Elementary Prospective Teachers, Hundreds Charts, Number Sense, and Technology

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Abstract

The purpose of this study is to describe how a group of prospective elementary teachers explored a hundreds charts, with the aid of calculators to expedite and verify some basic number sense. By the term "hundreds charts", we mean the integers one through 100 arranged in a 10 by 10 array.
This group of prospective elementary teachers was taking a mathematics methods course during the year of 2001 in Taiwan. The author demonstrated how to operate on the use of calculators. The prospective elementary teachers first need to see one to one correspondence between their own hundreds chart and the display of calculator after using the operations of 1 + + = = =…. After the instructor demonstrated what mathematical concepts come out of the hundreds charts, they were allowed to explore in small groups. The results of their explorations will be the focus of our study. This was the first time any of this group of prospective elementary teachers had seen a hundred chart. They struggled during the small group explorations. They were surprised that so many number sense ideas came out of this single chart after the group reports. They were fascinated by the power of calculators. In this study we will also report on the prospective elementary teachers views on the use of calculators and hundreds charts.

Introduction

Teaching mathematics for understanding has been the emphasis of the mathematics community in recent years. NCTM (1989, 2000) and Lee (2001, 2002) suggests that we should move away from rote memorization of facts and procedures to the development of mathematics concepts. Teaching mathematics for understanding should make connections among various representations of mathematical concepts by being investigated by students through problem solving or related real world activities. At the same time, Aspinwall (2001) concludes learning for understanding should be viewed as a meaning-making process that involves the learner in actively building connections between what is being learned and what is already known. Teacher-centered approaches to learning do little to diagnose conflicts between students' concept images and concept definitions because students are not asked to tell what they know; instead, they are asked to reiterate what teachers know or have said (p.91).

This study reports on how the researcher attempts to provide for the prospective teachers an opportunity in a mathematics methods course to learn about what we mean by teaching and learning mathematics for understanding. Through the activity of Hundreds Charts exploration and with the help of graphic calculators, the prospective teachers become self-enabled to construct, to discover, to expose and to gain a feeling about teaching and learning mathematics for understanding as well as a strong pedagogical content knowledge of number sense.

Literature Review

Concept Images and Concept Definition

Tall & Vinner (1981) formulated a distinction between mathematical concepts as formally defined and the cognitive processes by which students make personal meaning of concepts. The latter they called concept image and by this term meant the total cognitive structure learners create, which colors the meaning of the concept for the learners and includes all the mental pictures and associated properties and processes. On the other hand, concept definition is the form of words or symbols used by the mathematical community at large to specify the concept (p.90).

Number Sense

As stated by NCTM (1987), good number sense is characterized by well-understood number meanings, the ability to develop many relationships among numbers, and ability to recognize the relative magnitudes of numbers and the relative effects of operating on numbers. We believe the exploration of hundred charts with the aid of calculators will help prospective elementary teachers acquire a solid number sense.

Leutzinger and Bertheau (1989) stated that number sense permits students to make decisions concerning the relationships between numbers and enables them to give valid reasons for their decisions. It is the responsibility of the elementary school teachers to imbue students with a feel for how numbers are related to one another (p.112).

Zaslavsky (2001) states that children acquire number sense through pattern recognition and gaining familiarity with numbers. The more often children work with certain quantities, the easier it is for them to recognize and understand the value of those quantities. The senses transmit the relevant message to the brain. Numeration systems probably developed in the order; first gestures, then words, and finally symbols that we called numerals (p.313). Some researchers (Schneider & Thompson, 2000; Lee, 2001; Lee, 2002) think that a student who has good number sense has a good understanding of number meanings, number pattern recognitions, and numerical relationships.

Calculator as a Investigation Tool

Reys (1989) states that the calculator offers teachers and students a rich learning aid and describes an activity using calculator and a set of hundred-charts in teaching primary level students. In Reys' activity, students use the constant addend feature of their calculator to skip count by 2s, 3s, 4s, and 5s. For each count, the numbers displayed are recorded on one of the hundred-charts.

Waits and Demana (2001) recommend that teaching problem solving should 1) use paper-and pencil and then support the results using the technology, or vise versa; and 2) use manipulative and paper-and-pencil techniques during the initial concept development and use calculators in extension and generalizing phases (p.59).

Calculators can greatly facilitate with formation of concepts, by enabling students to perform calculations with speed and accuracy, so that concepts rather than computations become the focus of the students' attention. Calculators help students see that mathematics has value. Students using calculators find mathematics more interesting and exciting. The researcher and other authors have discussed the use of calculators for teaching and learning (Lee, 2001, 2002; Jones et. al. 2001; Waits and Demana, 2001).

Representation

NCTM (2000) reports representation is more than a process; it is a way of thinking and learning mathematics. Representations are powerful tools for thinking and give learners useful tools for building understanding, communicating information, and demonstrating reasoning. With different representations, we can examine students' understanding of certain mathematical concepts.

Based on Goldin & Shteingold (2001), a representation is typically a sign or a configuration of signs, characters, or objects. It can symbolize, depict, encode, or represent something other than itself. It has a two-way nature aspect. It can be divided into external systems and internal systems. Some external systems of representation are mainly notational and formal, such as our system of numeration. Other external systems are designed to exhibit relationships visually or spatially, such as number lines, graphs, and tables. On the other hand, the internal systems of representations can be verbal or syntactic, imagistic, formal notational, strategic and heuristic process, and individual affective systems

Sieve of Eratosthenes

Eratosthenes (276-194B.C.), a librarian of the great library in Alexandria, was one of the most brilliant men of the ancient world. He used his method of "Sieve of Eratosthenes"(Fig.1) to separate the primes from composite numbers. His method was; circle 2 and cross out all the multiples of 2, which leaves the next prime 3, and then cross out all the multiples of 3, which gave the next prime 5, and so on. Following this fashion he found the primes. The following table includes all the prime numbers less than 100, which are underlined.

①	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Figure 1: The Hundreds Chart with all the Prime Numbers Underlined

The Study

In the elementary school mathematics curriculum, Hiebert (1988) expected in-service teachers to do two types of written symbols; those that represent quantities (e.g., 1, 2, 1/2, 2.1) and those that represent actions or operations on quantities (e.g., +, -, ×, ÷) (p.336). In the Principle and Standards for School Mathematics (NCTM, 2000), we assume that students should know that "mathematics involves examining patterns and noting regularities' (p.262), that "statements need to be supported or rejected by evidence", and that "assertions should always have reasons" (p.56). In this study, we wish our group of elementary prospective teachers to be certainly experts in exploration patterns and regularities in hundreds charts rather than investigating in the relationships among the written symbols.

The Mathematical Constructing Framework

The mathematical constructing framework, used here for analyzing our subjects' data collections in the investigation of the hundred charts with the help of calculators, was adapted from Shaughnessy et al. (1996). They considered that describing, organizing and reducing, representing, as well as analyzing and interpreting data are critical ingredients for handling data.

Jones et al. (2001) based their work on Cucio (1987) and Moore (1997) and provided detailed explanations for this framework. They stated that describing data involves direct reading of data displays, recognizing graphical conventions and making straightforward connections between the original data and the display. Organizing and reducing data incorporates mental actions such as ordering, grouping and summarizing data. Representing data involves the construction of visual representations of data including representations that exhibit different organizations of the data.

Analyzing and interpreting data focuses on recognizing patterns and trends in data and on making inferences and predictions from the data. We use these four constructs as our framework for analyzing the data provided by our prospective elementary teachers. That is, we discuss their abilities to describe, to organize and reduce, to represent, and to analyze and interpret.

Design

The classroom activity of exploring a hundreds chart includes open-ended explorations, with emphasis on mathematical thinking, group work discussion and extended written tasks. The design of this activity includes three stages. The first stage is the whole class lecture mode. The instructor provides the hundreds charts for each prospective teacher and talks about the one to one correspondence between the numbers and the entries of the hundreds charts as well as the skip counting of 2, 3, 5, and etc. We have them color multiples of a number on a hundreds chart and look for patterns. We use the colored tiles to arrange many numbers in terms of several rectangular shapes, whereas prime numbers yield only one row of tiles. Then the instructor introduces the concepts of composite numbers, factors and multiples in the hundreds charts. Next, the instructor asks the small groups to find all the prime numbers within one hundred like the ancient mathematicians did the "Sieve of Eratosthenes". Then the whole class plays several times the Prime Numbers Bingo Game. One of the bingo cards would be like this (Fig. 2). The Prime Numbers Bingo Game is a game, where each prospective teacher randomly puts all the 25 prime numbers less than 100 into a 5-by-5 grids card. Then we draw a prime number at each time from a bag, and whoever first makes a connection of straight line of five numbers either vertically, horizontally or diagonally will win the game. This game will help the prospective teachers to recognize what the prime numbers less than 100 are and at the same time have fun to remember them.

2	47	23	53	3
17	5	19	61	41
31	59	7	83	67
71	97	79	11	89
13	73	43	37	29

Figure 2: A Prime Numbers Bingo Game Card

Finally, we talk about the sequences that can be produced from a hundreds chart. For instance, the natural number sequence, the even number sequence, the odd number sequence, and the triangular number sequence, etc.

The second stage is the small group discussion time. The prospective teachers are expected to explore all the possible mathematical concepts they can come up with in this hundreds chart. After

30 minutes discussion of whether they can find any mathematical concepts from a hundreds chart, each group reports their findings.

The last stage is an extended take home assignment for them to work on the hundreds charts (Fig. 3) to see what can trigger them to find some more mathematical concepts.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Figure 3: The Hundreds Chart Display

Subjects

This group of 32 prospective elementary teachers was in their final stage of their professional training. They were students in a university in the southern part of Taiwan. This was their final semester. They had to take this mathematics methods course in order to graduate. They were kindergarten education majors. They did not like mathematics and thought they were not good at mathematics either. That is the reason they postponed taking their mathematics methods course until their last semester.

Results

We will not present the results of how to use the calculators to generate sequences of multiples, carry out the constructions of the sieve of Eratosthenes, finding common multiples, least common multiple, and why there is no greatest common multiple but there is a greatest common divisor, because we covered these topics in formal lectures. We will present the following interesting list of mathematical patterns discovered by our subjects.

1. All the sums of vertices of the diagonal of any squares are 101. For example,

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1                10
   12       19               1+100=10+91=101


   82       89               12+89=82+19=101
91               100
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2. The sum of all the numbers on each diagonal of the 10-by-10 grids is 505.
   1+12+23+34+45+56+67+78+89+100=505
   91+82+73+64+55+46+37+28+19+10=505
3. Any adjacent numbers on the lines increasing from right to the left ↖ have differences of 11.

61	62	63	64	74-63=11
71	72	73	74	84-73=73-62=11
81	82	83	84	94-83=83-72=72-61=11
91	92	93	94	

4. Any adjacent numbers on the lines increasing from left to right ↗ have difference of 9.

1	2	3	4	11-2=9
11	12	13	14	21-12=12-3=9
21	22	23	24	31-22=22-13=13-4=9
31	32	33	34	

5. Any sums of same amount of numbers on the X shapes are equal.

1	2	3	1+12+23=36
11	12	13	21+12+3=36
21	22	23	

6. Any sums of same amount of numbers on ✚shapes are equal.

72	73	74	73+83+93=249
82	83	84	82+83+84=249
92	93	94	

7. Any sums of the numbers on the dots position of this shape ※ are equal.

61	62	63	64	62+63+92+93=310
71	72	73	74	71+81+74+84=310
81	82	83	84	
91	92	93	94	

8. The multiples of 11 form a diagonal line that is also a reflection line.
   11, 22, 33, 44, 55, 66, 77, 88, 99,

44	45	46	45 and 54
54	55	56	64 and 46
64	65	66	65 and 56

9. The sums of vertical numbers form an arithmetic sequence of difference 10.
   1+11+21+…+91=460, 2+12+…+92=470, … , 10+20+…+100=550

10. The sums of horizontal numbers form also an arithmetic sequence of difference 100.
   1+2+3+4+5+6+7+ 8+9+10=55, 11+12+…+20=155, … , 91+92+…+100=1055

11. The sums of the diagonal of 1-by-1, 2-by-2, … , and 10-by-10 grids form a quadratic relation sequence of 1, 13, 36, 70, 115, 1712, 238, 316, 405, 505.

Discussion

The role of calculators in this study is to enable prospective elementary teachers to investigate, explore, model, and to validate their findings. Without calculators the activities become difficult to complete because of class time limitations. Data presented by our subjects provide both the external and internal systems representations. In our study representation has an effect on subjects' learning of external representations and structured mathematical activities. We use hundred charts and calculators to discuss how subjects are representing number sense internally, for example, what basic patterns do they discover, how the number patterns are related to one another, what the structural relationships are that they develop, and how they connect different representations with one another.

The research mathematical framework presented has allowed us a more detailed look at the meanings that prospective elementary teachers can give from exploring the hundred charts with the help of calculators.

Describing Data

Due to the whole class lectures, small group discussion and group reports, most of the prospective elementary teachers were able to identify some mathematical concepts as we mentioned in our result section. However, their findings are more like individually describing some pop out ideas rather than systematic findings. For example, they produce one specific example on vertices sums of a specific square and think the searching is done. They have the ability to describe data found in the hundreds charts, such as the concepts of sums of two or more numbers or arithmetic sequences, but lack the ability to systematically identify all the representations of the same concept and have not been able to generalize, organize, and reduce data representations.

Organizing and Reducing Data

Some of the prospective teachers' concept images about finding the mathematical concepts in the hundred charts are limited by their number sense. They lack the ability to organize and reduce the data, but rather view the whole relationships of a specific concept as separated ideas. For example, one prospective teacher produced pages of the findings by listing pairs of numbers adding up to 101, rather than organizing and reducing it as one mathematical concept. As a consequence, there is only one mathematical concept to come from his/her own hard work.

Representing Data

In terms of representing data, some of the prospective teachers' view of data recognition is limited by their number sense. They can find the sums of correspondent two numbers on two diagonals equal to the number 101. For example, some prospective teachers provided 1+100=101, 10+91=101, 12+89=101, 19+82=101, 23+78=101, 28+73=101, 34+67=101, 37+64=101, 45+56=101, 46+55=101. However, they could not spot that if they rotate the diagonal counterclockwise, then any two numbers on the correspondent places sum to 101 also.

Analyzing and Interpreting Data

One prospective elementary teacher found out that the row sums form an arithmetic sequence of difference 100, and provided the following argumentation. Because

1+10=2+9=3+8=4+7=5+6=11……………………………①

11＋20＝12＋19＝13＋18＝14＋17＝15＋16＝31………②

The differences of ②－①, are 10＋10＝10＋10＝10＋10＝10＋10＝10＋10＝20.

So if when we add 11＋12＋13……＋20, we get 20×5＝100 more than 1+2+…+10=55. Based on this calculation we therefore can conclude that the sequence formed by row sums will increase each time by 100. This shows not only strong number sense but also being able to use previous knowledge to support findings.

Conclusion

Most of the prospective teachers used a combination of record-keeping, personal re-collection, concept images and concept definitions as the basis for their assessments. There is evidence that the prospective teachers used processes of systematic observations, conjecturing, generalizing and justifying assertions in their investigation. However, the results of their findings also suggested that prospective teachers might also be influenced by: a) seeing, or failing to see, patterns in responses and data, b) types and patterns of data being over-represented or under-represented in the their mental pictures, and c) inability to see and use all the details that occur in the hundreds charts.

Almost all the prospective teachers were interested in this activity. This was the first time any of this group of prospective elementary teachers had seen a hundreds chart. They struggled during the small group explorations. Because they did not know what was there to look for, but they were surprised that so many number sense ideas came out of this single chart after the group reports. Although the small group reports are limited in concepts, they are abundant in different representations of the same concept. The prospective teachers were fascinated by the power of calculators. A lot of the computations, whether looking for sums or differences, became simple. And using the calculator' program to find either the prime numbers or the composite numbers became easy. The concept of the Fundamental Theorem of Arithmetic was discovered naturally without much input from the instructor.

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