VA*Trans*: A Model of Self-Designed Learning Software That Helps Learners to Promote and Engage Themselves with the Visualization and Analysis Thinking

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Abstract:

A meaningful learning of mathematics commonly involves mental activities such as exploring, constructing, applying relationships in logical sequences and forming intuitive understanding about particular concepts. Many mathematics educators suggest that the success of these kinds of activities depends on how one engages himself/herself with two important types of mathematical thinking namely, *Visualization* (referred as V) and *Analysis* (referred as A). It is commonly said that effective combination, or correlation rather, of V and A will lead to effective and meaningful learning of mathematical concepts as well as other higher order thinking skills. The role of V and A is more imperative in the learning of mathematics topics that require more spatial configuration and analytical understanding.

This paper describes an attempt of making use the advantages of computer technology to assist learners to promote and engage themselves with these two types of mathematical thinking. A VA-oriented computer-based learning software called *VATrans* was designed and developed to act as a learning tool for students to learn a lower secondary level of Geometry of Transformations. The software was tested to a group of eight secondary students and a series of analyses of effectiveness was then performed accordingly. The analyses suggested that *VATrans* has shown its promising potential in helping learners promoting and engaging themselves with the VA thinking in that particular topic.

1.0 Introduction

Teaching of mathematics especially in secondary level is designed and planned to provide a practical learning experience for the students to understand mathematical concepts through a meaningful way. This is to be achieved by means of active involvement of mental activities such as searching, constructing, applying relations logically and constructing the understanding of concepts intuitively. These activities are commonly related with two modes of mathematical thinking, namely the *Visualization Thinking* (simply called Visualization or V) and *Analysis Thinking* (simply called Analysis or A). (see for example, Krustetkii, 1976; Vinner, 1983; Eisenburg & Dreyfus, 1986

and 1991; Sharma, 1991; Zazkis *et al.,* 1996). It is often said that the success of mathematics learning is related to one's ability to engage himself or herself with both V and A in a combined and correlated mode. For the purpose of simplicity, we would call this mode of mathematical thinking as *VA thinking*. The importance of VA thinking is more imperative in certain mathematics topics requiring more mental abstraction and space configuration such as Geometry of Transformations.
As mentioned earlier, both V and A are associated with mental activities in mathematics learning, the first is related to activities to generalize or form abstraction of concepts *intuitively* whereas the latter is related to activities to generalize or form abstraction of concepts *logically*. In this perspective, VA thinking can be simply described as mental activities to generalize or form abstraction of concepts *intuitively* and *logically.* The V, A and VA are discussed in details in Krustetkii, 1976; Eisenburg & Dreyfus, 1986 and 1991; Sharma, 1991; Zazkis *et al.,* 1996; Zimmermann & Cunningham, 1991).

On the other hand, the use of computer as a teaching and learning tool has brought a lot of changes to the teaching and learning process as compared to traditional approach. For instance, many mathematics educators utilized the high capability computer technology to generate and display graphical dynamic visualization to help learners understand concepts that require visualization as well as promoting such mental activities in the mathematics learning. The advantages and potentials of the use of computers as learning tool that helped students to construct and promote understanding of mathematics concepts have been described in detail by many mathematics educators and researchers (see for example, Dubinsky & Tall, 1991; Tall, 1991a and 1991b; Zimmermann & Cunningham, 1991; Zimmermann, 1991; Cunningham, 1991; Artigue, 1991; Zazkis *et al.*, 1996; Zaleha Ismail, 1997; Tan Wee Chuen, 2000).

This paper describes an attempt of making use the advantages of computer technology to assist learners to promote and engage themselves with the VA thinking. A VA-oriented computer-based learning software called *VATrans* was designed and developed to act as a learning tool for students to learn a lower secondary level of Geometry of Transformations. The software was tested to a group of eight lower secondary students and a series of analyses of effectiveness was then performed accordingly to see if its potential in helping learners promoting and engaging themselves with VA thinking in that particular topic.

2.0 The Framework of Design and Development of VA*Tran*

The VA*Trans* learning software was primarily designed and developed as a prototype that would provide a theoretical and developmental basis in the design of learning software that would assist learners to foster, promote and engage themselves with the VA thinking. For certain practical reason, VA*Trans* was designed and developed to serve this purpose in the learning of mathematics topic of Geometry of Transformations. With this in mind, it is vital to note that VA*Trans* is neither complete nor a ready-to-use learning software without designer's prescriptions.

The VA*Trans* prototype was developed by using the Visual Basic 5 (VB5). VB5 is an object-oriented programming language with high capability to provide a flexible programming and accurate calculations (Harrington, 1997). It works well in windows environment and is capable to develop interactive programs with its wizards and control tools (Wallace, 1995; Harrington, 1997).

Besides, VB5 also supports the database programming and can generate object transformation process visually and analytically.

The design of VA*Trans* prototype was focused on the promotion and engagement of learners with the VA thinking. As far as V thinking elements are concerned, this involved graphs, lines animations, images of shapes (constructed from 6 points), use of various colors to highlight the object, images and processes of an object transformation. As for the A thinking elements, *VATrans* comprised of generalization of the transformation properties, guided questions to analyze the image properties of each transformation and labeling of each point in a shape. A specially designed electronic form was also provided in each subtopic of transformations which would guide the users to generalize the relationship by constructing rules or equations of the properties and location of images under a particular transformation. In addition, *VATrans* was also equipped with data logging feature that would enable the researcher to record and retrieve every input provided by the users.

The *VATrans* was primarily divided into 2 main parts, namely Tutorial Part (TP) and Practice Part (PP). The parts we described here were actually the screen display designed in *VAT*rans. Such design was based on the consideration of the mathematics contents as well suggestions gathered from a series of discussion and with experienced mathematics teachers. Each of these parts comprised of three main topics of Geometry of Transformations, namely Translation, Reflection and Rotation. Specific examples of VA elements designed in the *VATrans* are shown in Figure 1 to Figure 5. The short descriptions of these examples are given below whilst the full details are available in Tan Wee Chuen (2000).

The TP of Translation comprises of two parts. The first part is Symbolic Translation. The objective of this part is to introduce the symbols of translation values through the visualization of the movements of various shapes and changes of the translation values. The second part is Translation itself. In this part, the students can explore and test their ideas by drawing the object and observe the translation process dynamically (see Figure 1). A special part was designed (embedded in the VA*Trans* itself) to guide the users to generalize their exploration analytically through the generalization of the meanings of symbolic translation, properties of translation and formulas of image location in two-dimensional coordinate system. This was to encourage them to discover and express symbolic patterns involving in Transformation (see Figure 2 and Figure 5). The PP in VA*Trans* comprises Translation, Reflection, Rotation and Inverse Transformation. The PP will enable students to test and check their ideas in each subtopic by drawing the object and image based on certain transformations input from students (see Figure 3, Figure 4 and Figure 5).

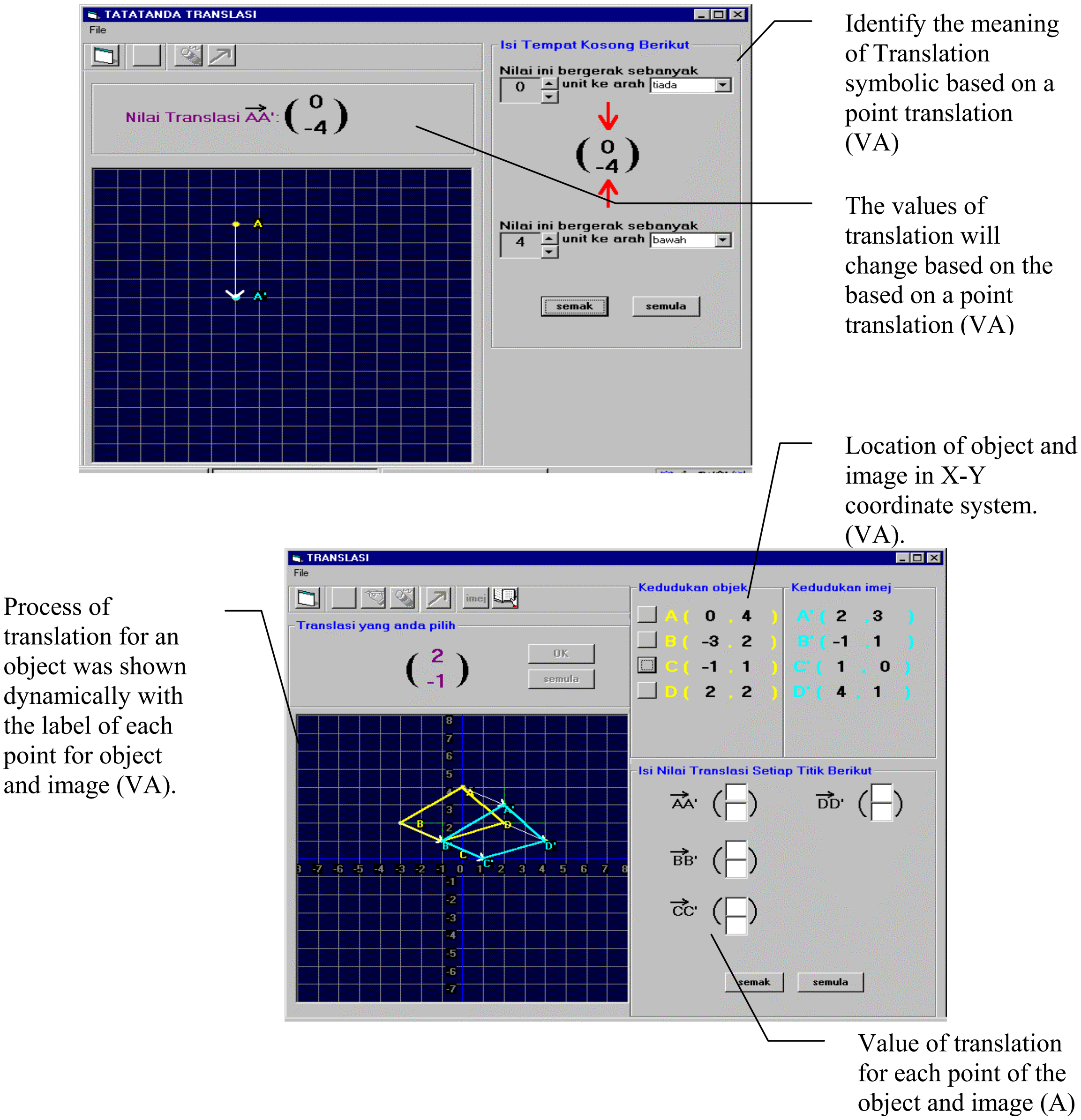


Figure 1: Tutorial Part of VA*Trans* in Translation

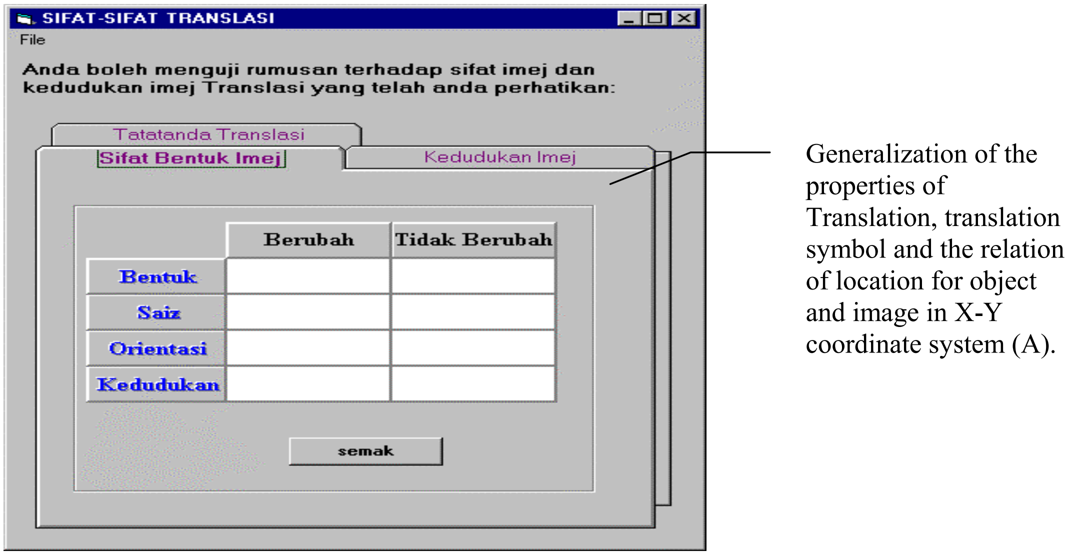


Figure 2: Generalization Part for Translation Tutorial System

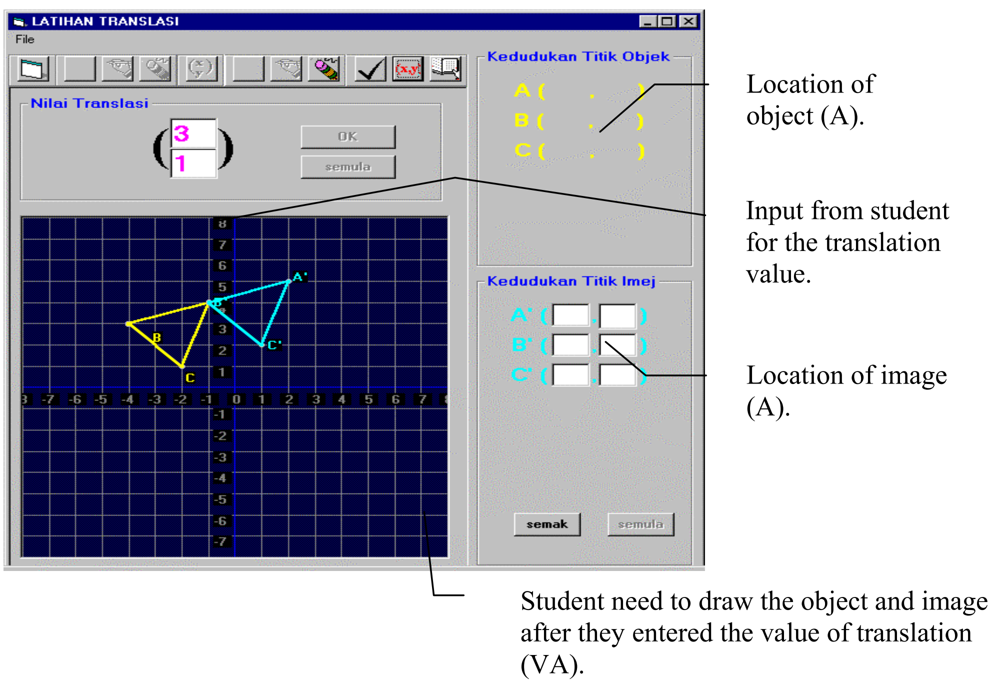
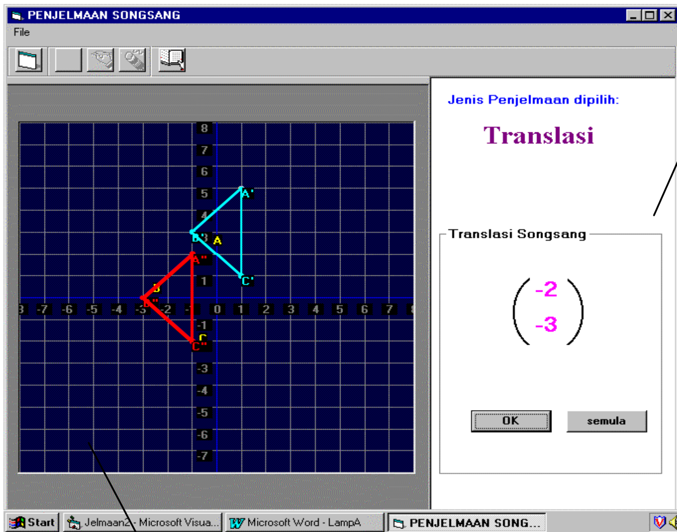


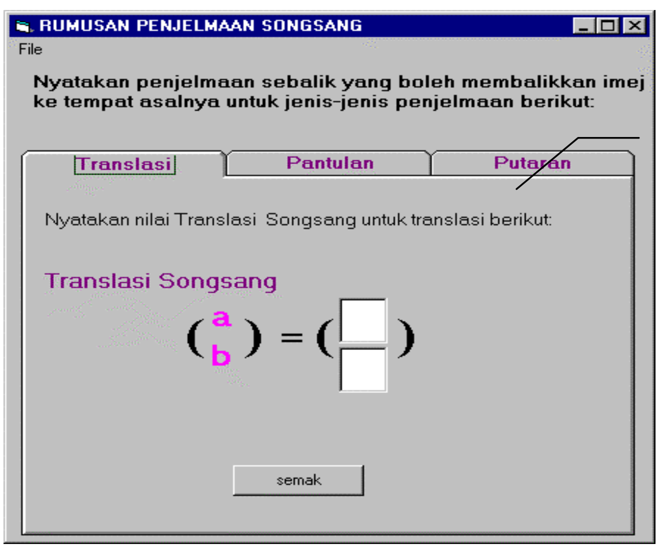
Figure 3: Translation Practice Part



Student entered the value of inverse translation (A).

The object and image of inverse translation with the labels for each point. (VA).

Figure 4: Inverse Translation



Generalization of the relation between the value of translation and inverse translation (A).

Figure 5: Generalization Part for Inverse Translation

3.0 Does VA*Trans* Work?

The Design of Analysis of Effectiveness

As mentioned earlier, VA*Trans* was tested out to a group of eight students from a selected secondary school. The primary objective was to study its potential in helping learners promoting and engaging themselves with VA thinking in that particular topic. The subjects were chosen from a pool of students who already have learnt the particular and voluntarily willing to participate in the study.

This analysis of effectiveness was in three consequential sessions, namely Session I, Session II and Session III. In Session I, the subjects were first welcomed, comforted and briefed with the necessary information about the study such us the objectives of the study, why they were chosen and what they were expected to do. They were then given a 30-minute paper and pencil test containing tasks that were designed to measure VA thinking in that particular topics. The test was then quickly checked and a series of interviews with the students were conducted subsequently to find out more about the VA thinking.

In Session II, the subjects were asked to explore the TP part of VA*Trans*. The interaction of the users with the activities designed in this particular part was observed and every input was recorded in the data logging. This was subsequently followed Session III which required them to go through the worksheets through the use of PP of VA*Trans*. Once they have completed these activities, they were asked to work on another set of paper and pencil test followed by a series of interviews. It must be notes that the reasons of having the test and interviews remained the same, i.e. to measure VA thinking but this time after they have undergone prescribed activities in these particular two sessions embedded in VA*Trans*.

All the information gathered from these various sources i.e. paper and pencil test, interviews and data logging were then analyzed using both quantitative and qualitative approach to see changes of VA thinking among the users prior and after the use of VA*Trans*. The VA*Trans* would be considered to be 'working and potentially good' if it could generate changes in the light of helping the users to promote and engage themselves with VA thinking in that particular topic. A brief description providing a basis on how the VA thinking was investigated is shown in Annex 1 (modified from Krustetkii, 1976; Vinner, 1983; Eisenburg & Dreyfus, 1986 and 1991; Sharma, 1991; Zazkis *et al.,* 1996; Tall, 1991a and 1991b; Artigue, 1991; Zimmermann & Cunningham, 1991) whilst the full details of them can be found in Tan Wee Chuen (2000).

The Results

For the purpose of simplicity, we will discuss the effectiveness of VA*Trans* by examining the changes related to VA thinking prior and after the use of VA*Trans.* Again, the full findings of this investigation are reported in Tan Wee Chuen (2000).

The investigation of information gathered from the paper and pencil test and the interviews strongly suggests that at the initial stage (i.e. prior the use of VA*Trans*), most of the students seem to be engaged with the VA thinking in a very clear dichotomous mode. As a result, most of the students were unable to solve the problems in reflection and rotation as they couldn't imagine the shape of image which certainly required them to think both VA modes. The lack of VA thinking was manifested by their complaints that these problems were hard to solve they could not perform appropriate mental imagination and there were no formula for them to find the image of reflection and rotation!

Specifically, the information gathered from the test suggested that many students focused on visual features and representation for a particular shape under a particular transformation displayed on screen. They focused on the changes of the physical feature such as orientations, location and shape of image after transformation. However, this phenomenon changed gradually during and after the use of *VATrans*, i.e. Session II and Session III where they gradually focused their attention on the process of transformation displayed on screen. In fact, they were able to describe the process of transformation and the changes of physical feature for image of transformation when they were asked to explain a particular transformation. For example, first they would describe the physical feature of image (i.e. V thinking according to our description as described in Annex 1), used color and labels of the object to keep track the process of transformation (i.e. A thinking according to our description as described in Annex 1). However, when the students were asked to solve the Inverse Transformation in worksheet, we found that there was a tendency for students to over-generalize their findings of the formulas they had formed or formulated. For instances, they tend to apply the pattern or formula they seen in Inverse Translation to Inverse Reflection and Inverse Rotation. Nevertheless, they seem to notice this as they work further with the prescribed activities and finally able to correct it. In particular case, when the students were asked to explain this situation, they initially thought that the formula could be applied in others Inverse Transformation. As they tested their idea in VA*Trans*, they realized that the have made errors out of such generalization. Typically, they resolved their errors from the feedback VA*Trans* and reattached meaning on the formula they generalized in worksheet. As a result, the students were able to describe correctly the Inverse of Transformation using the visual and analytical representations.

In general, we found that there was a significant shift among students in terms of VA thinking in learning the concepts and processes related to Geometry of Transformations, from being dichotomously visual or analytical to visual plus analytical. The shift became more imperative as they explored the activities prescribed in VA*Trans*.

4.0 Conclusion

By in large, this little attempt described in this short paper strongly suggests that it is feasible for us to practically make use of the advancement of computer technology to enhance or provide alternatives in the learning processes in mathematics. Of course, VA*Trans* is far from complete; neither is self-sufficient to replace the roles of mathematics teachers. However, the writers are hopeful that VA*Trans* could provide a theoretical and developmental basis in the design of learning software that would assist learners, especially to foster, promote and engage themselves with the VA thinking in the process of mathematics learning.

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Annex 1: Summary of The Basis of Investigation on V, A and VA Thinking Amongst Students During The Learning of Geometry of Transformations*

Types of Thinking	Components	Descriptions
V	V_1	View or explain the transformation process as the whole physical of certain shape(s)
	V_2	Construct or represent the mental images of transformation externally (e.g.: drawing by using external medium such as paper and computer screen)
	V_3	Construct or represent the mental images of transformation internally (e.g.: imagination in minds)
	V_4	View or explain the transformation process based on its properties such as location, orientation, size and distance of the objects and images
A	A_1	View and manipulate a particular transformation with points that formed the shape
	A_2	View and use the color, label or symbol to manipulate a particular transformation
	A_3	Apply rules or algorithm syntax (e.g. formula) to manipulate a particular transformation
	A_4	Display the tendency to view a particular transformation in various small parts

Notes:

* A learner is classified as to be practicing VA thinking if he/she engaged themselves with at least two components of V_i and A_i simultaneously