The Effect of the Use of Technology to Explore Functions(1) ~ Visualization of Data on Learning Functions ~

Kyoko Kakihana[*], Chieko Fukuda[**], Katsuhiko Shimizu[***]
Tokyo Kasei Gakuin Tsukuba Junior College[*],
Hakuoh University[**],
National Institute for Educational Research[***]
kakihana@cs.kasei.ac.jp[*]

Abstract: In this article, an integrated approach to the learning of functions is introduced, and the effects of students' learning activity and of the use of technology when students learn functions in an integrated way were examined in classroom experiments. Integrated learning of functions refers to an integration of functions with real world situations and data as well as to an integration of various kinds of function using expressions, graphs and numbers. In these activities, technology is effective in helping students to visualize the situation of functions.

1. Background

These days everything changes so fast and it often happens that things at the tertiary level learnt in school often become too old in a very short time. At the top of this, many professors complain that the achievement level of students in mathematics classes is getting lower with students therefore unable even to compute basic calculations such as fractions (Okabe, T. and others, 1999). It is necessary that students are able to think and make judgments depending on the situation. NCTM Standards emphasize that algebra in grades 9-12 includes both the study of relationships among quantities and the study of structure. Each might be studied separately, whereas powerful mathematics can be attained when these perspective are integrated (NCTM, 1998, P.281). For students living such a changeable world, developing these abilities to integrate modeling real phenomena with functions and being able to analyze from an overall perspective various types of patterns and functional relationships are vital importance.

The fact is, however, that many students find the learning of functions difficult, consequently, dislike it. There are many reasons for this in Japan. For one, in the school curriculum, linear functions and quadratic functions, cubic functions and so on are taught separately and students feel always not to have digested any of them (Steen, 1990, p.4). Another reason is that in schools arithmetic and algebra have always been dominated by the goal of training students to manipulate numerical

and algebraic symbols (Fey, 1990, p.62).

As a first step toward solving the above problems, in this study, we set up classes whose objective was an integrated learning of functions. The hypothesis of this integrated learning of functions is as follows. At first, students will explore the characteristics of functions by observing behaviors in a graph and find patterns of the data in integrated ways. Thereafter the ability to use various kinds of functions in the real world will be developed. Computer technology today can produce graphs of data and functions to fit data very easily. And it makes possible for students to grasp and compare these functions visually (Shimizu, K, 1997, P.19), or to verify them from the numerical table by software. In the integrated learning of functions, through technology such as exploratory graphing software, students will be able to learn how to interpret technological representation and how to use the technology effectively and wisely, as emphasized by NCTM Standards (NCTM, 2000, p.37).

The purpose of this study is to introduce an integrated learning of functions in a classroom trial and to study the effects of the students' learning activities of functions in an integrated way and of the technology such as exploratory graphing software in these activities.

2. Integrated Learning of Functions

By an integrated learning of functions we mean the following: (1) an integration of functions with data and a real-world situation and (2) an integration of various kinds of functions with expressions, graphs and numbers. Exploration of patterns of data and functions are the mainly targets of students' learning activity (fig.1).

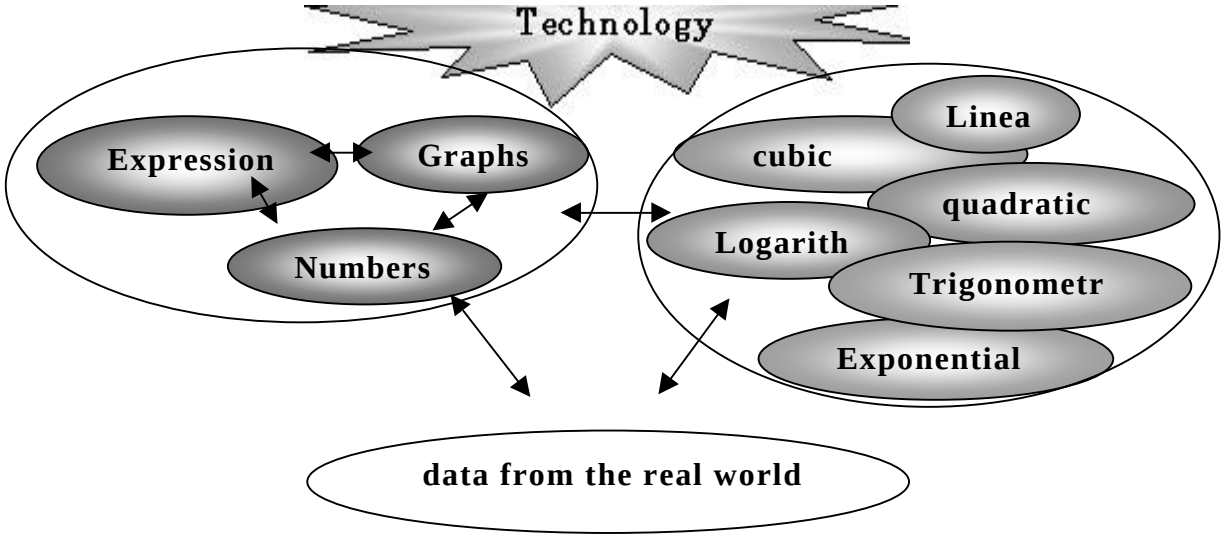


fig.1 Integrated Learning of Functions

As the result of integrated learning, students could observe expressions, graphs and numbers at the same time while numbers in an expression or data are changed and could connect them to each other and analyze data. Thus they come to understand the connections between symbolic, graphic and numerical forms of the same ideas (Fey, 1990, p.75). In this way, various kinds of functions are connected and categorized according to the behavior of each function, for example the tendency gradually increases or decreases and the variable "n" of an expression is either an even or an odd number. As the result, students find the hidden pattern of data (Steen, 1990, p.4) and are able to explain or express the pattern of data by using functions. Graphs, expressions and numbers help students to explore the behavior of the function. Technology such as exploratory graphing software provides students with the tools they need to observe the behavior of functions and data at the same time through the display of expression, graphs and numbers on one screen. In this approach, real-world data are recognized and analyzed through various kinds of functions. This approach is widely used throughout science and industry to portray the behavior of a variable (Steen, ibid, p6).

3. Method

The possibility of integrated learning of functions using the exploratory graphing software "Calculus Unlimited" was tested in two junior college classes. Through the teacher's observation and students' worksheets and questionnaire, the extent to which students' learning activities were integrated as a result of using technology was analyzed. Also, how their activities were integrated was analyzed.

4. Exploratory graphing software "Calculus Unlimited" (http://www.visual-math.com/)

"Calculus Unlimited" was developed for exploring functions visually in 1996 by J.L. Schwartz, and M. Yerushalmy. It is used for learning and teaching algebra, trigonometry, pre-calculus and calculus, coherently and consistently by starting from the concepts of function and variable. Students are able to observe functions by their symbolic expressions, their Cartesian graphs and their numerical values at the same time on a screen (fig2.1). This software has many powerful features to

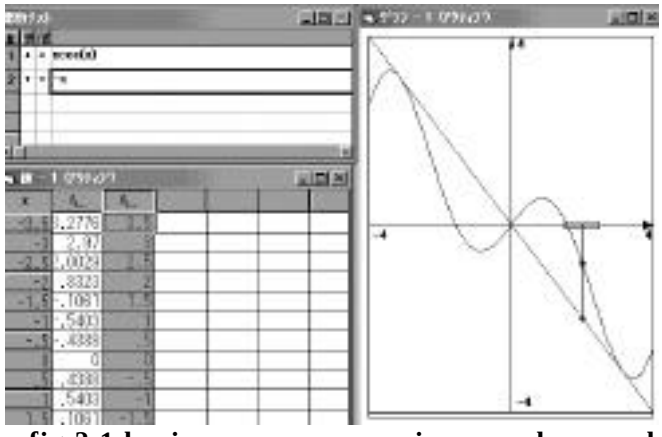


fig.2.1 basic screen: expression, numbers and graphs are observed on one screen.

explore functions. Here, three of them are shown for their value in exploring the behavior of functions. With the first one, when symbols in an expression are changed, the effects of the change on graphs and values are observed and explored. In the same way, when graphs are transformed by shifting, stretching, squeezing and reflecting, the effects on symbols and values are observed and explored (fig.2.2).

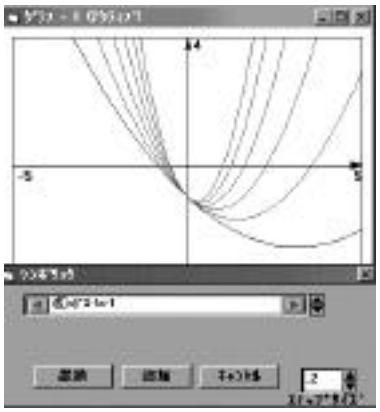


fig.2.2 graphs are stretched by symbol

With the second one, functions are calculated visually (fig.2.3). When two functions are given, students can observe

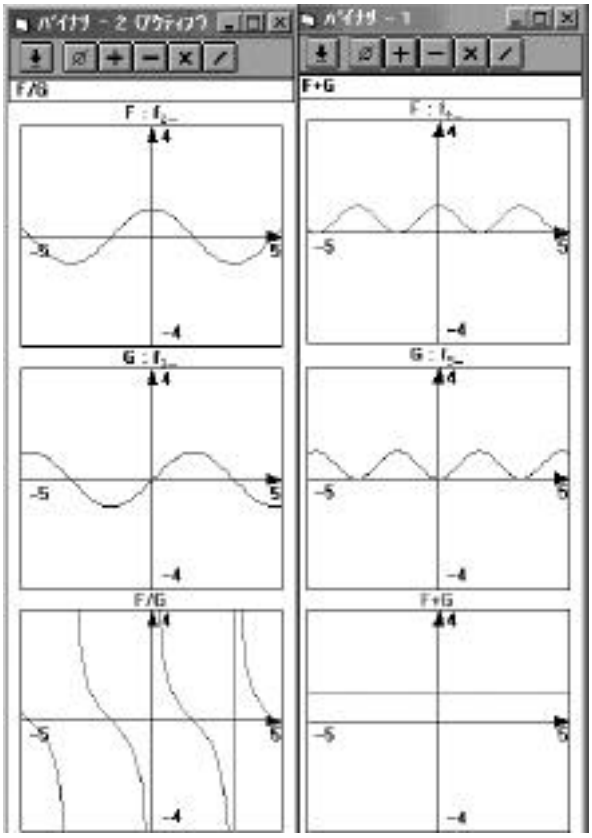


fig.2.3 calculation of two given functions

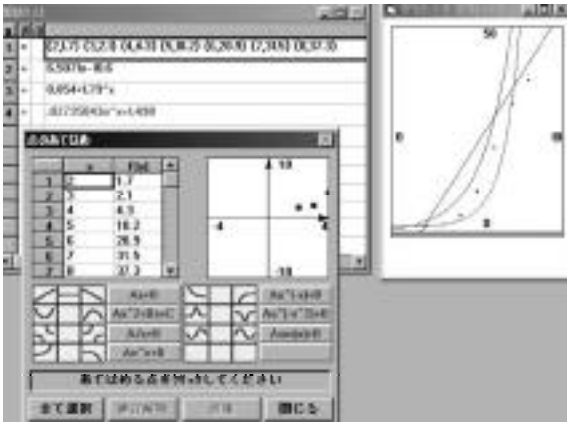


fig.2.4 fitting curve

results of calculations of these in visual form. With the third one, a student can select a function from the visualized list of functions to fit data (fig.2.4) and explore functions in order to find the best one. The third one was used in this study, especially.

5. Results

We conducted the study in two junior women's college classes (19,20 years old).

Class A was taught in 1999 and consisted of twenty-two girls. Class B was taught in 2000 and consisted of thirty-two girls. Each class was made up of three lessons, each lasting 90 minutes.

Task 1: Categorize functions on a list of functions

Task 2: Gather real-world data and find functions to fit the tendency of these data. After finding a function, calculate the prediction of future data and evaluate the fitness of selected functions.

(1) Students' activities in task 2 were as follows.

Case of Eiko in Class A(fig.3.1);

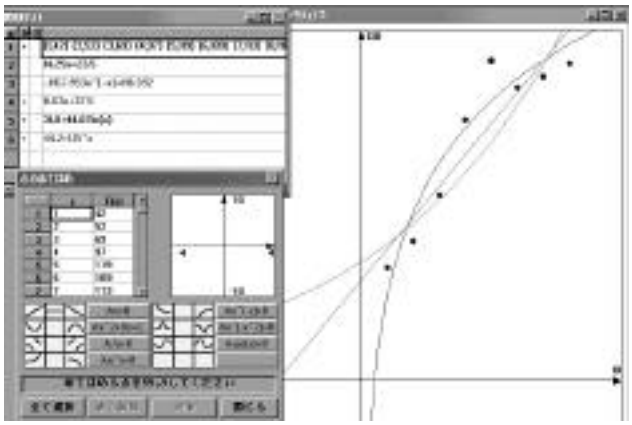


fig.3.1 Eiko's screen

She gathered the data of sales' change of industry for waste from the internet. She tried some functions (a logarithms function, an exponential function and a linear function) to check whether to fit data. She evaluated as follows, "At the beginning point data goes up steeply and then gradually goes up very slowly. I don't think these data will go up continuously. Therefore, the logarithms function fits well to these data". She added, " I learned logarithms function in high school but have forgotten it. Now I am very glad to have been able to use it"

Case of Keiko in Class A (fig.3.2)

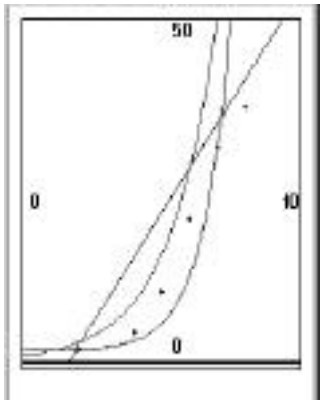


fig3.2 Keiko's graph

She gathered data of numbers of cellular phone subscribers and she explained, "From 1992 to 94, the number did not increase so much, but from 95 to 97, it went up rapidly. Therefore, the exponential function seems to fit well to the data. But I think, in the real world it will become common for everybody to own a cellular phone, so the number will not increase any more and so the data will not fit this function". She tried to connect the tendency of data with the shape of graph, expression and the real world.

Case of Ayumi in Class B(fig.3.3)

She gathered data of numbers of travelers to foreign country for years since 1979. She said "I tried the quadratic function, the cubic function and the exponential function to check whether to fit this data. During these 15 years all functions seem to fit the data. But when I calculated to predict the number ten years from now, I found very interesting things. By using the cubic function the number was getting

down, and by using the exponential function it was twice the value of the quadratic function. When I analyzed functions under a limited area of a graph, every function displayed the same curve. But when I calculated the prediction of data by using each function, they were very different. It was very interesting to find the function which fitted best for the future."

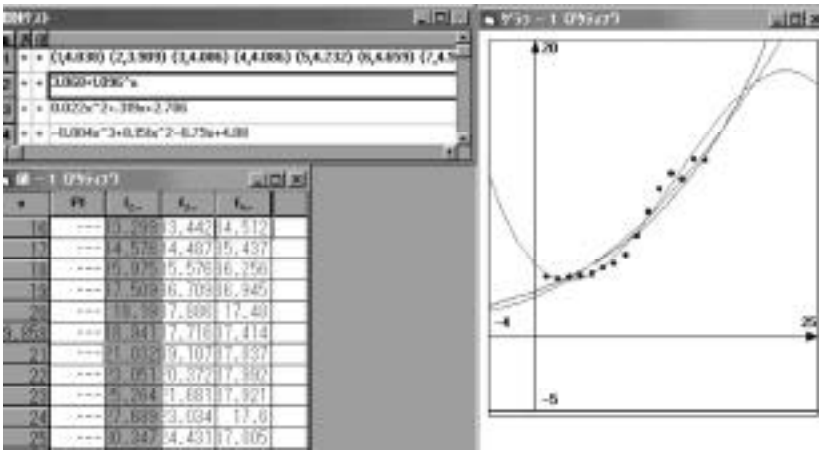


fig.3.3 Ayumi's screen

Case of Makiko in Class B

She told, "I thought the function which fit best was a linear function. But I tried a cubic function and a quartic function. I was surprised to find that the quartic function seemed to fit best. But I thought the number of data was too small to decide which function would fit best."

Students gathered many kinds of data, for example the change in the amount of children's pocket money, of population of people who are older than 100,etc. from the internet or an almanac and explained various kinds of functions by observing graphs. One girl said, "It was exciting to get many interesting results when I tried to fit various kinds of functions for data gathered by myself."

(2) Results of Questionnaire

The questionnaire is conducted to students to identify the effects of this teaching approach.

A. For students in Class A to the question as to how they felt about learning functions after the three lessons, sixteen out of twenty-two students answered, "My view of functions has changed."

B. For students in Class B, fifteen items each for case H and N were given.

Case H: How did you think in high school?

Case N: How do you think now?

Students were asked to select one from four levels and a score was given for each level: 4 = "Strongly Yes", 3 = "Yes", 2 = "No", 1 = "Strongly No". For each question, the scores for each answer were calculated and analyzed by paired t-test.

The results are following (fig.4). In the following results, figures in parentheses mean the average of each item of H and N (H-->N)

The significance of difference between case H and N was shown in the following items.

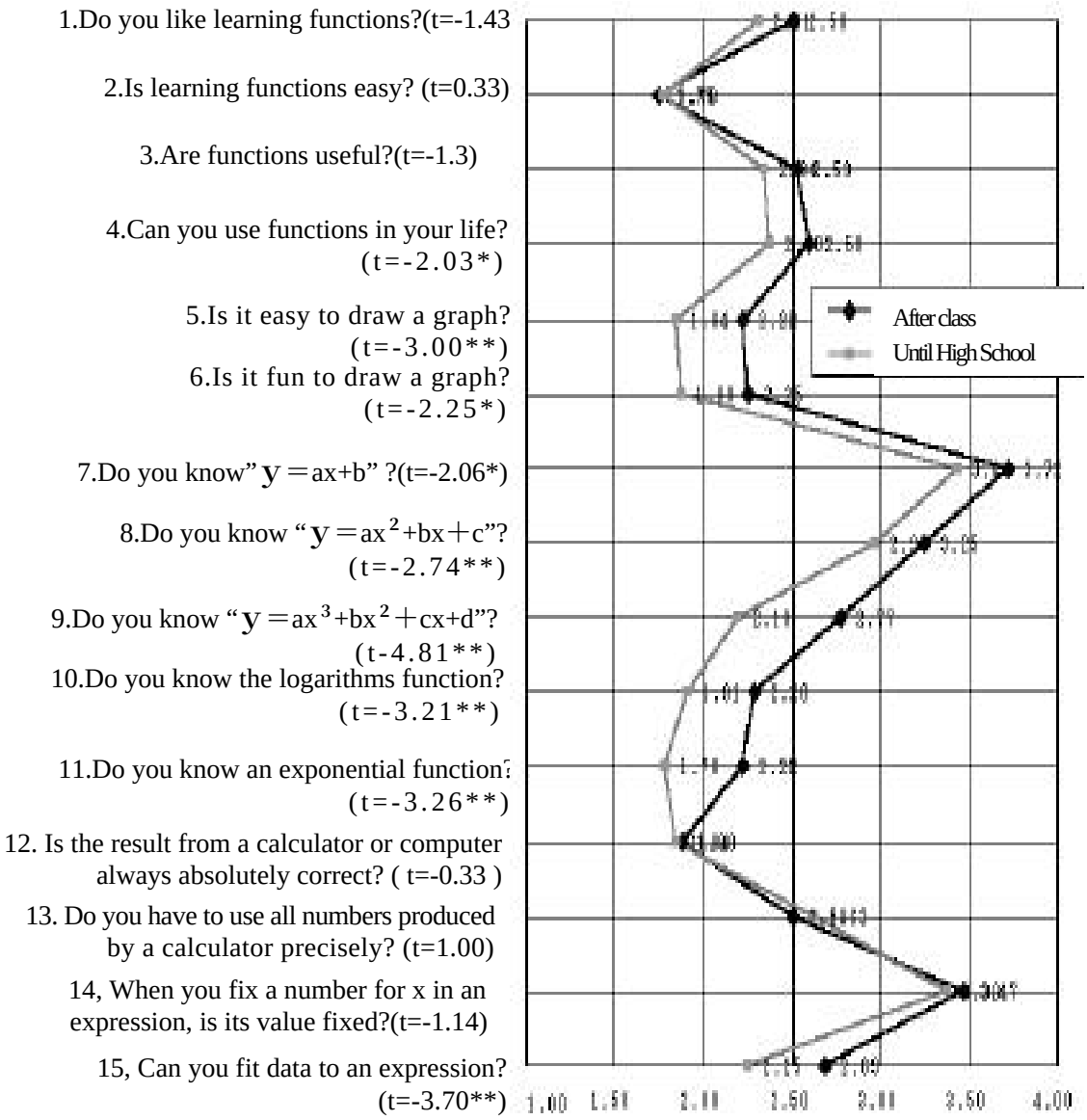


fig.4 Questionnaire Results

(t :t-value,*:5%,**1%)

4: Can you use a function in your life? (2.38-->2.59,t=-2.03,p<0.05)

5: Is it easy to draw a graph? (1.84-->2.22,t=-3.00,p<0.01)

6: Is it fun to draw a graph? (1.88-->2.25,t=-2.25,p<0.02)

7.8.9.10:Do you know the form of function such as..... ?

Especially in the average for $y=ax^3+bx^2+cx+d$ (2.19-->2.77, t=-4.81, P<001), logarithm function (1.91-->2.28, t=-3.21, P<0.01) and exponential function (1.78-->2.21, t=-3.26, p<0.01) the difference between case H and N was evident.

15:Can you fit a function to real-world data? (2.25-->2.69, t=-3.70, p<0.01)

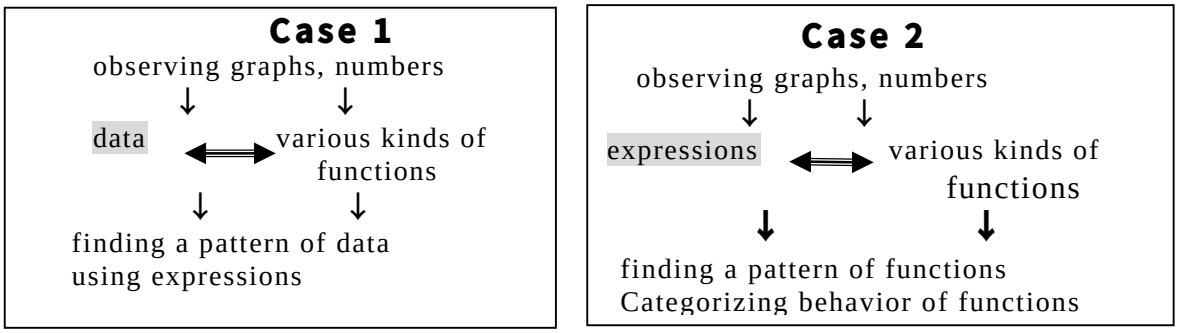
Results of 5 and 6 could be the effects of technology.

The difference was not shown between case H and N for the following items: “1: Do you like learning functions?” (2.31-->2.50,t=-1.43) and 2:Is it easy to learn functions?”(1.78-->1.75,t=0.33). However from the teacher’s observation, it was shown that technology was helpful in drawing a graph and students seemed to feel better with the technology for learning functions. For example, Mariko tried to complete task 1 to categorize functions which she had not tried to do without technology and she commented, “If I could have used a graphic calculator before, learning of functions might have been interesting.” And another girl, Yuka told, “I don’t like learning functions. But if I could have used graphic software in my high school, I might have not disliked it so much”.

6. Discussion

I. The Effects of Integrated Learning of Function

(1) Through these activities, two cases of students’ learning and using functions were found. Case 1 involves integration of data with various kinds of functions by observing graphs and numbers. Case 2 involves the integration of an expression with various kinds of functions. Students can control values in an expression and then observe its effect on a graph. And also students analyze the behavior of functions visually and categorize functions.



Case 1 was shown in students’ activity on task 2. In this case, students integrate data with various kinds of functions and found a pattern of data and used expressions, even though they did not know the definition. In addition, students are able to analyze behaviors of various kinds of functions with real-world situations. Through these activities, students try to interpret graphic representations intelligently and to understand the connections among symbolic, graphic and numerical forms of the same ideas (Fey, 1990, p64). In this case, a teacher can also give students other ideas such as a growth curve that is not taught in the school curriculum but serves as a very important function in the real world. In this activity, students are able to connect activity of data analysis with learning of function. Case 2 was shown in students’ activities on task 1. In Case 2, students

controlled values in an expression and then by observing a change of a graph, they analyzed the behavior of functions visually and were able to categorize the characteristics of functions in the list given on task 1. In this case, students also are able to analyze functions by connecting expressions with graphs, and then reconstructing the characteristics of functions by themselves (Kakihana, Fukuda & Shimizu, 2000).

II. The effects of technology

(1) Through the activities of task 2, students used the technology as potential function generator (Zbiek, 1998, P.193) to fit curve to real-world data and were able to test a various functions visually to find the function fits best. Students used the technology to grasp and verify visually characteristics of functions, and estimated value on the graph. By estimating other ordered pair of function from the graph, students found the perceptual misconception by themselves which is created from the effects of scale and the limited viewing window inherent in computer displays (Fey, 1990, p.75) and they made sure the values calculated by technology with situation of real world. In these activities with integration of graphs and numbers, students learned how to interpret technological representation (NCTM, 2000, P.37).
(2) Students gathered various kinds of data from the internet and found that there were several kinds of functions they could to fit these data. During these activities, the use of exploratory graphing software made it possible for students to integrate and connect the data analysis and functions. The results of our questionnaire and interviews showed that these activities changed students' bad feeling to better for learning functions and made them active.
(3) The activities in this research would not have accomplished without technology, because technology helped to draw a graph very easily and quickly. Also, it helped students to calculate values to explore behaviors of data and to explore the change of graph in order to fit a curve without knowing the rigid definition of a function.

7. Conclusion

The effects of integrated learning of functions and technology are as follows:
(1) Students could explore and explain or argue about patterns of data using various kinds of functions. Students could try also to apply various kinds of functions, even though they have produced from the results of a calculator and are not taught in the school curriculum. As the results students would be motivated to learn new kinds of functions.
(2) Students could think about linear function, logarithms and so on not separately but also connect the behaviors of each function through the shapes of graphs and patterns. The feature to calculate functions in visual form (Yerushalmy,1999) was not used in this activity, but this feature must make students think of the system of calculation of functions as patterns and find the system of functions by

themselves. These activities could lead to informal development of intuition of patterns.
(3) Now students gather data from internet and are able to use data of real world for learning functions. In the future, students could argue about their patterns of functions and their own prediction with friends through the internet and deepen their activities. Through these activities, students' ability of analyzing real phenomena by integrating data with functions will be developed.

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