Evaluating and Improving Quantitative Literacy
~The Case of High School Students in Japan ~

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Abstract: In this study at a Japanese high school, students' quantitative literacy and especially their ability to utilize algebra and functions were evaluated. Most of the students are following a Humanities track rather than a mathematics or science track. The framework and questions of the study were based on a 1992 NAEP assessment (Dossey, 1997), but we evaluated our results taking the Japanese curriculum and daily learning environment into consideration. The purpose of this evaluation is to better understand Japanese students' performance compared to U.S. students and to offer some suggestions for assisting students to attain a higher level of ability. Some students were interviewed and they stated they were trying to improve their quantitative literacy through technological activities.

1. Introduction

Literacy now includes both a document aspect and a quantitative one (Dossey, 1997). Steen emphasizes that quantitative literacy is the new literacy for a data-drenched society (Steen, 1999).

Quantitative literacy means different things to different people. We can find some of their ideas in "Why numbers count" (Steen, 1997) or "What's QL/QR?" (Briggs, web site) (Fig.1). In all of them, there is the common aspect that literacy means the ability to use mathematical skills to cope with the practical demands of everyday life. Furthermore, it's said that the heart of quantitative literacy is real world problem solving (Henry, 1997).

Moreover, in reforms to the Japanese national curriculum standards, it is emphasized that students' ability to reason should be developed and for that purpose the problem-solving approach to learning will be positively adopted (Japanese Ministry of Education, 1998). Nevertheless, when many Japanese high school students meet problems in their daily lives which are open-ended or have many methods of solution, they are at a loss. They know how to solve equations, but they cannot formulate equations. They know a linear function but cannot connect it with real phenomena. It seems that they are not good at adapting mathematics to real

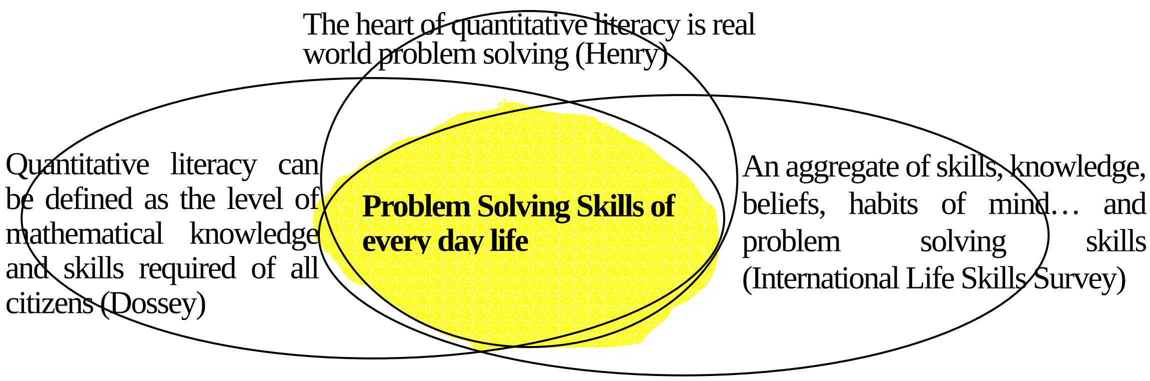


Fig.1 Some concise definitions of quantitative literacy

problems, and it should be noted that this ability is still not evaluated much in Japanese schools.

In the U.S., this kind of literacy has been investigated before, and the evaluating method and the result have been reported in detail (Dossey, 1997). Then, using the problem-solving research performed in the U.S., we investigated Japanese high school students' quantitative literacy.

2. Purpose

The following three points were investigated.

1) How much quantitative literacy does a Japanese high school student have?
2) What features characterize their performance compared with U.S. students?
3) What kind of changes would be performed in the environment where technology can be used?

3. Methods

The National Assessment of Educational Progress (NAEP) study focused on 12th grade students and their ability to apply mathematics to problem-solving. The study used a sample question in Dossey's report on reforming the teaching of mathematics. The evaluation levels for the question were clearly described.

Task-1

Question: The original problem was translated into Japanese and used in our research (Fig.2).

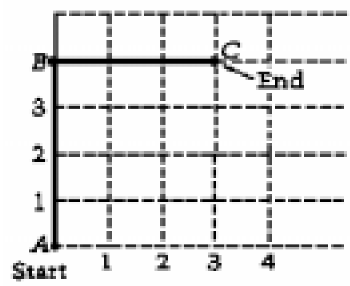
Subject: 76 students in the second grade of high school (16-17 year old students).

The high school at which they study is an average level Japanese high school. Most of these students are not taking a concentrated math and science course load.

Date and time limit: March, 2002. Students were given 30 minutes to answer the following question.

Problem: This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all your work.

The darkened segments in the figure above show the path of an object that starts at point *A* and moves to point *C* at a constant rate of 1 unit per second. The object's distance from point *A* (or from point *C*) is the shortest distance between the object and the point.



Question: In the space below, complete the following steps.

a) Sketch the graph of the distance of the object from point *A* over the 7-second period.

b) Then sketch the graph of the distance of the object from point *C* over the same period

c) On your graph, label point P at the point where the distance of the object from point A is equal to the distance of the object from point C.

d) Between which two consecutive seconds is the object equidistant from points A and C?

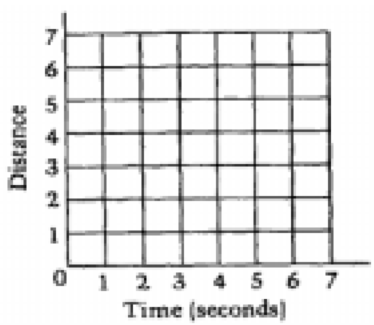


Fig.2 Original question (NAEP)

Task-2

Question: The original problem's mathematical content is oriented toward algebra and functions and the question proceeds along these content areas. But, other modeling and solution methods can be considered. As the ability to choose a model for this problem is also important, we extended the question and tested a new group of students (Fig.3)

Subject: 79 students in the second grade of high school (16-17 year old students). They are different students from those in Task-1 but they are also not taking heavy math and science courses.

Date and time limit: June, 2002. Students were given 20 minutes to answer the following question.

Problem: (see Fig. 2)

Question:

a) Explain the distance of the object from point *A* over the 7-second period. You may use drawings, words, and numbers in your explanation.

b) Then explain the distance of the object from point *C* over the same period.

c) Mark point P in the figure. Point P is where the distance of the object from point A is equal to the distance of the object from point C

d) Between which two consecutive seconds is the object equidistant from points A and C?

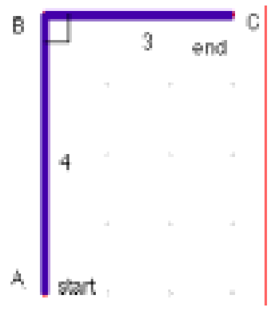


Fig.3 Extended question

Interview

A total of four students from Task-1 or Task-2 were selected and interviewed. While we asked them how to solve this program, some technical suggestions were given to them.

4. Results

Task-1

1) The NEAP rubric evaluation scale and The U.S. and Japanese scores are as follows (Fig.4).

Score & Discription
Extended Complete, correct answer
Satisfactory Both graphs are correct. P is located correctly or the time of equidistance is correct.
Partial At least one graph is correct. P is located incorrectly. OR both graphs incorrect, but P is located correctly.
Minimal At least 2 points are plotted correctly on at least one graph that is not just a reiteration of the position graph; i.e., a plot of distance versus time.
Incorrect/Off Task The work is completely incorrect, irrelevant, or off task.
Omitted Item No answer

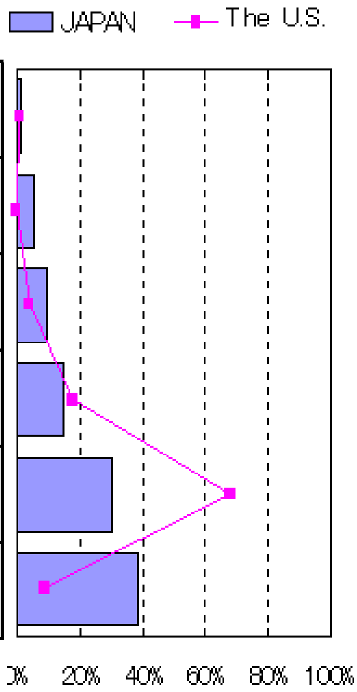


Fig.4 Result of Task-1

2) The percentage of correct answers is almost the same, but Japan has many students of a higher level.
3) The no answer rate is 38.5%, much larger than the U.S. rate.
4) 16% of students used the Pythagorean theorem to calculate distance. Compared with Task-2, it is a low value.
5) There were many graphs in which only the first and last points were calculated and then connected with a straight line even though this portion of the graph was a curve. Although some students were investigating the middle points, since they could not get the approximate value of the square roots, they connected the first and last points in a straight line.
6) The value of square root was inaccurate, some students got confused (Fig.5).
7) A student expressed a curve with a linear formula and solved point P by using simultaneous linear equations (Fig.6).

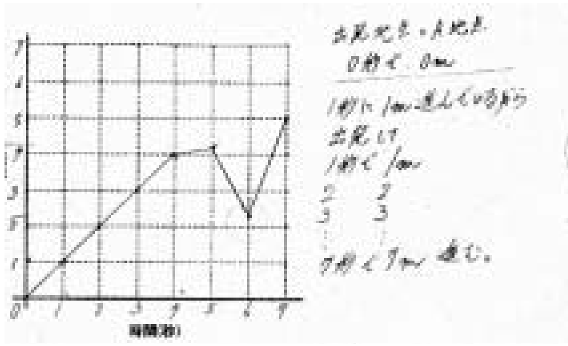


Fig.5 Students confused by incorrect square root

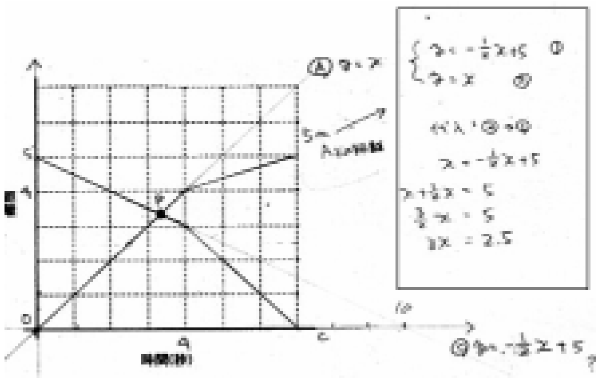
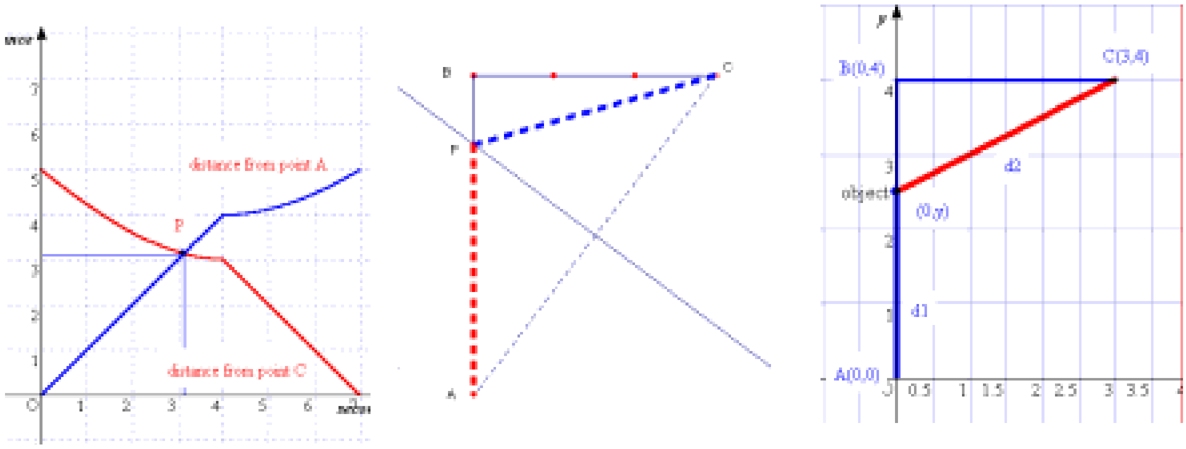


Fig.6 the simultaneous linear equations

Task-2

1) Before this investigation, the following three were expected as modeling and the solving method of this problem (Fig.7). The result was categorized by the way of approach (Fig.8).
2) Since how to solve was chosen freely unlike Task-1, the rate of no answer exceeded 42% and Task-1 further.
3) One student gave no answer but she had erased her incomplete correct solution. We interviewed her.
4) From the words "the equal distance", one student remembered the perpendicular bisector and considered a solution using geometry. Although no correct answer was achieved, it was interesting to see how the problem was modeled mathematically. We discussed this with the student.
5) About two months ago, the students had learned the formula which expresses the distance for two points. Although it was expected they would use the analytical geometry-method, no one used it.
6) The rate of students who calculated one or more distances by Pythagorean theorem is 32%. This higher rate seems to be related to the fact that the students had recently studied this point in class. However, most students only calculated the distance between A and C. The other distance value cannot be calculated easily since the object is moving.



1) Algebra and function

A table -> graph

-> read an intersection

OR

->formula of the graph

->simultaneous equations are solved

2) Geometry

perpendicular bisector of AC

-> the intersection of a way and this perpendicular bisector

3) Analytic geometry

Using formula of distance for two points

Object's position (0 - 4 seconds) (0,y)

Distance from A (0,0) to object

$$d = \sqrt{(0-0)^2 + (y-0)^2}$$

Distance from C (3,4) to object

$$d = \sqrt{(0-3)^2 + (y-4)^2}$$

Solving an equation.

Fig.7 The three solving methods we expected

How to approach

			%
Algebra and function	They made a table	17	20
	They made a table and plotted	3	
Geometry	Only one used this approach		1
Analytic geometry	No one did it		0
Not clear	They connected point A with point C	14	37
	They calculated distance between A and C	12	
	They expressed a change of distance	11	
No response	Three student computed something and erased all of them		42

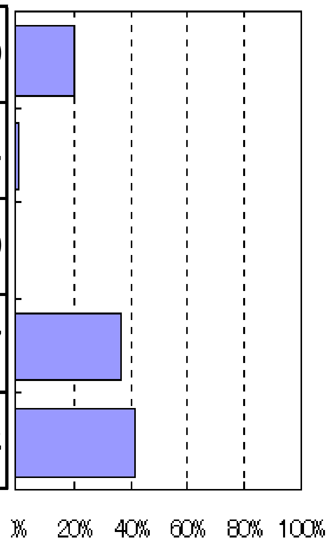


Fig.8 Result of Task-2

Interview - T= Teacher; S = Student

1) <u>The student who wrote nothing on the paper.</u>

T: Did you understand the shortest distance?

S: The length between two points connected by the straight line.

T: How do you write it by a figure?

S: (Figure was drawn.)
T: Why didn’t you explain?
S: I didn't know how to explain it.
T: While moving from B to C, how much of a change in distance is there?
S: Although the distance becomes longer and longer, I don't know how to calculate it.
T: Do you know the Pythagorean theorem?
S: Yes.
T: (connecting B and A with the object’s position at the 5 second point)
What do you think about this triangle?
S: $4^2 + 1^2 = ?^2$.

2) <u>The student who stopped although the table was made and a part of the straight line had been drawn.</u>
T: Why did you stop?
S: I did not know the value of $\sqrt{17}$.
T: Do you estimate it roughly?
S: It's more than four.
T: You understand approximately. If you use a calculator, you can calculate not only 4, 5, 6, and 7 seconds but also the times between them.
(We took many points and plotted them on a spreadsheet.)
S: Is this a parabola?

3) <u>The student who made a geometric model</u>
S: I imagined the same size circles drawn from A and C (Fig.9).
T: Where is point P?
S: The place where two circles touch. I think it's necessary to just draw a perpendicular bisector. P is good anywhere, if it is on a perpendicular bisector.
T: He is not only walking this way.

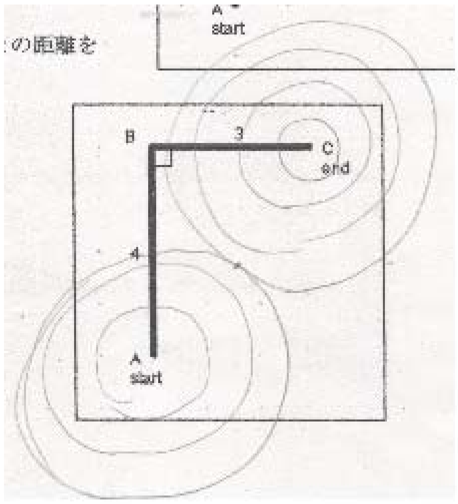


Fig.9 Geometric model

4) <u>The student who had changed her passive knowledge into active knowledge.</u>
She solved it by simultaneous equations (see Fg.6).
T: What did you think of this method?
S: I had solved such a problem before, and remembered using simultaneous equations.
T: Is this answer right?
S: Strange! It should not become the same distance within 1 second. (She did not look at the graph, although it was available. She did not use the graph for her explanation.)
S: I see! My calculation was incorrect.

(She tried to recalculate it.) 3.33 second. It's OK.

T: Is it a straight line? Because you know how to calculate a distance, let's investigate this period in more detail.

(She calculated and put her results into the spreadsheet)

S: It's drawing the curve a little.

(She put many lines in the figure and confirmed it.)

S: But since I do not know the formula of such a graph, I cannot solve this problem.

T: What do you want to do?

S: I want to look for the point that has the same distance.

T: Then let's calculate many distances and investigate them.

S: Is it near this point? I can approach this point closer and closer (Fig. 10). At same time, two points of the graph are also approaching. The point where the two graphs intersect is the solution of the equation.

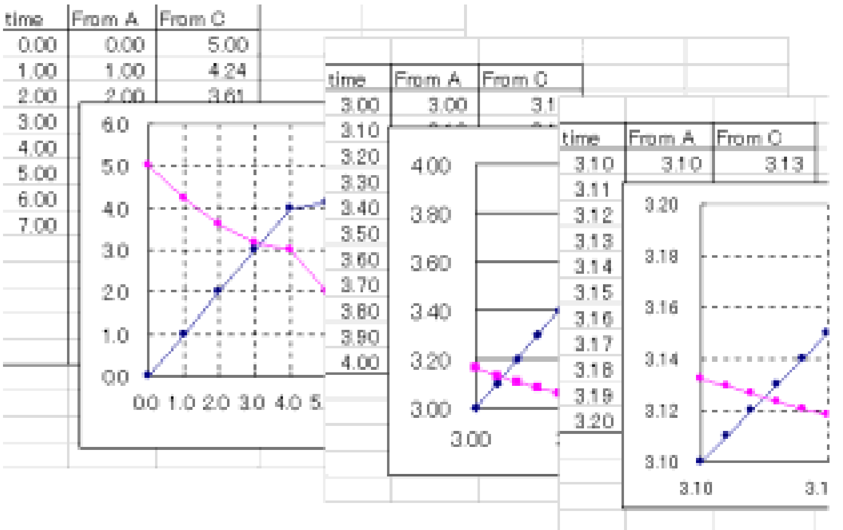


Fig. 10 understanding to solving an equation.

S: Although I recognized that solving the equation requires transforming the equation to the form "x=", I though the meaning is like this. If I use this method, many kinds of problems can be solved even though their formulas are not known or their formulas are more complicated.

5. Discussion

1) Although the Pythagorean theorem is known, students cannot use it where it should be applied. Even though some point of knowledge or method has been learned in a recent class, most students cannot put it to use. Even if it is applied immediately after the class, the rate of retention is low.
2) In the results for Japanese students, there are many students who provided no answers (Task-1: 38.5 %; Task-2: 42%) or did not write their idea clearly. While solving the problem, they had no firm confidence in the method they were trying to use. Some of them erased the solution before they finished it. In the case of Task-2, many students did not understand how to approach the problem and they were puzzled. However, if a solution method was suggested beforehand, as in Task-1, the students were able to achieve a remarkable level. The passive approach to problem-solving was sometimes obvious.
3) When taking up the two above mathematical problems in Japanese high school classes, they are usually solved with a system of equations, rather than drawing graphs and reading intersection P from the graphs. One student actually took this approach, even if she did not obtain the solutions. Since a deductive method is emphasized, most students think they must use equations to solve the problems. The fact that graphs can also be used confuses students and is one reason why they sometimes provide no answer or

don't have confidence in their idea.

4) In Japanese math classes, students are taught to solve problems using the best and clearest method. The students then memorize that method. Therefore, students believe that there is only one right method which must be used so they rarely apply the trial-and-error method. It is necessary to further evaluate how students solve problems, and to encourage them to more freely apply their ideas instead of responding passively in most situations.
5) Japan is ranked fairly high in math related research done by organizations such as IEA (National Institute for Educational Policy Research, 2001). The experiment described in this paper does not necessarily reflect the situation in all of Japan. The Japanese high school students sampled in this study were from only one high school and they are not taking extra math or science courses. However, if quantitative literacy is necessary to apply mathematical knowledge and skill in a real life problem, it cannot be said that quantitative analysis is truly taught in Japan.
6) In Japan, because almost all lessons follow the simultaneous style, there are few opportunities for the students themselves to acquire active knowledge by the discovery method. They tend to be given a lot of knowledge and that influence is reflected in the first half of this paper. Then, in the interview, we tried to change the passive knowledge given by the lesson into active knowledge by using technology.
   Active knowledge is demonstrated by;
   - performing correlation with other knowledge;
   - performing various expressions;
   - connecting with many examples of application;

   The girl in the previous example solved an equation using various approaches, substituted many numerical values and explored many graphs. She reconstructed the meaning of solving equations. She changed the meaning from transforming the equation in the form "x=" to looking for the point where two points take the same value. We can see this as an example of a student developing an essential and flexible understanding by herself.
7) While students were solving the problems, they sometimes lost their confidence and stopped trying even if they had made a good start. They were unsure how to begin complicated calculations or they were confused by wrong calculations. During the interviews, the following cases were recognized. When the value of a square root was determined by a calculator, students were able to achieve a deeper level of solution to the problems or they had more confidence in their solutions. They investigated more deeply using their own ideas and by examining feedback from their calculations. Moreover, calculator software which provides an easy graph to work with gave students more confidence to try new approaches.

6. Conclusions

1) Japanese high school students are not good at adapting mathematical knowledge to an actual problem even if they have it. Their knowledge is passive, not active. Although, according to PISA(PISA, 2000) a Japanese students' ability to apply mathematics has been ranked number one in the world, it seems that there are few students who are actually capable of using mathematical knowledge in real life situations.
2) In Japan, there are many students who do not have confidence in their own ideas and who are weak in

communicating their ideas and expressing their opinions. In problem- solving, they will use the stereotyped method which has been taught to them in school rather than trying to think for themselves. We think that their attitudes are also influenced by their teachers' evaluation style and how the teachers manage their classes.

3) The following cycle was recognized when students used simple technology such as calculators or graphing tools:
   - The students have a new idea
   - They confirm it with the technology
   - They are pleased to receive some reaction and has more confidence
   - They have further interest
   - They go to the next step

   It's possible that the students will then progress to the next step by themselves.

4) Simple technology had become the tool which changed passive knowledge into active knowledge, and the tool which supported students' own active attitude.

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