Mathematics History Is Another Door for Using Technology in Education

The change of belief in mathematics via exploring historical text with technology in the case of undergraduates

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Abstract

A number of researchers expect that the innovative use of new technology alters traditional technology. This study focuses on traditional or lost technology, used in the history of mathematics, as well as new technology. It discusses the aims of using technology in teaching mathematics history to experience hermeneutics in mathematics history. The epistemological necessity of using technology is discussed from the socio-historical-cultural perspective. In the last part of the paper, this paper gives an example of the evidences that show how use of both types of technology for teaching history of mathematics changes students' beliefs about mathematics as human enterprise. To illustrates the change, this study used the LEGO mechanics and DGS (Dynamic Geometry Software), observation of undergraduate students in the class of mathematics education methods for pure-math or informatics majors and students' descriptions of how their belief of mathematics had changed.

Introduction

From the perspective of Marshall McLuhan's Media Revolution (1962), today's IT revolution must influence mathematics education reform as the past media revolution, from manuscript to type printing, influenced the Scientific Revolution. We, as mathematics educators, cannot avoid the Media Revolution just as the medievals could not. Indeed, there are several movements in the world which are trying to push the reform. For example, in ATCM99, Chow Wai Man and Peter Jones (1999) discussed how the new mathematics curriculum in Hong Kong recommends using technology. Percy Kwok (1999) analyzed IT issues in education with the classifications of pedagogical characteristics. At the same time, there are several research studies which describe the features of technology in comparison to traditional technology. In ATCM, Barry Kissane (1999) discussed features of CAS (computer algebra system) which bring new symbol sense, Ida Ah Chee MOK (1999) illustrated that the availability of technology never promises a panacea. Masami Isoda and Akihiro Matsuzaki (1999) illustrated that we could not simply alternate the old technology with the new technology, but that the cognitive roles of the old technology must be changed. Those research studies focused on the cognitive role of technology for learning or exploring mathematics.

This research also discusses this problematic transition which must occur when introducing new technology but focuses more on the role of traditional technology for teaching history of mathematics in the following context. The teaching of the history of mathematics has been incorporated into mathematics education. The Japanese high school mathematics curriculum from 2003 begins to emphasize the history of mathematics for teaching mathematics as human activity and culture, as well as it emphasizes introducing technology. In teaching mathematics history with technology, several research studies exist. Maria G. Bartolini Bussi developed Java applets as well as mathematical mechanics (http://www.museo.unimo.it/theatrum/). David Dennis (1997) discussed the using of DGS for interpreting history. Masami Isoda (2000a) used LEGO mechanics as well as DGS for teaching history. These research studies illustrate that these tools are useful for teaching mathematics as a human enterprise. The aim of this paper is to illustrate this significance in the case of teaching history using both traditional and new technology and to show how this use of technology changes students' beliefs concerning mathematics.

Hermeneutics in Mathematics History

Chinese used calculating rods in the age of Nine Chapters (AD 100 at least) and they had developed their algebra system using their rods. In the age of pre-history, physical instruments had a major role for influence culture. For example, Li-Yu Fu (2000) illustrated the teaching of ethno-mathematics using aborigine's bow in Taiwan. In ATCM98, Masami ISODA (1998) illustrated the power of DGS to understand the text of Descartes in 1637 (we refer to later). In the case of history, both old and new technology is necessary for interpreting history and gaining on appreciation of mathematical enterprise.

For distinguish historical interpretations with other mathematical appreciation, we want to compare the L'Hopital's Weight Problem (1696) with the VCE's Art Gallery Problem (Victoria Certificate of Education 1994, http://www.bos.vic.edu.au/vce/vcegate.htm).

L'Hospital's Weight Problem: Let F be a pulley, hanging freely at the end of a rope CF which is fastened at C, and let D be a weight. D is hanging at the end of the rope DFB, which passes behind the pulley F and is suspended at B such that the points C and B are on the ropes that do not have mass; and one asks at what place the weight D or the pulley F will be.' (Maanen.1991)

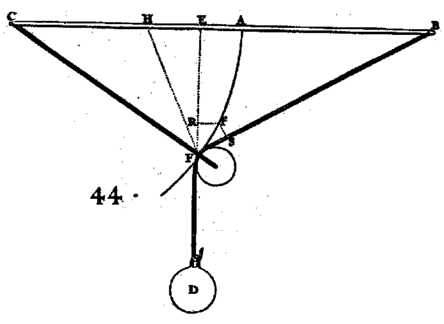


Figure 1. L'Hospital 1696

L'Hospital solved this problem by both of calculus and geometry. To interpret the reason why L'Hospital preferred this problem in his first book of calculus is very important. We want to refer to it later.

Maanen (1991) discussed the classroom activity of this problem on the setting of mechanics. If we do not use the mechanics, we prefer DGS for exploring the position of D. The locus of D is given by DGS (figure 2, used GSP on Cassiopeia Computer Extender). As in Maanen's setting, suppose we determine these four points as *C(0,0), E(x,0), B(6,0), D(x,y)* and given *CF=1* and *BF+FD=10* (figure2).

Then, *ED* = $f(x) = 10 - \sqrt{37-12x} + \sqrt{1-x^2}$

If we use CAS, we find the graph of *f(x)* is figure 3 (used Maple on Cassiopeia).

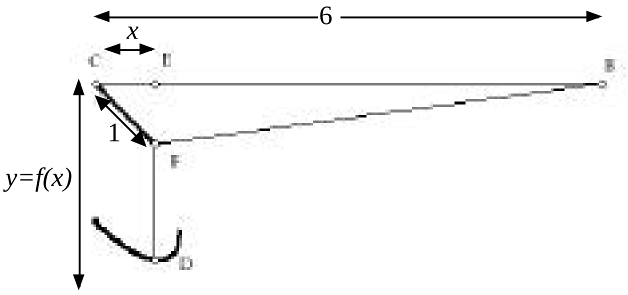


Figure 2. The locus of D

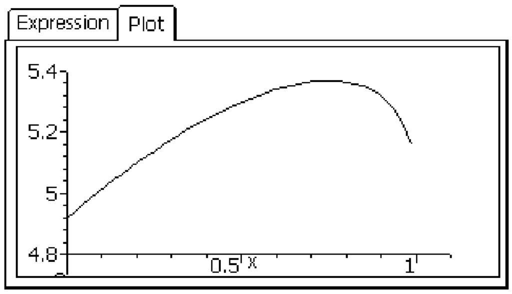
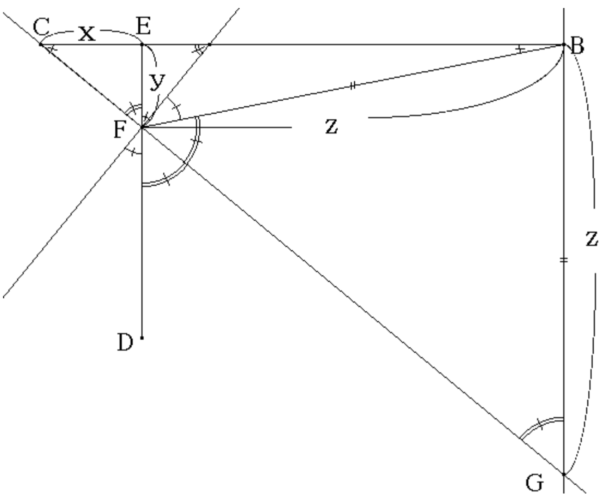


Figure 3. The graph of *f(x)*

The correspondence between the locus of D on DGS and the graph of f(x) on CAS shows validity of f(x). We can use CAS for getting a maximum value;

$$f'(x) = 0, then, 12x^3 - 73x^2 + 36 = 0 \qquad \therefore x = \frac{3}{4}, MaxF(x) \approx 5.37$$

From the text by L'Hospital, we could know that he also discussed the geometric solution. If we use DGS, we know the maximum case (figure 4, used Cabri II) and get simultaneous equations;



$$x^2 + y^2 = 1$$
$$(6-x)^2 + y^2 = z^2$$
$$x : y = 6 : z$$

Figure. 4

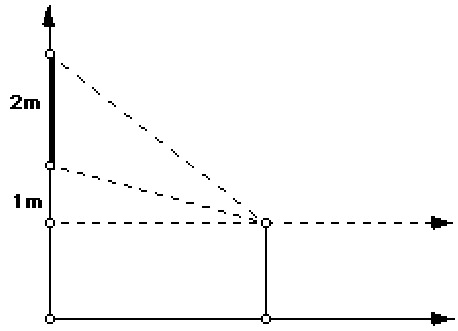
From these equations, we get the equation:

$$12x^3 - 73x^2 + 36 = 0$$

This equation already emerged from *f'(x)=0.* The readers must then recognize this result as demonstrating the validity of calculus because both methods show us the same result. This problem is a very good problem because it shows the power of different mathematical representations through recognizing the correspondences. This is a good example to show the power of multiple representation function in innovative technology. But we could find similar examples in other resources. Indeed, the Art Gallery problem shows us similar correspondences.

The Art Gallery problem; A room in an art gallery contains a picture which you are interested in viewing. The picture is two metres high and is hanging so that the bottom of the picture is one metre above your eye level. How far from the wall on which the picture is hanging should you stand so that the angle of vision occupied by the picture is a maximum? What is this maximum angle? (VCE1994)

This problem also can be solved by both methods of geometry and calculus. Figure 5 (by GSP on Cassiopeia) is the solution which used the locus point ('the distance', 'the angle of vision'/10) and figure 6 (by Maple on Cassiopeia) is the graph of the function. We could also find correspondences between the locus and the graph. We should know that this is also a very good problem because it has same feature of L'Hospital's problem. For discussing the mathematical appreciation that comes from the awareness of correspondences between different mathematical methods, each of the problems is as important as the other one.



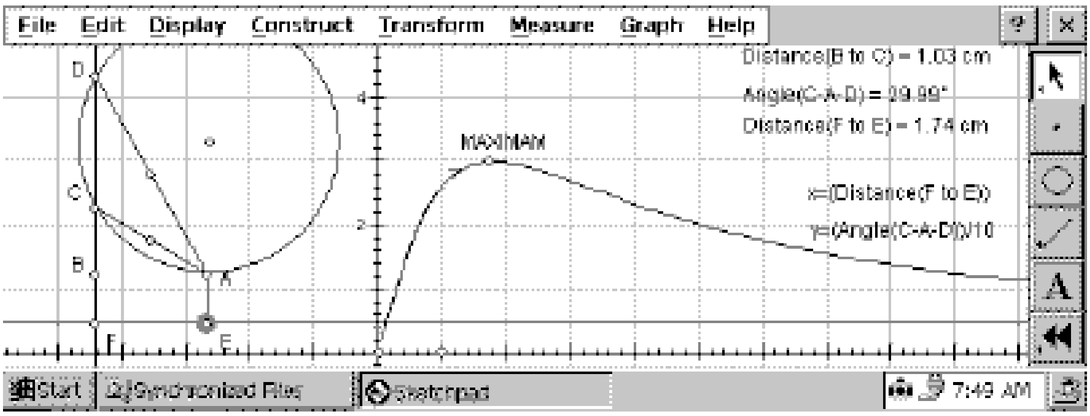


Figure 5. The locus of the point ('the distance', 'the angle of vision'/10)

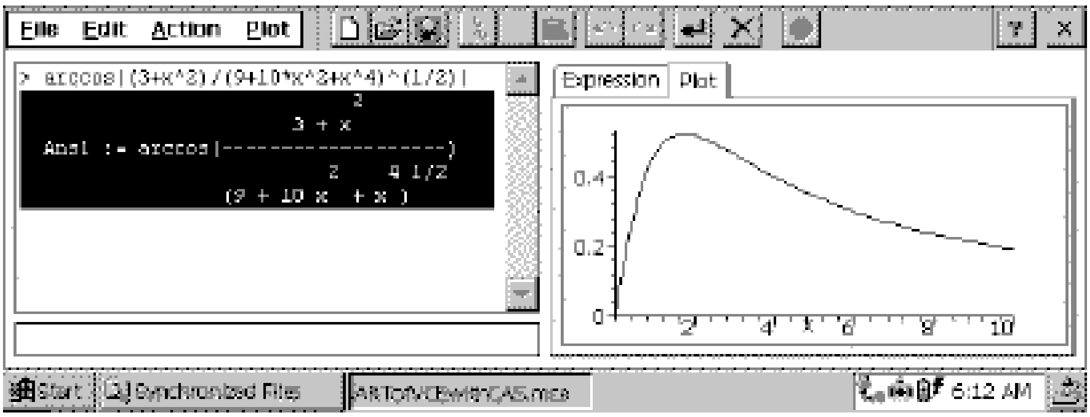


Figure 6. The graph of the function

The advantage of L'Hospital's weight problem is that the problem gave us the opportunity to interpret the historical text on the historical context. We could explain the reason why L'Hospital (or the real author, Jean Bernouli) used this problem in the first textbook of calculus in which he tried to show the validity of his new

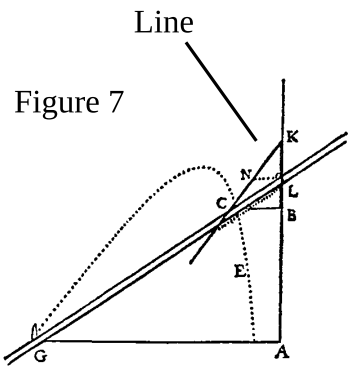


Figure 7

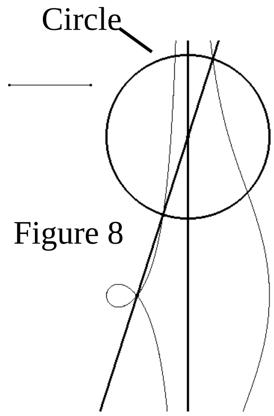


Figure 8

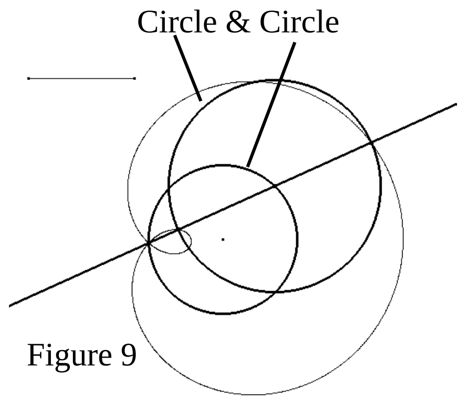


Figure 9

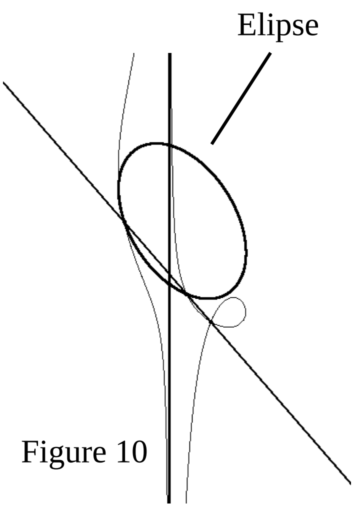


Figure 10

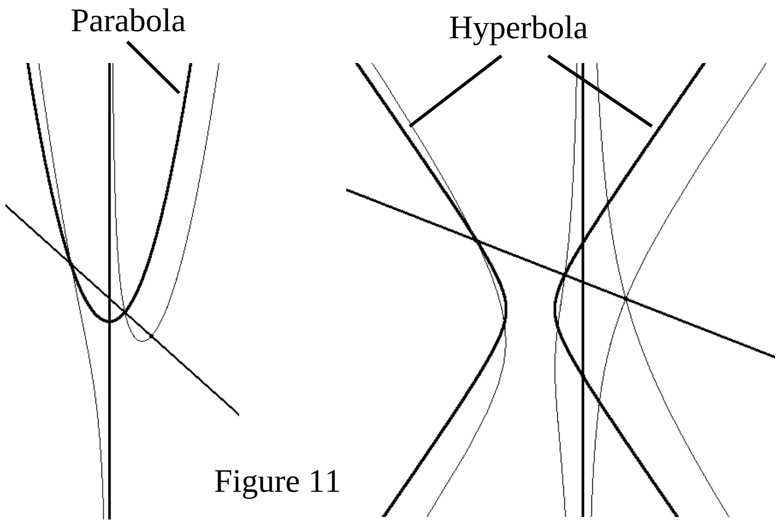


Figure 11

method, calculus, through solving the problem which is solvable by the traditional method, geometry. Because calculus was an unknown, new and powerful mathematical method, he had to prefer this problem for showing his new method's validity.

Historical problems, texts and tools give us the chance to experience the hermeneutics in mathematics history (Jahnke, 1994., Isoda, 2000b). This kind of interpretation, the hermeneutics in this case, is very important for appreciating mathematics as human enterprise. L'Hospital's problem is not easy for today's high school students who only prefer the paper and pencil approach because it includes the solution of a third degree equation[1]. In this case, DGS and CAS help students to solve the problem. If students solved it by both methods they could reach this interpretation. This is a case where modern technology helps to experience the hermeneutics in mathematics history.

A Supporting Theory for the Use of Historical Technology

The Vygotskian perspective, or socio-cultural perspective (eg. Otte, 1991), about mediational means (Wertsch 1991) including physical tools like mechanics and psychological tools like mathematical representation, support

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[1] L'Hospital used that the CB is a solution of the equation. Thus, high school students could solve the equation.

the use of technology in teaching mathematics history. Indeed, the socio-historical-cultural perspective (Vygotskian perspective) describes that each mediational means has its own embedded historical-cultural functions and restrictions (Wertsch 1991).

For example (Isoda 1998, 2000b), Descartes used mechanical tools or reasoned with the metaphor of mechanics. In his Geometry (1637), he discussed his skepticism against the ancients' restriction of tools, using only ruler and compass, for geometry and he overcame the restriction via his algebraic method. After showing that geometric construction problems, such as obtaining a segment, could be solved by his method, he discussed the power of his methods when used for curves, about which ancients such as Pappus did not employ. In his discussion, we find that his method of problem posing was deeply mediated with mechanics as follows.

First, he applied his methods to ask about the curve (figure 7, 1637) made by linkage mechanics using a line:

> 'Suppose the curve EC to be described by the intersection of the ruler GL and the rectilinear plane figure CNKL, whose side KN is produced indefinitely in the direction of C, and which, being moved in the same plane in such a way that its side KL always coincides with some part of the line BA, imparts to the ruler GL a rotary motion of about G'

After showing that it is a hyperbola by use of algebraic representation, he changed the condition as follows:

> 'If CNK be a circle having its center at L, we shall describe the first conchoid of the ancients (figure 8 by quaters) while if we use a parabola having KB as axis we shall describe the curve which is the first and simplest of the curves required in the problem of Pappus (figure 11 by quaters).'

It is difficult for us to interpret this sentence without knowing the mechanical tool. Through changing the conditions (or parts of mechanics), he found other curves using the linkage mechanics. We are not sure Descartes actually used these mechanics but we are sure that he had the metaphor of mechanics and DGS (figure 8 to 11, used Cabri II) helps us to imagine the existence of the metaphor.

The importance of this paragraph is strengthened through the interpretation of a well known commentary by Pascal used against Descartes in Panse:

> '79. Descartes - We must say summarily: "This is made by figure and motion," for it is true. But to say what these are, and to compose the machine, is ridiculous. For it is useless, uncertain, and painful. And were it true, we do not think all Philosophy is worth one hour of pain.' (used Japanese translation by Youich Maeda 1978).

Against the algebraization of geometry by Descartes, Pascal tried to keep the spirit of the ancients' geometry. This confrontation is related to the difference of mediational means between mechanical tools and psychological tools. It tells us that these mathematicians' activity is one of human enterprise. It also implies that each mediational means has own historical-cultural functions and restrictions. That is the reason why it is necessary to use historical technology for interpreting history of mathematics. And also, in some cases, modern technology enables us to interpret history.

A Case Study; *Students change their belief via teaching history with technology.*

For illustrating that mathematics history is another door for using technology in education and discussing the effect of technology, we want to present the case study that shows how undergraduate students changed their belief of mathematics involving history by using historical and modern technology.

Four lesson hours in mathematics education method class were used. The aim of lessons is to give students the chance to experience hermeneutics in mathematics history, interpretation of historical text with technology on the historical context, and to know mathematics as a human enterprise. Students who attended the class were pure math or informatics students in the undergraduate program. Before the lessons, they did not have the experience of attending a class of mathematics history. They knew only names of famous mathematicians as the names of theorems. A few of them had read books about mathematics history but did not have any experience reading original or translated historical texts. They did not know the pantograph as the drawing tool and did not have any experience using DGS. The sequence of lessons was as follows.

First lesson:	Using an original picture of Schooten, students explored the locus which were draw by the compass of ellipse.
Second lesson:	Students learned how to construct the locus using DGS.
Third lesson:	Students drew the compass of ellipse using DGS.
Fourth lesson:	Students read the texts of Descartes' 'Geometry' and 'Rules for the direction of the mind', draw the locus using DGS to understand the meaning of the text and then, read the text of Pascal's 'Panse' and 'Spirit of Geometry', and discuss their interpretation.

Students met the historical technology.

In the first lesson, the teacher presented Schooten's picture (1646, figure 12, see Maanen 1992) and asked students to guess the locus of E. Many students drew some curves and others drew some segments (see figure 13). Then, teacher asked students to make the mechanics of figure 12 using LEGO. Students made it and drew locus. Through the process of making it, students soon began to change its conditions. Students discussed how the curves were changed depending on its conditions (figure 13&14). The teacher explained that the locus is an ellipse just in case of AB=AD. Students tried to consider the case once more but the time left was too short to find the proof.

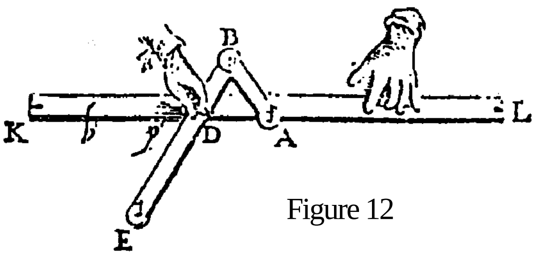


Figure 12

After the lesson, students described their opinion as follows:

> I could understand using mechanics more than only imaging locus in my mind. Until using it, I understood ellipse by the equation and *I had never considered how to construct it.*
>
> We could not use tools until we know how to use it. *I realized that to design tools and explore the way of using them are the issues of mathematics itself.*
>
> *Teacher began the lecture that the ancients used to use rope to draw a circle and an ellipse. I could experience the similar situation via the LEGO: The ancients must have constructed their knowledge of mathematics through this kind of exploration.*
>
> I never experienced such interesting construction in math lesson like this. I really wanted to

demonstrate the reason why an ellipse was drawn.

I never thought that the linear motion could produce an ellipse

These opinions expressed the effect of LEGO mechanics for exploring curves and implied that their views of mathematics had changed as well as their concepts of curves. And especially, the italic parts of the opinions indicated that they interpreted the using of mechanics from historical perspectives which must be similar to the perspectives of the ancients or Descartes.

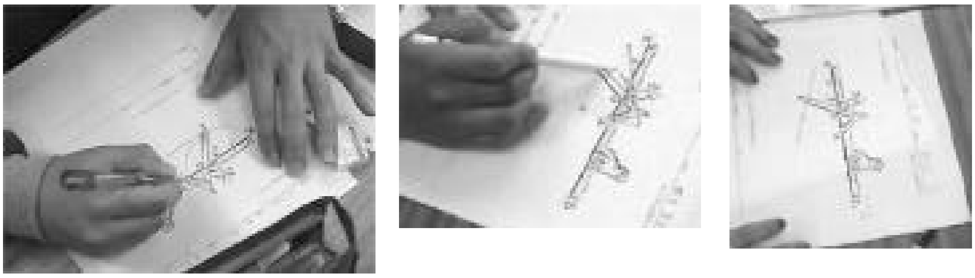


Figure 13. Students presumptions before making LEGO

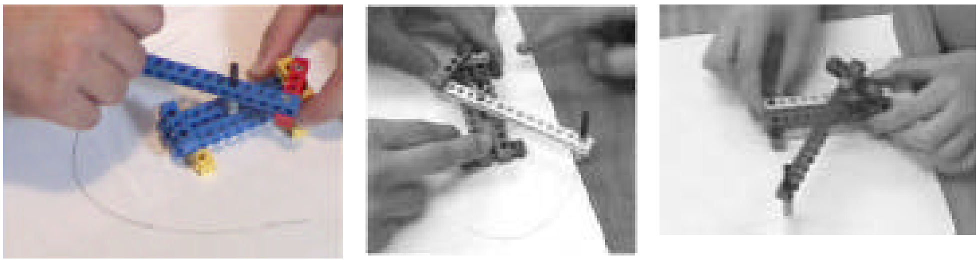
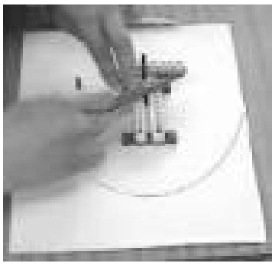


Figure 14. The cases of ellipse



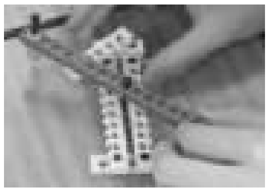


Figure 15. The cases of non ellipse

The change of belief via exploring with technology.

In the second and third lessons, students learned the construction by DGS, and then, they began to construct the mechanics on DGS. At the fourth lesson, students read the text of Descartes, explored the meanings with DGS and read the text of Pascal. After discussing their opinions, they described their opinions as follows.

For me, mathematics was symbolized by the words ‘memorize theorems’ and ‘knowing the ways of calculation’. I only learned a few parts of history, but I could know that those mathematicians’ ideas and their ways of explorations are far from today’s mathematics. The reason why the ancients preferred mathematics for the initiation of philosophy must be mathematics was not difficult or mathematics was integrated enough to understand. Unfortunately, the image of today’s

> mathematics is too hard for the people. Through using tools, we can demonstrate in the classroom how ancient mathematicians explored comprehensible mathematics.
>
> Before the lessons, I thought today's school mathematics was the most refined and thus, the most simple mathematics. But I experienced that the ancient mathematics and tools had specific reason which should be used and easily understandable. This is a new perspective for me.
>
> I did not think about the way of construction any other ways but in the textbook. I learned that the historical textbook and tools tell us a lot of unknown methods. And through the interpretation of history, I could know other aspects of mathematics.
>
> People imagine problem solving from the word 'mathematics'. Although, I only learned some history of mathematics, I know the importance of knowing the roots of problems and ideas.
>
> I believed that to deduce from the assumption to conclusion is the formal way of mathematics. Through the lessons, I learned that the analysis of mechanics by which to try to find the solution and the reasoning of other representations in the case of obstruction. From the history, I learned another face of mathematics.

These opinions expressed that those four lessons strongly impressed them to reconsider their belief of mathematics as a human enterprise. At the same time, some opinions implied that their belief about mathematics before the lessons was not appropriate for future mathematics teachers who could teach mathematics as human activity. Unfortunately, in the case of Japan (see TIMSS), these inappropriate opinions exist even in the undergraduate math majors. Thus, teaching history with both traditional and new technology for pre-service teachers is effective for the change of belief in mathematics.

Conclusion

The aim of this paper is to illustrate the significance of using technology in the case of teaching mathematics history; specifically, mathematics as a human enterprise through giving students the chance to experience the hermeneutics in mathematics history. Both historical and new technologies are useful tools for knowing the imbedded socio-historical-culutral perspective in mathematics. In the last part of the paper, the effect of using both technologies in teaching history was expressed by the students voices after the lessons. Students were impressed with their changes of belief in mathematics as a human enterprise. Teaching history with both traditional and new technologies for pre-service teachers is effective for the change of belief in mathematics, and, it could be said, necessary for future mathematics teachers.

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