# The Effect of the Use of Technology to Explore Functions (2)
## ~ Educating Mathematical Literacy for the Users of Mathematics ~

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**Abstract:** A teaching approach of functions was planned that focuses on educating mathematical literacy for the "users of mathematics". The "users of mathematics" refer to people who are convinced of mathematical ideas with their own words and can apply these ideas to various situations in everyday life. In this teaching approach, students are encouraged to solve problems with trial and error or to conduct experiment by using technology etc. Learning situation to stimulate students' interests and intuitions was prepared, and they investigated various kinds of functions with their own context, by using graphing software. In this article, a classroom trial of this teaching approach is described in detail. Also, the effects of students' learning activity in this situation for users of mathematics and of the use of technology are examined. The fact that an activity using technology in this approach resulted in students' autonomous learning is also shown.

### 1. Educating Mathematical Literacy for the Users of Mathematics

If we suppose a distinction between people who will use mathematics in future everyday life (users of math) and people who become professional mathematicians, the difference of these two targets of mathematics education seems to be less noticed in mathematics education. But in recent highly information-oriented and high technological society, a speed of advancement of mathematical knowledge become very rapid, and at the same time mathematical literacy in daily life becomes more important, then those different aspects are needed to be taken into account in mathematics education.

"Dozens of informative books echo and amplify the historical dichotomy of mathematics as academic and numeracy as commercial"(Steen,1999,p.10). The aspect of users of math can be found in many recommendations for education[1]. In line with an importance of its education, what users of math can do is as follows. Users of math occasionally manipulate concrete material and experiment themselves,

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[1] "Mathematics Counts"(Cockcroft,1982), "Intelligent use of Mathematics and Ability of Thinking"(Fujita,1991), "The New Literacy" (Steen,1997).

which leads to inductive mathematical thinking, therefore they can understand mathematics by themselves. Moreover, they should try to make a use of mathematical skills in their everyday life, with trial and error if they need. And learning mathematics will differ from traditional one that depends on proofs. Then, it is possible to learn by activities of practical experiment and experience. "There are many cases that students can experience mathematics more powerfully by computer." (Fujita,1991,p.71). And Fujita said, "Their understanding with satisfaction is consistent with mathematical concepts.".

## 2. Background

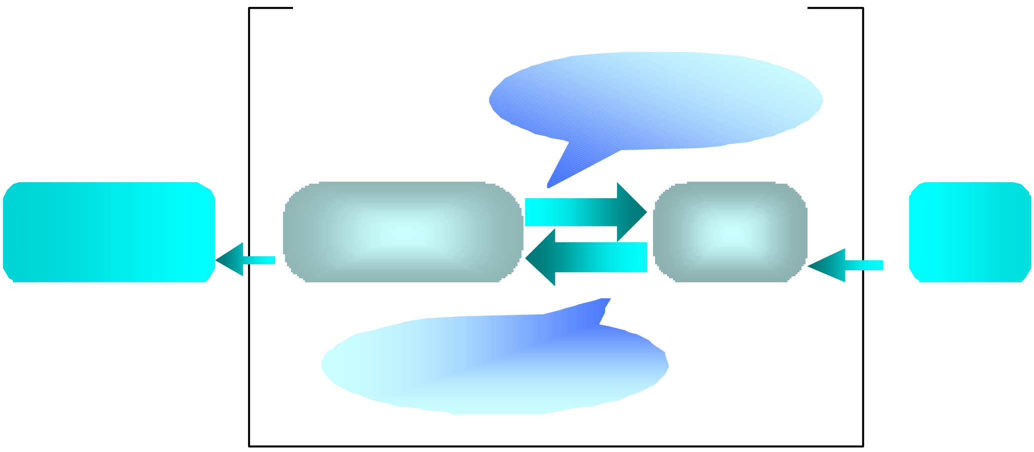
In upper secondary school, learning functions is the most important theme, but they do not feel enough a power of functions that are used in a various situations for users of math. Based on private teaching experience, the reasons might be follows.

a) Students spend a lot of time to practice transforming symbolic expressions, then they cannot combine behaviors of data or phenomenon behind functions to expressions.

b) They feel learning of functions a routine work, as a result, they are not interested, and they do not want to explore functions in their context.

In order to improve those situations, it will be very effective that students visually manipulate various kinds of functions with technology. In this classroom trials, two kinds of approaches to learn functions with technology are designed (fig.1).

<u>Approach 1)</u> Students control numbers in expressions of several functions and then they observe its graph visually. Students understand the behavior of functions through this activity.



<u>Approach 2)</u> Students image the shape of graphs, and they try to draw a graph of functions by graphing software. They have to distinguish graphs tried, and they choose a proper expression for their own one. In this case, someone can do it by algebraically, but others can fit expressions to graphs by trial and error and finally choose correct expression.

When they learn in traditional class, the Approach 1) is more often used than 2). But for education of users of math, Approach 2) should be more emphasized than 1), and also both should be in a balance. Because, in everyday life it is important that we get some data, find the pattern, express it and predict or guess.

## 3. Method

Right after the unit of "figures and equalities (including equalities of circle)" had just finished, the following classes including two tasks were planned.

Subjects: 43 students in the second grade of upper secondary school (16,17 years old).

Lessons: Three lessons of 50 minutes each were used for this trial.

Software: "Grapes[2]" (fig. 5,6,7). This software do not only draw a graph for a given expression, but also has powerful features, such as when parameters in an expression are changed, the effect of the change of graphs can be observed and explored.

Task 1: This task is planned for Approach 1). Following expressions are given (fig.2), students observe a graph, search the behaviors of functions visually and categorize functions with their characteristic common to some of functions. These categories are of their own criterions.

Task 2:

This task is planned for Approach 1) or 2), especially 2). They decide and explore their own themes in which they are interested when categorizing functions.

Through teachers' observations and students' worksheets, we analyze what activities students have and how students' activities are helped with technology.

**Theme: categprize functions** class no name

Categorize those functions at your own aspect. You could find many relations among functions, then check in upper table and write their relations in lower table and reasons if possible.

| no | functions |
|---|---|
| 1 | $x-y+2=0$ |
| 2 | $2x-2y+4=0$ |
| 3 | $x^2+y^2=1$ |
| 4 | $2x^2+2y^2=2$ |
| 5 | $x^2+y^2=9$ |
| 6 | $x^2+y^2=4$ |
| 7 | $x^2-y=0$ |
| 8 | $x^2-(y+3)=0$ |
| 9 | $x^2-y-3=0$ |
| 10 | $(x-1)^2-y=0$ |
| 11 | $x^2-2x+y^2-3=0$ |
| 12 | $(x-1)^2+y^2=4$ |
| 13 | $x^2+y^2-2y-3=0$ |
| 14 | $x^2+(y-1)^2=4$ |
| 15 | $x^2-2x+y^2-2y-2=0$ |
| 16 | $y=2x^2-4x+6$ |
| 17 | $y=x^2+6x+3$ |
| 18 | $y=x^2+4x+3$ |
| 19 | $y=\frac{1}{2}x^2-x+\frac{5}{2}$ |

| no | functions | a | b | c | d |
|---|---|---|---|---|---|
| 1 | $y=x+2$ | | | | |
| 2 | $y=-x+2$ | | | | |
| 3 | $y=x-2$ | | | | |
| 4 | $y=-x-2$ | | | | |
| 5 | $y=-2x+2$ | | | | |
| 6 | $y=2x+2$ | | | | |
| 7 | $y=\frac{1}{2}x+2$ | | | | |
| 8 | $y=x+4$ | | | | |
| 9 | $y=x^2-2x+3$ | | | | |
| 10 | $y=x^2+2x+3$ | | | | |
| 11 | $y=-x^2-2x+1$ | | | | |
| 12 | $y=x^2+2x-1$ | | | | |
| 13 | $y=-x^2+2x-3$ | | | | |
| 14 | $y=(x-1)^2+2$ | | | | |
| 15 | $y=2x^2-4x+4$ | | | | |
| 16 | $y=2x^2-4x+6$ | | | | |
| 17 | $y=x^2+6x+3$ | | | | |
| 18 | $y=x^2+4x+3$ | | | | |
| 19 | $y=\frac{1}{2}x^2-x+\frac{5}{2}$ | | | | |

## 4. Results

Task 1:

(1) Examples: How students categorized functions.

[2] It is developed by Tomoda, K. in 1998 and improved to "Version 5.61a" in 2000.

First, students tried to categorize without technology, some of them were at a loss of what to do. Then, students used "Grapes" and visualized. After some observations, they started to categorize. As seen in fig.3, student K.M. could recognize common characteristic but could not represent well. Student S.F. could find some functions' groups that were categorized with parallel of liner functions or same shape of quadratic functions, but he could not connect with them. And student H.O. recognized translation that is common throughout functions.

(2) Students' comments.

| most general character | | | | | | | | | | | translation |
|---|---|---|---|---|---|---|---|---|---|---|---|
| common charactors of functions | parallel | parallel | parallel | parallel | same shape | same shape | parallel | parallel | parallel | same shape | same shape |
| $y=x+2$ | * | * | | | | | * | | | | |
| $y=-x+2$ | * | | * | | | | | * | | | |
| $y=x-2$ | * | * | | | | | * | | | | |
| $y=-x-2$ | * | | * | | | | | * | | | |
| $y=-2x+2$ | * | | | * | | | | | * | | |
| $y=-2x-2$ | * | | | * | | | | | * | | |
| $y=x+4$ | * | * | | | | | * | | | | |
| $x^2+y^2=9$ | | | | | * | | | | | * | |
| $y=x^2+2x-1$ | | | | | | * | | | | | * |
| $x^2-4x+y^2-2y-4=0$ | | | | | * | | | | | * | |
| $y=x^2-3x+3$ | | | | | | * | | | | | * |
| student | K.M. | S.F. | | | | | H.O. | | | | |

When one student wanted to display a pair of equivalent expressions on the screen, for example $y=x+2$ and $2x-2y+4=0$, he said "The software could not draw the second graph.". Later, he commented that he had not noticed one graph was on the other. Another student said, "In past lessons we learned functions always only with expressions. But now a circle appear on a screen, I realize it is a circle.". Other comment "As I control numbers in an expression, the graph change together with it. Otherwise, I can understand how to control numbers according to a graph I want to draw.". Students can realize a connection between graphs and expressions and have a deep interest.

Task 2:

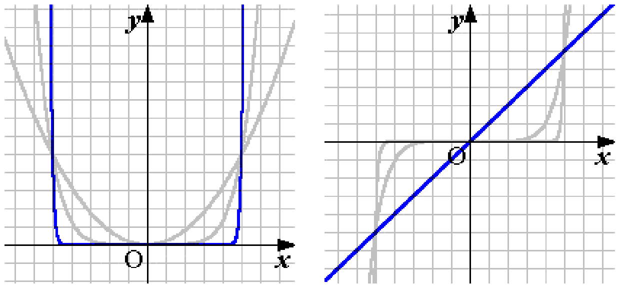
(1) Case of M.F. (fig.4)

From the form of circle equations, she got an idea to increase the degree of x and y of $x^n + y^n$ . She actually tried the idea by "Grapes". She was interested in what happened in graphs, and next tried about $y = x^n$ . In textbook she had learned following functions $y = x^2$ , $y = x^2 + c$ , $y = x^2 + bx + c$ .

But now she can explored $y = x^2$ , $y = x^3$ , $y = x^4$ with "Grapes". She explored various kinds of functions that she wanted. Her intuitions that "as $n$ increased, the change become rapid" and "the shapes of graphs can be grouped into odd orders and even orders" are very important.

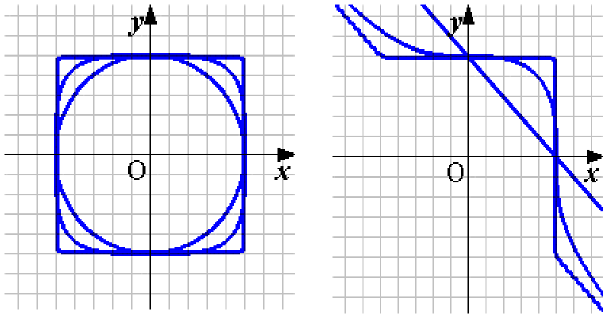
(2) Case of K.K. (fig.5)

He tried to draw a tangent to the circle $x^2 + y^2 = 9$ . He drew them at random, then confused. As he noticed there were many tangents, he fixed the gradient of tangent was one, and translated this line. At this time, he used this software's feature to change parameters, and he got the value of intercept 4.3. "What is this?" He may try to calculate the value algebraically, or may fix another condition and explore.



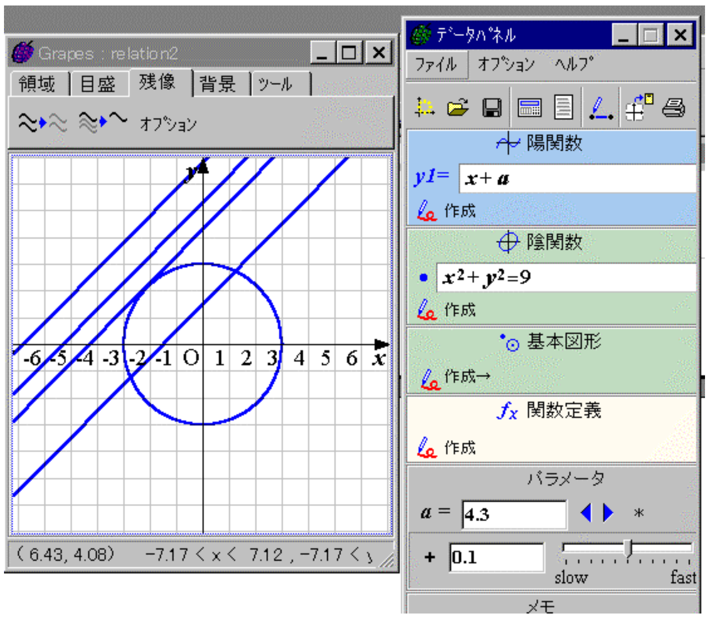
(when $n$ is even numbers) (when $n$ is odd numbers)

(2) $y = x^n$



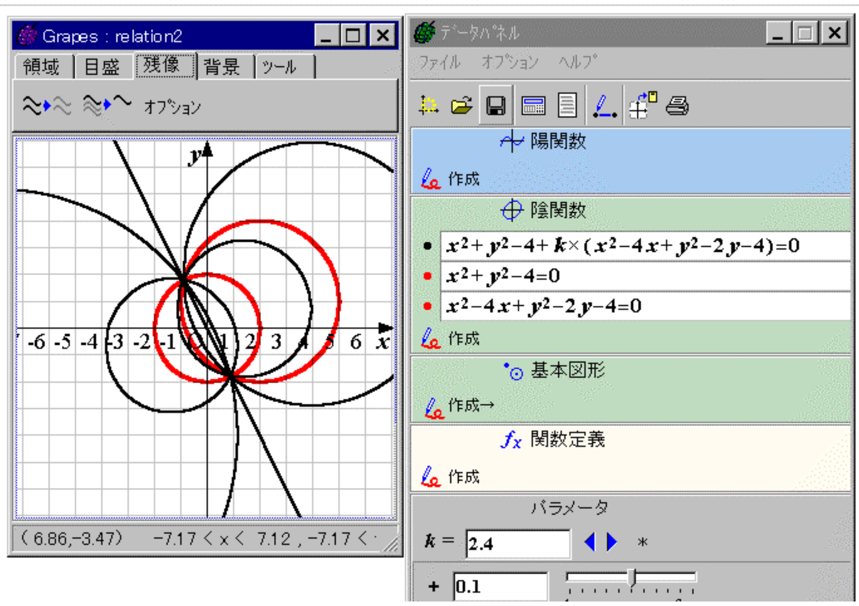
(when $n$ is even numbers)(when $n$ is odd numbers)

(1) $x^n + y^n = 1$



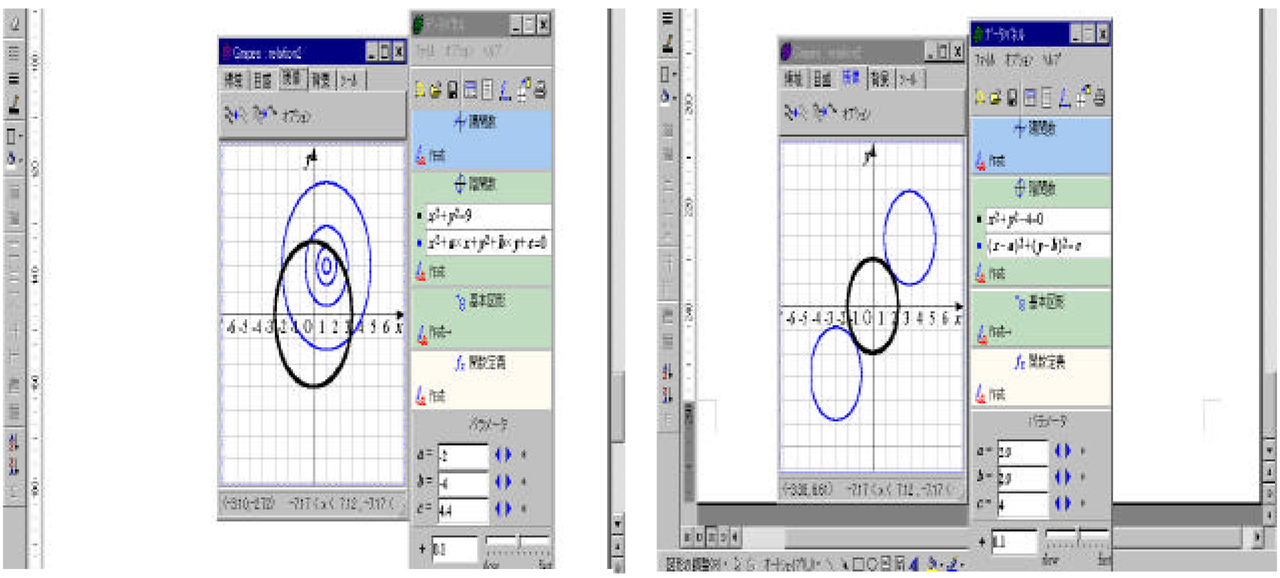
(3) Case of T.K. (fig.6)

His problem, drawing a graph passing at the intersection of two circles, is one of the textbook's exercises. In previous class he had wanted to solve it, but he did not understood its meaning and he could not solved it. Even after he had learned the typical solution, he had not been convinced. He tried it again by "Grapes", and then understood its meaning and solution. He used this software's feature to change parameter, he said "As I change parameters, the graph moves about.". In fact, its movement is very fun. At that time, this problem came to be his own.




(4) Other problems students posed (fig.7).

All of them solved their own problems at their own levels. One student chose his theme in a textbook, the other picked his theme because he wanted to draw a beautiful design on a screen visually. When they explored their themes, an expression of what they wanted is obtained at once algebraically, also it can be obtained by the trial and error.



## 5. Discussion.

1. The Effects of the Learning of Functions.

(1) They understood functions more deeply by multiple representations or categorizing.

As seen in the student's comment, in most of lessons he had learned functions mainly with expressions, but in this approach functions were represented by expressions, dynamic graphs and data. One student was convinced of behaviors of equivalent expressions by visualizing. "One line of research on how people learn emphasizes the helpfulness of making multiple representations of the same idea and translating from one to another."(AAAS,1993,p26). Now, students use technology for connecting them together, but later he will be able to do in his head.

Student H.O. noticed a similarity between two relations in categorizing functions. Also student M.F. noticed a similarity between two function groups in his exploration. "Mathematical thinking often begins with the process of abstraction ."(AAAS,1990,p.19). In this sense, categorizing process is useful, and relations between each function are realized. In addition, "they strengthen belief in the correctness and underlying unity of the whole structure"(AAAS, ibid. p.16). This activity was helpful for students to understand functions deeply enough to use them in real world.

(2) They tried to use functions independently and to explore inductively.

As users of mathematics, an attitude to try to use functions is important. It is most important that the attitude arose in students' activities. They slipped out a passive learning in which the procedures were predetermined and "correct" answers were expected. And student M.F. started to explore functions that she had not learned yet. This activity is highly motivative, and can go beyond curriculum. Also it urges them to develop informal intuition. They decide their own themes, and solved them in their own ways.

Experiment or trial and error, which are found in some student's activity, is also a powerful and important method for users of math. "A useful series of manipulations has to be worked out by trial and error."(AAAS,1990,p.20). After trial and error, student K.K. finds a rule and explores by the rule. He finds out a intercept 4.3 and try to verify it in algebraic way.

(3) They could experience to search and use the tool more effectively.

Student K.K. and other students adapted this software's feature to change parameters in their own problems. Users of math need to select the best method for solving their problems, instead of given method in textbooks. For that purpose, they need to experience this kind of mathematical inquiry.

2. The Effects of the Use of Technology.

(1) They were interested in exploring functions.

The reason why students learn on their own initiative mostly due to use of technology. Student T.K. did not understand the meaning of the textbook's problem and were not interested in it until he draw graph using technology. One student said, "I can stretch and squeeze graphs by changing the number in an expression.". In his words, we feel the use of technology excite his interests.

(2) They became confident to solve the problem and their abilities.

Though they pose a problem, if they have nothing to help solving it, they will lose their interest. "Computer technology continues to help solve previously daunting problems."(AAAS,1990,p.18). One student said "After I drew many graphs, patterns came out. I become to be able to control expressions in order to obtain graph that I want.". They become confident to solve the problem, if they use technology.

(3) They could explore mathematics by trial and error.

As Fujita said, practical experiment and experiential activity are powerful and important method for users of math. In this trials, students need to draw many graphs at once, the use of technology has made trial and error activity possible.

## 6. Conclusion.

In our learning of functions with the goal of which is Users of math, those effects are as follows

(1) Students could understand functions more deeply.

(2) Students could explore independently.

(3) Technology plays an important part in understanding and exploring.

A knowledge which is given is weak and not flexible. Through this activity, in which students require action not just observation, they could construct knowledge themselves. In this meaning, it is important that a lot of independent exploring experiences like this are adapted into classes.

## Reference

American Association for the Advancement of Science (1990), Chapter2 "The Nature of Mathematics" in "Science for All Americans", Project 2061, Oxford University Press. pp.15-22

American Association for the Advancement of Science (1993),Chapter2“The Nature of Mathematics" in "Benchmarks for Science Literacy", Project 2061, Oxford University Press. pp. 23-38

Cockcroft Wilfred H.(1986), ‘Mathematics Counts Report of the Committee of Inquiry into Teaching of Mathematics in Schools under the Chairmanship of Dr W H Cockcroft’, London: Her Majesty’s Stationary Office, pp.1-11

Fujita, H. (1991), "Intelligent use of Mathematics and Ability of Thinking", Mathematics Seminar,1991,Aug.,(pp.68-71)

Fukuda, C., and Shimizu, K. (1998); A Study on New View of Mathematical Literacy in 'Science for All Americans', in a Proceeding of 22nd conference of

Japan Society for Science Education, pp.259-260
Kakihana, K., Fukuda, C. and Shimizu, K. (2000); The Effect of Visualization of data on Learning Function-A Use of the Software and advanced Calculator to Explore functions-, in a Proceeding of the 24th conference of Japan Society for Science Education (in print).
Steen, L.A. (1990), Pattern, in "On the Shoulder of Giants- New Approaches to Numeracy -", National Academy Press,pp.1-10
Steen, L.A. (1997), 'The New Literacy', in Steen, L.A. "Why Numbers Count - Quantitative Literacy for Tomorrow's America -", College Entrance Examination Board
Steen, L.A. (1999). NUMERACY: The New Literacy for a Data-Drenched Society. Educational Leadership,1999,Oct.,(pp.8-13)
Tomoda, K., http://www.osaka-kyoiku.ac.jp/~tomodak/grapes/