The Use of Technology in a College Mathematical Modeling Class

Zhonghong Jiang
Florida International University

Abstract

This article describes the use of various technologies in a mathematical modeling course designed for preservice mathematics teachers. Two detailed examples are given to show how the students enrolled in this course use the Internet, graphing and curve fitting software, and spreadsheets to collect, represent and analyze data, and to build mathematical models. Other technologies such as statistics software, dynamic geometry software, graphing calculators, and Calculator-based Laboratory (CBL) are also used to stimulate the students' mathematical modeling and reasoning insights and their learning interest. The technology tools enable the future teachers to appreciate the power of mathematics that helps them understand the world.

Introduction

National conferences and committees have increasingly advocated an emphasis on problem solving and mathematical modeling. In its *New Goals for Mathematical Sciences Education* (1983), the Conference Board of the Mathematical Sciences advised that the changing nature of mathematics required teachers to continually upgrade their knowledge and skills through advanced study and suggested mathematical modeling as an area of study. In 1989, the National Research Council issued its *Everybody Counts: A Report to the Nation on the Future of Mathematics Education*, which warned an urgent need for teaching reforms that include an emphasis on model building. In 1989, the National Council of Teachers of Mathematics (NCTM) formulated a specific plan of action, *Curriculum and Evaluation Standards for School Mathematics*, stressing the importance of mathematical modeling as a facet of problem solving. More recently, the NCTM and the Mathematical Association of America (MAA) published a series of activity or text books on mathematical modeling, such as *Mathematical Modeling in the Secondary School Curriculum* (Swetz & Hartzler, 1994) and *Mathematical Modeling in the Environment* (MAA, 1998). Despite these repeated recommendations and exhortations, however, little effort has been expended in preparing secondary school teachers to use mathematical modeling techniques and situations effectively in their classrooms (Swetz & Hartzler, 1994). For many years, the mathematics community placed its highest value, at least implicitly, on "pure mathematics," and it continued to educate the bulk of both undergraduate and graduate students with scant attention to mathematical modeling of real world problems (Hadlock, 1998). It is time to change this questionable situation in the mathematics preparation of our preservice teachers.

To provide the preservice teachers with quality mathematics education, we (other mathematics educators and I) have focused on curriculum changes at Florida International University. One of the important changes is designing and implementing a new, standards-based, mathematical modeling course for secondary school mathematics preservice teachers. The purpose of this course is to provide future teachers with the knowledge and experience that enable, motivate, and encourage them to solve real-world problems through mathematical modeling. The course can help the prospective teachers construct their content knowledge from a perspective that involves rich connections among mathematics, science, and real-world situations. It features innovations derived from the national mathematics education standards. It has been and will continue to be offered at the junior level, and is team-taught by a group of faculty using exemplary teaching strategies.

Technology Integration

Advanced technology is increasingly pervasive in everyday life. More and more educators believe that the use of technology can effectively facilitate the teaching and learning of mathematics. This belief has been reinforced since the coming out of the innovative technologies in mathematics education including dynamic geometry software such as the Geometer's Sketchpad (Jackiw, 1995), computer algebra software such as Mathematica and Maple, new spreadsheet programs such as Microsoft Excel, graphing calculators such as TI-83 and TI-92, and a variety of other powerful electronic tools. These technologies are highly interactive so that whenever a student's actions yield a reaction on the part of the machine, it in turn sets the stage for interpretation, reflection, and further action on the part of the student. With these technologies, one can make powerful resources immediately available to aid thinking or problem solving, provide intelligent feedback or context-sensitive advice, actively link representation systems, and generally influence students' mathematical experience more deeply than ever before (Kaput and Thompson, 1994). In addition, the rapid computing speed of computers and graphing calculators can free students from tedious calculations and allow them to concentrate on conceptual understanding. By opening a new, colorful world to the students, technology can greatly motivate the students, stimulating their stronger interest in mathematics. Based on these considerations, the NCTM Standards (1989) emphasize the effective use of technology as one of the chief features of the reform curriculum. In recent years, many research studies (Choi-koh, 1999: Dixon, 1997; Jiang, 1993; O'Callaghan, 1998; Thompson, 1992) have provided evidence supporting the belief that students could be benefited by the use of technology.

Technology is a natural tool for mathematical modeling. It would be less than optimal and sometimes difficult for us to teach and for students to explore mathematical modeling without using technology. Therefore, the use of technology is emphasized in this course. Calculator-based Laboratory (CBL) with multiple sets of probe-ware is used for data collection. Computer software (such as Mathematica and statistics software) and graphing calculators are used for data analyses and curve fitting activities. Other computer applications such as the Geometer's Sketchpad and Microsoft Excel spreadsheet are also used to stimulate the future teachers' mathematical modeling and reasoning insights and their learning interest. These technology tools enable the future teachers to appreciate the power of mathematics that helps them understand the world. Using the graphical and numerical representations together, the future teachers can interpret situations both visually and numerically. This helps them formulate and refine problems (if the problems do not arrive neatly packaged), investigate problems from multiple perspectives to gain further insights, and articulate problems clearly enough to build mathematical models. When they experience difficulties in the problem posing and solving processes, constructed computer situations can help them develop ideas and strategies to approach solutions. These computer situations are usually difficult for the future teaches who lack sound understandings of the problems to construct by themselves in the first place.

In the following sections, examples will be given to show how we, as well as our students, used technology in the modeling class taught in the Fall 2001 semester.

The Use of Graphing and Curve Fitting Software

Some basic mathematical structures that lend themselves to modeling are graphs, equations (formulas) or systems of equations or inequalities, digraphs, index numbers, numerical tables, and algorithms (Swetz & Hartzler, 1994). Functions/equations and their graphs turn out to be the mathematical structures most frequently used for modeling. To that end, the graphing and curve fitting software becomes very important or even indispensable in mathematical modeing. A good example of this aspect is a modeling task assigned to the students enrolled in the modeing class. The task was to build a mathematical model for the world population growth based on the data from the web site www.census.gov/ipc/www/worldhis.html, which gives historic estimates of world population from 10000 B.C. to 2000 A.D. To make better sense of the population growth, the students used Physics Analysis Workstation (PAW), an interactive graphing and curve fitting system (*http://paw.web.cern.ch/paw/*), to construct the graphic representations of the data.

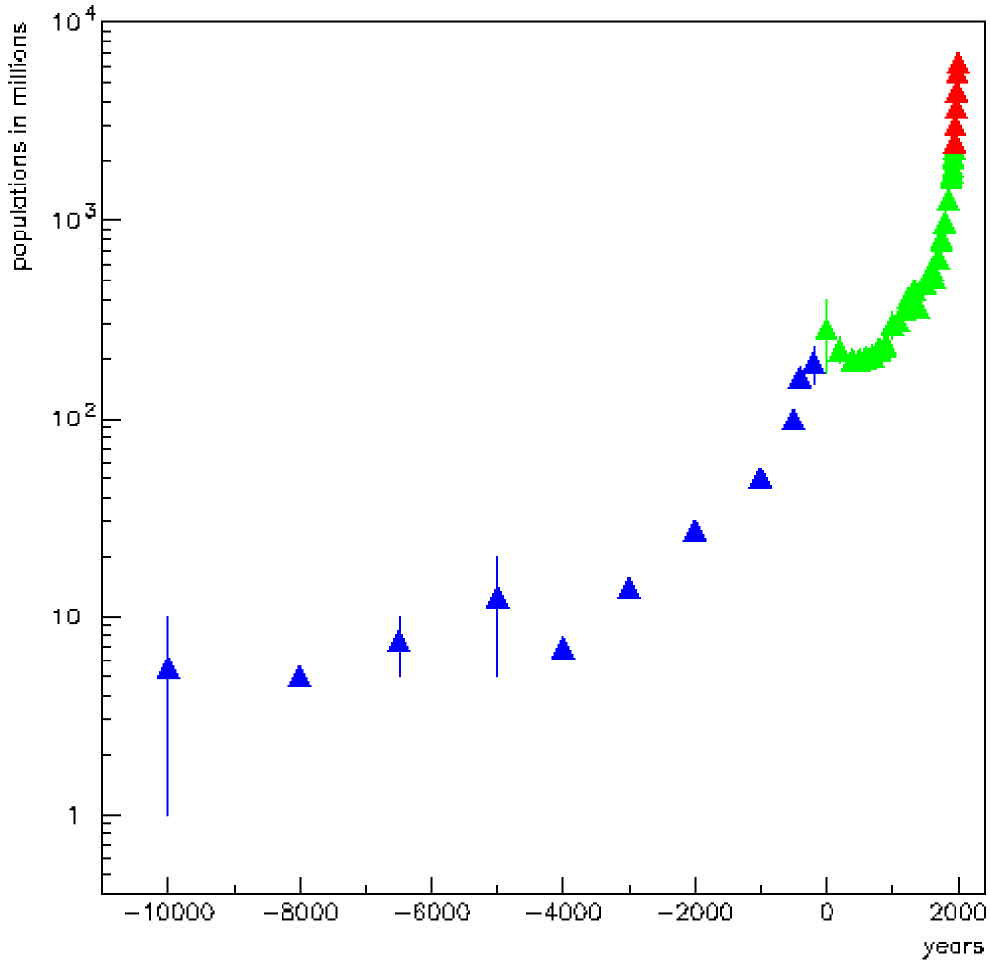


Figure 1. Population growth from 10000 B.C to 2000 A.D.

In the graph displayed in Figure 1, the blue section shows the world population from 10000 B.C to 1 A.D., the green section shows the population from 1 A.D. to 1950 A.D., and the red part shows the population from 1950 A.D. to 2000 A.D. From the graph, the students clearly visualized that the population of the world had changed slowly until around 1800 A.D., and it rapidly grew during the last 50 years.

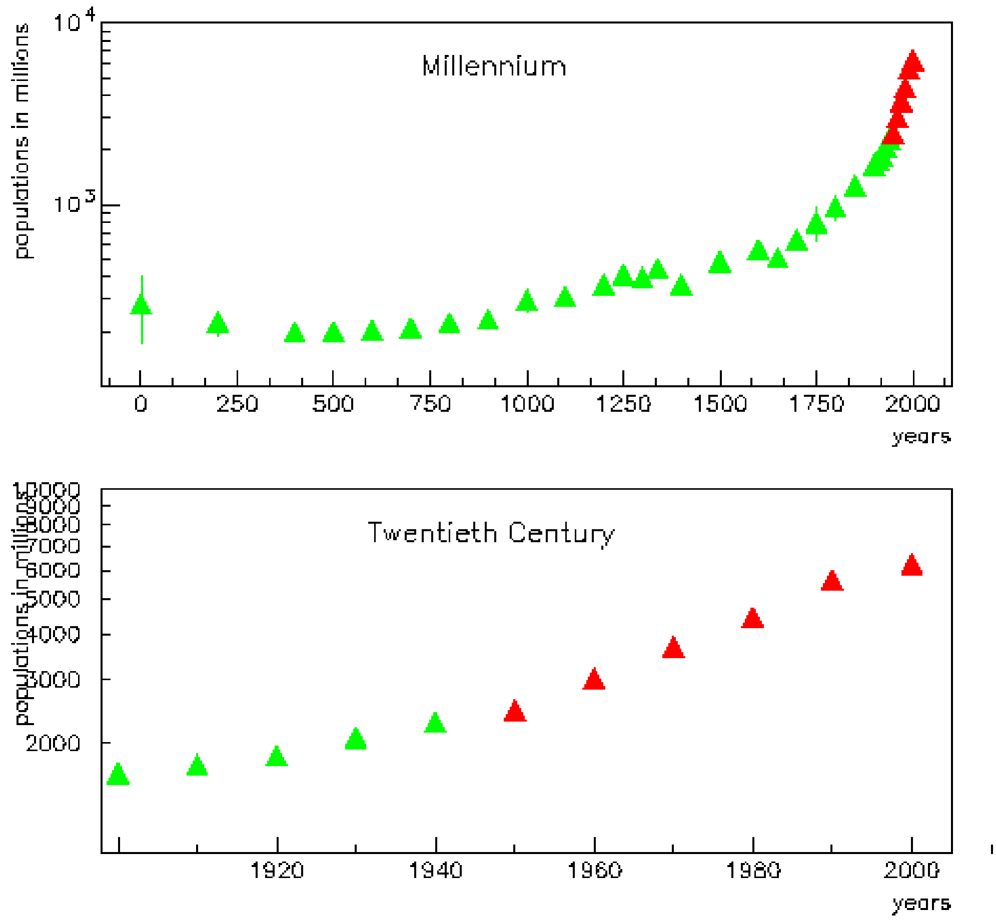


Figure 2. Population growth during the period from 1 to 2000 A.D. and the 20th century

In the two graphs displayed in Figure 2, the students could visualize the population change during the period from 1 A.D. to 2000 A.D. and that during the twentieth century.

Analysis of data usually involves fitting the measured data to a model in order to make some predictions about the system under investigation. The students decided to use an exponential function and a quadratic function to model the data. In PAW the fitting algorithm solves the related equations to determine the best-fit parameters for the chosen functions based on some data and a linear model. With the help of PAW, the students got to know that for the exponential function $\exp(p_1 + p_2 t)$, the best-fit parameters were $p_1 = -29.263$ and $p_2 = 0.1902E-1$; and for the quadratic function $p_0 + p_1 t + p_2 t^2$, the best-fit parameters were $p_0 = 0.14449E+7$, $p_1 = -1537.2$, and $p_2 = 0.40893$. The related graph is shown below:

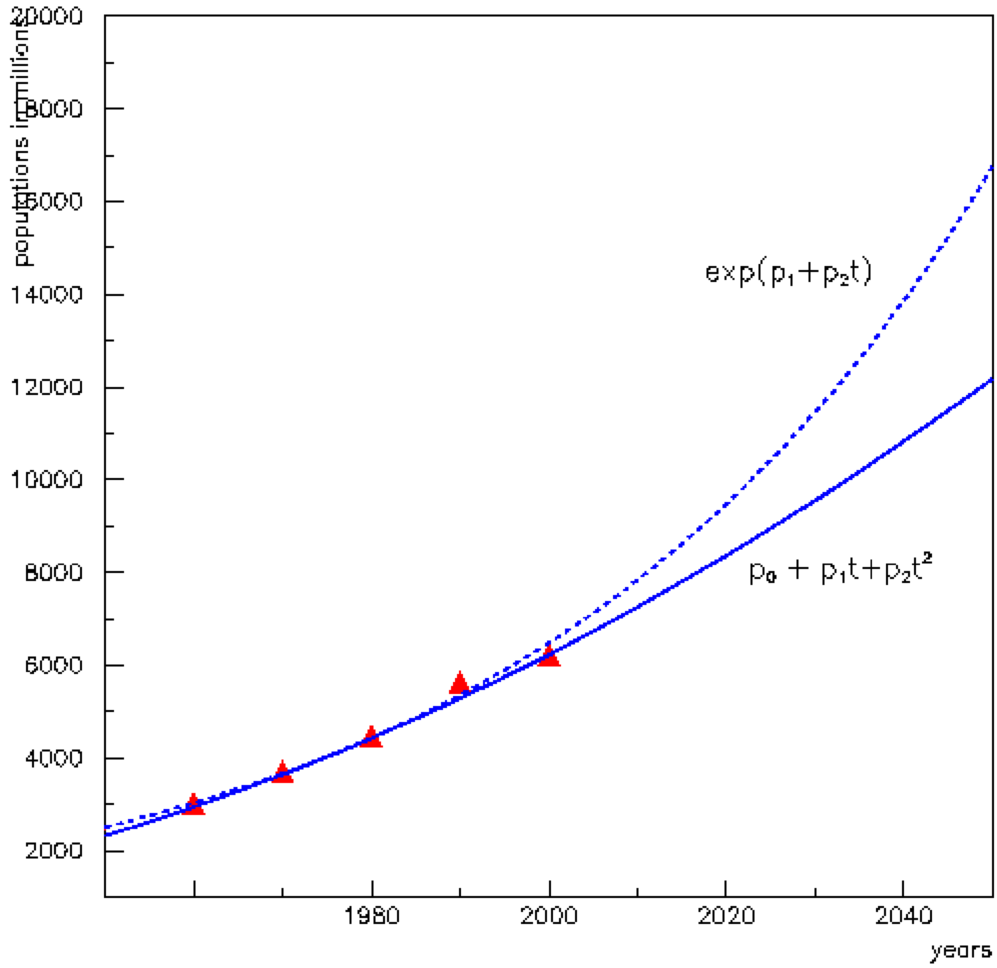


Figure 3. Curve fitting and extrapolation

With the exponential and quadratic functions mentioned above, the students were able to predict the population of the world in the years 2010, 2020, 2030 and 2050. Using the Mathematica software for numerical calculations, the students' predictions are listed in the following table:

	Year 2010	Year 2020	Year 2030	Year 2050
Calculated with the exponential function.	t=2010 Pe = 7841.61*10^6 Population: around 7.84 billion	t=2020 Pe = 9484.37*10^6 Population: around 9.48 billion	t=2030 Pe = 11471.3*10^6 Population: around 11.47 billion	t=2050 Pe = 16781*10^6 Population: around 16.78 billion
Calculated with the quadratic function.	Pq = 7246.09* 10^6 Population: around 7.25 billion	Pq = 8353.97* 10^6 Population: around 8.35 billion	Pq = 9543.64* 10^6 Population: around 9.54 billion	Pq = 12168.3* 10^6 Population: around 12.17 billion

From these calculations and also from the graph shown in Figure 3, the students realized that during the period 1950-2000 these two functions give close results, but as the value of t increases, the difference between Pe and Pq grows. Thus they understood that when making predictions, one must be careful. To them, it seemed to be a good idea to predict the number of people living on earth within the range of Pe and Pq values.

The Use of Spreadsheets

In the modeling class, every student was required to complete projects, including a term project. Below is one of the term projects that our students completed in the class. In this project, the Microsoft Excel spreadsheet program was used to both present and implement the modeling ideas.

Earthquake Modeling Project

Description: *High-rise buildings are susceptible to displacements during earthquakes. It is normal for the last floor of a thirty-storied building to sway as much as 30 cm (11.8 in) during peak earthquake accelerations. Although buildings have multiple degrees of freedom, we will focus on a model with only one degree of freedom, which will represent the lateral sway of the last floor. The structure of the building acts as an enormous spring whose magnitude depends on the construction material. For simplification purposes, we have modeled the earthquake forces, Q as a sinusoidal time-dependent function:* $Q(t) = Q_1 * \sin(W*t)$

Driving Question: *How is the behavior of a building under the effects of an earthquake modeled?*

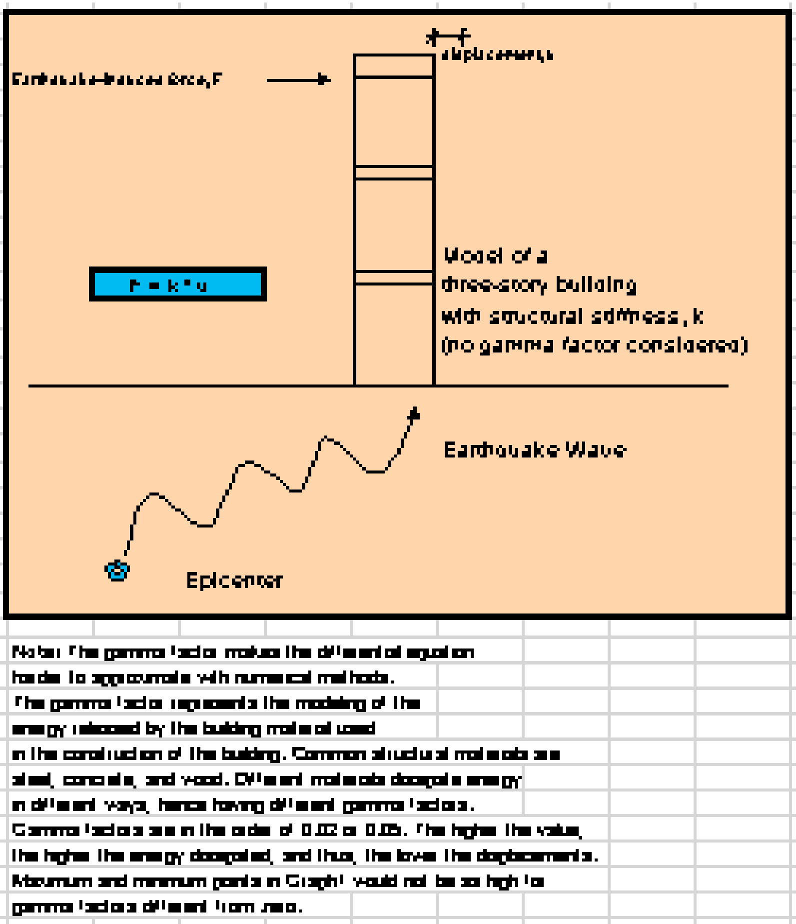


Figure 4. An Application of phisical formula F = ku.

Theory: *High school students are usually exposed to problems, which deal with the following basic notions of force: F = ma, F = cv, and F = ku, where m = mass of building, a = acceleration, c = coefficient of damping, v = velocity, k = stiffness of building, and u = displacement. F= ku is illustrated in Figure 4.*

A more complex problem that includes the three effects can be solved through the use of differential equations:

$Q(t) = ma + cv + ku$, *or*

$Q(t) = mu'' + cu' + ku.$

For the case of an undamped system, the equation simplifies to:

$Q(t) = mu'' + ku.$

Numerical Method: *In order to adapt this type of problem to the high school level, numerical methods provide an appropriate solution, using three components of displacement, u:*

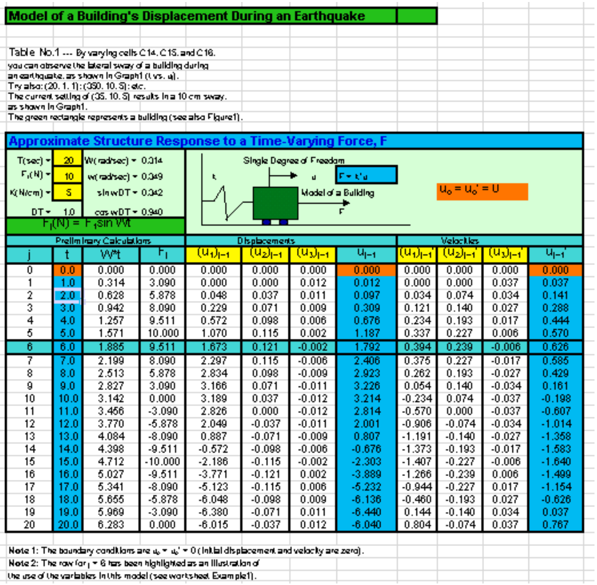
$u = u_1 + u_2 + u_3$, *where*

$(u_1)_{j+1} = u_j \cos(w*dt_j) + (v_j/w) \sin(w*dt_j)$

$(u_2)_{j+1} = (Q_j/k)(1 - \cos(w*dt_j))$, *where* $Q_j = Q(t_j)$

$(u_3)_{j+1} = (dQ_j/(kw*dt_j))(w*dt_j - \sin(w*dt_j)).$

(The variables involved are: w = natural frequency of building, T = period of vibration, $dt_j = t_{j+1} - t_j$*, time interval (fraction of T),* Q_1 *= force coefficient,* $dQ_j = Q_{j+1} - Q_j$*, time-varying force, and* v_j *= velocity at j.) The numerical method can be implemented very well using a spreadsheet (see the figure on the left below). The graph on the right below shows the positions of an object at the top of the building at different times. If one rotates the graph 90˚ counterclockwise, he can see that as represented by the graph, the top of the building is moving sideways.*



Model of a Building's Displacement During an Earthquake

Table No.1 --- By varying cells C14, C15, and C16,
you can observe the lateral sway of a building during
an earthquake, as shown in Graph1 (t vs. u).
Try also: (20, 1, 1); (350, 10, 5); etc.
The current setting of (35, 10, 5) results in a 10 cm sway,
as shown in Graph1.
The green rectangle represents a building (see also Figure1).

Approximate Structure Response to a Time-Varying Force, F

T(sec) = 20 W(rad/sec) = 0.314
F_1(N) = 10 w(rad/sec) = 0.349
K(N/cm) = 5 sin wDT = 0.342
DT = 1.0 cos wDT = 0.940
$F_i(N) = F_1 \sin Wt$

Preliminary Calculations				Displacements				Velocities			
j	t	W*t	F_i	$(u_1)_{i+1}$	$(u_2)_{i+1}$	$(u_3)_{i+1}$	u_{i+1}	$(u_1)_{i+1}'$	$(u_2)_{i+1}'$	$(u_3)_{i+1}'$	u_{i+1}'
0	0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1	1.0	0.314	3.090	0.000	0.000	0.012	0.012	0.000	0.000	0.037	0.037
2	2.0	0.628	5.878	0.048	0.037	0.011	0.097	0.034	0.074	0.034	0.141
3	3.0	0.942	8.090	0.229	0.071	0.009	0.309	0.121	0.140	0.027	0.288
4	4.0	1.257	9.511	0.572	0.098	0.006	0.676	0.234	0.193	0.017	0.444
5	5.0	1.571	10.000	1.070	0.115	0.002	1.187	0.337	0.227	0.006	0.570
6	6.0	1.885	9.511	1.673	0.121	-0.002	1.792	0.394	0.239	-0.006	0.626
7	7.0	2.199	8.090	2.297	0.115	-0.006	2.406	0.375	0.227	-0.017	0.585
8	8.0	2.513	5.878	2.834	0.098	-0.009	2.923	0.262	0.193	-0.027	0.429
9	9.0	2.827	3.090	3.166	0.071	-0.011	3.226	0.054	0.140	-0.034	0.161
10	10.0	3.142	0.000	3.189	0.037	-0.012	3.214	-0.234	0.074	-0.037	-0.198
11	11.0	3.456	-3.090	2.826	0.000	-0.012	2.814	-0.570	0.000	-0.037	-0.607
12	12.0	3.770	-5.878	2.049	-0.037	-0.011	2.001	-0.906	-0.074	-0.034	-1.014
13	13.0	4.084	-8.090	0.887	-0.071	-0.009	0.807	-1.191	-0.140	-0.027	-1.358
14	14.0	4.398	-9.511	-0.572	-0.098	-0.006	-0.676	-1.373	-0.193	-0.017	-1.583
15	15.0	4.712	-10.000	-2.186	-0.115	-0.002	-2.303	-1.407	-0.227	-0.006	-1.640
16	16.0	5.027	-9.511	-3.771	-0.121	0.002	-3.889	-1.266	-0.239	0.006	-1.499
17	17.0	5.341	-8.090	-5.123	-0.115	0.006	-5.232	-0.944	-0.227	0.017	-1.154
18	18.0	5.655	-5.878	-6.048	-0.098	0.009	-6.136	-0.460	-0.193	0.027	-0.626
19	19.0	5.969	-3.090	-6.380	-0.071	0.011	-6.440	0.144	-0.140	0.034	0.037
20	20.0	6.283	0.000	-6.015	-0.037	0.012	-6.040	0.804	-0.074	0.037	0.767

Note 1: The boundary conditions are $u_0 = u_0' = 0$ (initial displacement and velocity are zero).
Note 2: The row for j = 6 has been highlighted as an illustration of
the use of the variables in this model (see worksheet Example1).

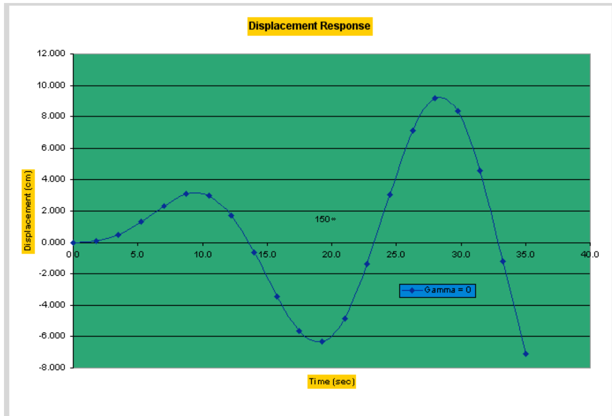


Figure 5. Numerical and graphical repreeentations used for modeling.

Questions: *The variables used in cells C14, C15, and C16 in the spreadsheet are variables T,* Q_1*, and k. The original input is (20, 1, 1). Change the spring constant to 10, simulating a ten-times stiffer building (20, 1, 10).*

By how much does the displacement increase or decrease? (Answer: decreases by 10).

Change (20, 1,1) to (80,1,1). Notice the amplification. What is the interpretation of results such as displacements of 1000 cm? (Answer: the building collapses).

What design consideration should be taken when designing a building, which will be occupied by senior citizens? (Answer: the building should be stiffer).

What is an invariant in this model? (Answer: the mass of the building. However, by the relation w = sqrt(k/m), one can simulate different masses).

Conclusion

In this paper, I only used two examples to show the ways of using technology in our mathematical modeling course for the preservice secondary mathematics teachers. As a matter of fact, there are numerous possibilities for students to use a variety of technologies to enhance their mathematical modeling capabilities. To show the benefits of using technology in mathematical modeling, course assessment is very important. In addition to traditional assessment techniques such as quizzes, exams, library research reports, and activity write-ups, alternative assessments based upon many alternative learning strategies should be used to measure what the student has actually learned, and obtain information for improving course design and/or instruction. Among these alternative approaches are artifacts from the open inquiry projects, observations on how individual students approach problem solving and mathematical modeling without help, and interviews with individual students to assess aspects of their learning experience that cannot be revealed effectively or efficiently through other methods.

References

Choi-koh, S. S. (1999). A student's learning of geometry using the computer. The Journal of Educational Research. Washington, D.C. 92, 301-11.

Conference Board of the Mathematical Sciences. (1983). New Goals for Mathematical Sciences Education. Report of a conference sponsored by CBMS, Airlie House, Warrenton, Virginia, November 1983. Washington, D.C.: CBMS.

Dixon, J. K. (1997). Computer use and visualization in students' construction of reflection and rotation concepts. School Science and Mathematics. 97, 352-8.

Hadlock, C. R. (1998). Mathematical Modeling in the Environment. Washington, DC: The Mathematical Association of America.

Jackiw, N. (1995). The Geometer's Sketchpad (Software). Berkeley, CA: Key Curriculum Press.

Jiang, Z. (1993). Students' learning of introductory probability in a mathematics microworld. Unpublished doctoral dissertation, The University of Georgia.

Kaput, J. J. & Thompson, P. W. (1994). Technology in mathematics educational research: The first 25 years in the JRME. Journal for Research in Mathematics Education, 25, 676-684.

Laborde, J., and Belleman, F. (1994). Cabri Geometry II (Software). Dallas, Tex.: Texas Instruments.

Mathematical Association of America. (1998). Mathematical Modeling in the Environment. Washington, DC: MAA.

National Council of Teachers of Mathematics. (1989). Curriculum and Evaluation Standards for School Mathematics. Reston, Va.: The Council.

National Research Council (1989). Everybody Counts: A Report to the Nation on the Future of Mathematics Education. Washington, DC: The Council.

O'Callaghan, B. R. (1998). Computer-Intensive Algebra and students' conceptual knowledge of functions. Journal for Research in Mathematics Education, 29, 21-40.

Swetz, F. and Hartzler J. S. (Eds. 1994). Mathematical Modeling in the Secondary School Curriculum. Reston, Va.: National Council of Teachers of Mathematics.

Thompson, P. W. (1992). Notations, conventions, and constraints: Contributions to effective uses of concrete materials in elementary mathematics. Journal for Research in Mathematics Education, 23, 123-147.