) view that paper and pencil calculations are necessary for mental training and need to be separately examined. . We agree though, with others (for example, Pierce, 2001) that there is a need to develop algebraic insight and this development should be assessed.

Imperatives with the introduction of CAS are a reexamination of the goals of mathematical education and the skills to be taught. Some of the papers we discussed provide useful starting points for these debates. We would argue that the benefits of the inclusion of CAS by far outweigh the potential downsides even though we acknowledge that those who are going to benefit most might be those who have the greatest mathematical ability while others will have to contend with new difficulties and impasses. Care must be taken to minimise the difficulties through thoughtful didactics. While learning to use a CAS is potentially difficult and might add to the learning burden of the user, widely documented tradeoffs are that the technology can support inquiry and conjecture and can expand students' problem-solving repertoire.

In the end, we need to acknowledge that mathematics is more than the manipulation of symbols which is all that a CAS can provide for us. However, we concur with Peschek and Schneider (2001) that not using CAS can be construed as a denial of the technological development of mathematics and of society and is difficult to justify to today's youth.

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