The ATοMIC Projects - Applications To Mathematics Incorporating Calculators

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The AToMIC (Applications to Mathematics Incoporating Calculators) and Sub-AToMIC (Subsequent Applications to Mathematics Incoporating Calculators) projects were two initiatives of the Queensland Association of Mathematics Teachers, in Australia, that aimed to capture and promote elements of "best practice" teaching and learning in relation to mathematics and graphing calculator technology. These projects were initiated and conducted by practicing classroom teachers to meet the needs of students in and teachers of Years 8 to 12 with a particular focus on the integration of applications of mathematics into students' learning experiences. Through these projects teachers developed ongoing networks that supported the sharing of classroom activities that were created by teachers for teachers. These activities also served as the basis for a program of state-wide teacher professional development.

Introduction

The call for the inclusion of technologies such as computers and graphics calculators as a necessary adjunct to the teaching and learning of mathematics has been echoed in the literature and curriculum documents for at least the last decade (eg, Barrett & Goebel, 1990; Demana and Waits, 1990). The potential for technology to enrich mathematics curricula, teaching, and learning has been noted in documents such as the NCTM *Standards* (National Council of Teachers of Mathematics, 1989, 1991), which explicitly advocate the use of technology in secondary mathematics classrooms. In Australia it is similarly recommended that all students have ready access to appropriate technology as a means to support and extend their mathematics learning experiences (Australian Association of Mathematics Teachers, 1996).

Despite long term attention, the use of computers and other technologies in school mathematics classrooms has been restricted, until recently, by economic, social and practical constraints (Kissane, Kemp and Bradley, 1995; Kaput 1992). At the same time other commentators have insisted that “to expect that schools and teachers can continue to exist apart from serious technological support is hopelessly myopic” (Kaput & Thompson, 1994, p. 682).

The circuit breaker to the tension between the need for the students and teachers to have free access to technologies, and what schools are able to provide, appears to be the development of affordable hand held devices which have become known, variously as graphing, graphical and graphics calculators. A number of reasons have been put forward for the now favourable disposition of a growing number of Australian school systems toward the use of these devices. However, their increasing acceptance appears to be strongly related to the very good value for money they offer in relation to computing power verses cost when compared to a computer equipped with appropriate mathematics specific applications. Graphics calculators represent a one stop shop with respect to the type of facilities presently required by a typical secondary mathematics student at a price point where a class set can be purchased for the price of a single computer.

Two initiatives of the Queensland Association of Mathematics Teachers (QAMT), the AToMIC (Applications To Mathematics Incorporating Calculators) and Sub-AToMIC (Subsequent Applications To Mathematics Incorporating Calculators) Projects, have developed tasks that make use of graphing calculators to learn and apply mathematics in life-related contexts. The aim of both projects was to create materials that provide students and teachers with learning experiences appropriate to both Senior Secondary (AToMIC) and Junior Secondary (Sub-AToMIC) mathematics classrooms and also to act as a starting point for the development of assessment tasks. While both projects were considered successful, from the point of view of the quality of the resources produced, the unique aspect of both of these projects was the process by which tasks were developed which was based on a high level of involvement from practicing classroom teachers.

Background

The Australian Context

The growing acceptance, in Australia, of the need to provide students with access to mathematically enabled technologies, at all times, is evidenced by changes in policy with respect to assessment practices in each of the states and territories. Students in Victoria (1997) and Western Australian (1998) now have assumed access to graphics calculators in their state-wide assessment programs. South Australia has also declared its hand by notifying schools, in 2000, that there will be assumed access to technology in their exams for 2002. Queensland has always had a policy of non-mandatory unrestricted access. Other states and territories are still considering their position.

The Queensland Context

The impetus for the two AToMIC projects lay in the need to develop support materials for teachers in the wake of the new mathematics syllabuses developed by the Queensland Board of Senior Secondary School Studies (BOSSSS) in 1992, and implemented by 1995. These syllabuses included direction to make use of technology in the teaching and learning of mathematics and to incorporate applications of mathematics into teaching and assessment programs. In response to this initiative, the QAMT instigated the first of two projects, the ATоMIC Project, that aimed to develop rich learning experiences and activities for Year 11 and 12 students that were based around graphing calculator technology. A second follow up project, the Sub-ATоMIC Project, aimed at providing similar activities and tasks for Year 8 to 10 students, was initiated in response to the request of members who were seeking similar resources to support calculator active teaching and learning in the lower secondary classroom.

The AToMIC Project

The AToMIC (Applications To Mathematics Incorporating Calculators) Project was QAMT's first attempt at teacher centred resource material development. The AToMIC Team involved 22 secondary and tertiary educators from across Queensland, and consisted of participants from regional centres, including Mt Isa, Rockhampton, Townsville, Toowoomba, Sunshine and Gold Coasts, as well as the capital city, Brisbane. Participants gathered at a writing workshop held at Hillbrook Anglican School in Brisbane in mid 1997 where they learnt how to develop and publish resources with the assistance of a TI-83 calculator, TI GraphLink software, and a computer. The resulting activities were trialled at state, national and international mathematics education conferences throughout 1997 and the feedback gathered during these sessions was used to further inform the development of the project materials. These activities were finally released as a book titled The AToMIC Project (Geiger, McKinlay and O'Brien, 1997)

The Sub-AToMIC Project

The Sub-AToMIC project also involved teachers in the development of learning experiences that integrate the use of graphing calculator technology. This project was different from its predecessor, however, because materials were developed over the internet, via email, rather than through a face to face meeting. It was initiated through the development of six draft tasks that were distributed to the Sub-AToMIC group as Word 6 RTF files. Participants then trialled and /or commented on the materials and this feedback was used to further develop the draft tasks and eventually to create new ones. All new tasks were distributed among the group for critisism.

Participants in this project include volunteers from the original AToMIC team and also self nominees who responded to an invitation posted to the QAMT list serve. The process of task development via the internet was also extended to a national audience though the publication of a paper describing the project in the Australian Association of Mathematics Teachers (AAMT) Virtual Conference in 1998. All members of that conference were also invited to contribute to the project.

An editorial panel was finally responsible for “working” each activity to achieve consistency of quality and style. The final set of activities was also published in book form - The Sub-AToMIC Project (Geiger, McKinlay and O’Brien, 1999).

Project Aims and Task Development

The aims of both projects are summarised below:

- The development of resources specific to the needs of Australian teachers who wish to make use of graphing calculator technology to enhance the teaching of mathematics
- A sharing of “ best practice” learning experiences which can act as models and bench benchmarks for relevant teaching/learning strategies for years 8 to 10
- The sharing of activities that may be used as the basis for meaningful assessment that offer an alternative to traditional pen and paper tests
- The enhancement of teachers skills in the use of graphing calculator technology
- The foundation of an on line support community with a principal interest in the enhancement of the teaching/learning of mathematics through the use of technology
- The publication of a collection of activities so that examples of “best practice” in this field can be shared more widely with the community at large.

With these aims in mind both project teams were given the following guidelines for the creation of activities.

> *Activities should take a problem centred approach to the teaching of mathematics in partnership with graphics calculator technology. Each activity is organised around a problem statement that can be used as:*
>
> - *An unfamiliar application of mathematics students have recently studied*
> - *A way of introducing a topics, or as a focus problem around which a unit of work is organised*
> - *A practice activity used to reinforce a new idea or technique studied within a topic.*
>
> *Activities should be designed around the use of calculators because they are a better way of solving a particular problem not merely for the sake of using a new technology. This is because the facilities offered by a calculator mean students can now solve problems they would not have previously thought of attempting. Further, each activity should contain:*
>
> - *a problem statement; a solution(s); and an extention(s)*
> - *a problem statement that addresses students and a solution section addresses teachers*
> - *enough information to lead a non-novice through the activity but not to the extent of key pressing instructions. Directions should be menu rather than key oriented.*

- *screen dumps used often enough so that a user can check they are still on the right track.*

Extensive keystroke outlines should be avoided in an attempt to provide materials that are less calculator specific and so relevant to students and teachers no matter what type of calculator they use.

An Activity

An example activity drawn from the Sub-AToMIC Projects follows.

How Fast at Sydney 2000?

Much excitement has been generated over the prospect that the Olympics will be held in Sydney in the year 2000. Australian athletes are of course particularly excited about the opportunity of performing in front of their home crowd. This is because of the advantage the "lift" such a crowd always gives an athlete which will make Australians more than usually competitive at these games. This advantage will not be enough on its own, however, and all athletes must set a goal to meet by the time of the Olympics. The 100 metre sprinters will be aiming to run times in the vicinity of those predicted for that year. The tables below are the world records for the 100 metre sprint for both males and females since the turn of the century.

Female

Year	Time (sec)	Name	Country
1934	11.7	Stanislava Walasiewicz	POL
1937	11.6	Stanislava Walasiewicz	POL
1948	11.5	Fanny Blankers-Koen	HOL
1952	11.4	Marjorie Jackson	AUS
1955	11.3	Shirley Strickland	AUS
1961	11.2	Wilma Rudolph	USA
1965	11.1	Irena Kirzenstein	POL
1968	11.08	Wyomia Tyus	USA
1970	11.0	Chi Cheng	CHN
1973	10.9	Renate Stecher	DDR
1973	10.8	Renate Stecher	DDR
1983	10.79	Evelyn Ashford	USA
1984	10.76	Evelyn Ashford	USA
1988	10.49	Florence Griffith-J.	USA

Male

Year	Time (sec)	Name	Country
1912	10.6	Donald Lippincott	USA
1920	10.4	Charles Paddock	USA
1930	10.3	Percy Williams	CAN
1936	10.2	Jesse Owens	USA
1956	10.1	Willie Williams	USA
1960	10.0	Armin Hary	FRG
1968	9.95	Jim Hines	USA
1983	9.93	Calvin Smith	USA
1988	9.92	Carl Lewis	USA
1991	9.90	Leroy Burrell	USA
1991	9.86	Carl Lewis	USA
1994	9.85	Leroy Burrell	USA
1996	9.84	Donovan Bailey	CAN

(Data from World-Wide TRACK & FIELD STATISTICS On-Line - http://www.uta.fi/~csmipe/sport/index.html)

Tasks

1. Use the above data to predict the goal times for the male and female 100 metres sprint at the Sydney Olympics in the year 2000.
2. You should notice that the times for female sprinters is falling more quickly than times for male sprinters. Predict when female sprinters will run the 100 metres as fast as male athletes.
3. Discuss your findings in relation to the limitations of your models.
4.

Possible Solutions

To look at a graph of the data, the statistics for male and female athletes can be entered into lists via the STAT menu and Edit sub-menu (Figures 1 and 2). It is a worthwhile activity for students to determine a "line of best fit" without the use of regression. However, if your aims include finding the most appropriate model by encouraging students to experiment with a range of appropriate functions, or if you want to show students a method of validating their estimated equations, then the regression approach is a useful tool for this purpose.

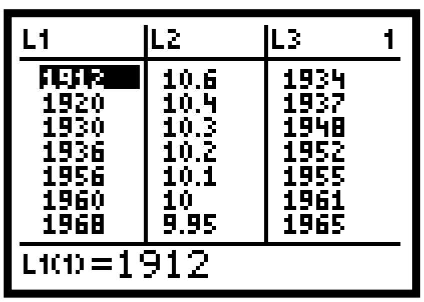

Figure 1

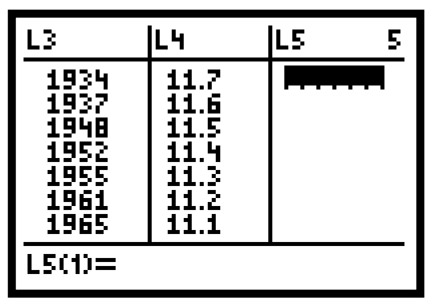

Figure 2

Task 1

By looking at the scatter plots for both data sets (Figure 3), accessed from the STAT PLOT menu, it would appear that linear functions will be reasonable models for both data sets (especially at Years 9 and 10). Sensible first estimates of the gradients of both lines can be based on carefully selected pairs of points from each data set. A value for the y-intercept can be obtained by substituting one on the selected points into the equation y = mx+c with the estimated gradient replacing m and solving for c.

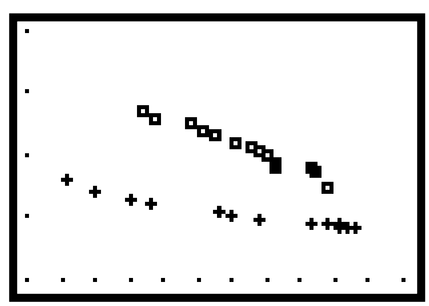

Figure 3

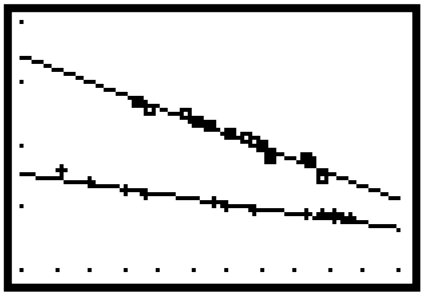

Figure 4

This first approximation of a model can then be entered into the calculator via the Y= menu, plotted against the appropriate set of data, and then be fine tuned until a satisfactory model is produced. Two "eye-balled" lines, $y = -0.008x + 25.75$ and $y = -0.21x + 52.35$, for female and male performance respectively, appear in the Figure 4 (left).

The TRACE function can then be used to determine the predicted times for the race in the year 2000. Alternatively a more precise estimate can be obtained by pressing the TRACE button, positioning the cursor on the line whose equation corresponds to the race being investigated, then typing 2000 followed by ENTER. The calculator will return an estimate based on the model for the data set of interest. In this case the predicted times are 10.35 seconds and 9.75 seconds, for the female and male races respectively (Figures 5 and 6).

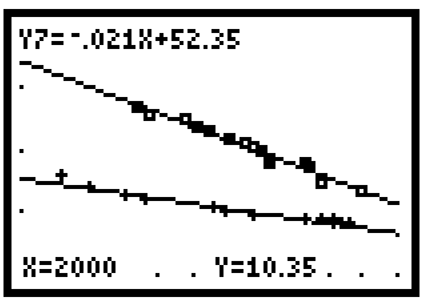

Figure 5

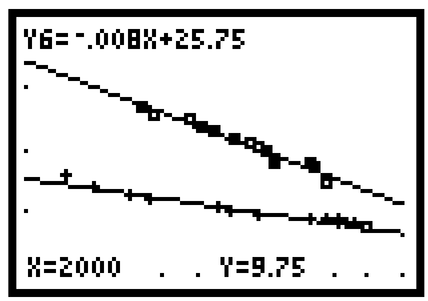

Figure 6

Task 2

By altering the viewing window (Figure 7) for the graphs above it is possible to view the intersection point of the two equations (Figure 8)

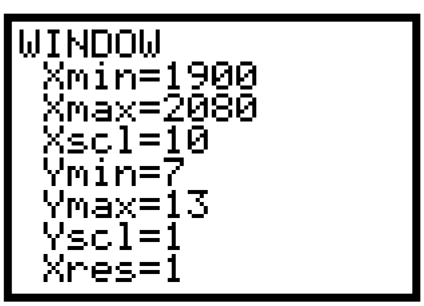

Figure 7

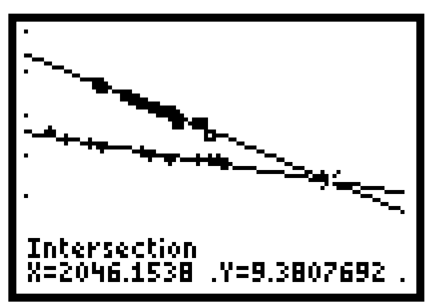

Figure 8

The point of intersection can be found by selecting "5. intersect" from the CALC menu (2nd TRACE). The calculator will request that the intersecting curves be identified (move the up or down button then push ENTER) and then ask for a guess (entered numerically then ENTER or move the cursor to the intersection then ENTER). These models predict that men and women will be will be running the 100 metres sprint in 9.38 seconds by 2046.

Task 3

These results are of course subject to the vagaries associated with model building. While the models chosen are reasonably appropriate up to the year 1996 they must break down as times for this distance approach the absolute limit of human performance. Some judges believe we are close to this limit now. Clearly, linear models will break down fairly quickly. No one, for example, will ever run 100 metres in 0 seconds as predicted by there models.

Extension

1. Repeat tasks 1 to 3 using other non-linear models.
2. Investigate the same questions for the men's and women's marathons (data is available form the same website footnoted on page 1).

Conclusion

One of the criticisms leveled at past attempts to incorporate applicable mathematics into school programs is that too often the problems have appeared contrived and not genuinely life related. This critisism can be related to the difficulty of writing problems that involve the application of mathematics, in a non-trivial way, without the calculation power available via computers of graphing calculators. The advent of the graphing calculator now makes such power readily available and accessible to students and teachers and so the challenge can now be shifted to the development of teacher support materials that take advantage of the capabilities of these devices.

The success of the AToMIC projects can be measured at a number of levels.

Firstly, a set of quality resources was produced in a form that teachers found useful, were consistent with syllabus objectives and were appropriate for the audience for who they were designed. Evidence for this can be found in the number of occasions that these activities have been used as the basis for school based assessment items in Queensland.

Secondly, teachers involved in developing and trialling tasks have been unanimous in their positive support of both projects as professionally enhancing activities for participants.

Thirdly, these projects have provided the basis for a professional development program, on the effective use of graphing calculator technology to teach and learn mathematics, conducted by QAMT throughout Queensland since 1998.

Finally, these projects have acted as a catalyst for the involvement of regional branches of QAMT in a common state-wide endeavour. It has provided the foundation for the development of a support network for those teachers directly involved in the project or who have been involved in workshops associated with the project. This has proved to be particularly helpful to those teachers who work in geographically remote communities or who are professionally isolated for other reasons.

It has been a project whose processes have been at least as rewarding as its products.

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