TRENDS IN ASSESSMENT IN CALCULUS ON THE INTRODUCTION OF GRAPHICS CALCULATORS

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Graphics calculators were allowed in tertiary entrance examinations in Western Australia for the first time in 1998. In this paper we describe changes in the style of questions used in the tertiary entrance examination for Calculus. A critical view of the examination papers in Calculus over four years which involved considering the difficulty and other characteristics of questions, highlighted the increased role of diagram upon the introduction of graphics calculators. This paper explores this development in relationship to questions on rectilinear motion and on the graphing of rational functions. The degree of difficulty of the questions in both topics has been virtually unchanged. However, the availability of the graphics calculator has led to a change in both the solution processes and in the expectations on students. Our analysis illustrates that the inclusion of technology active questions is only one aspect, and not necessarily the most important one, of adjusting assessment to suit the presence of the calculators.

Introduction

Graphics calculators were allowed in tertiary entrance examinations in Western Australia for the first time in 1998. This paper is part of a longitudinal study evaluating the impact of the calculators on Calculus. It flows out of a critical analysis (Mueller & Forster, 2000) of the Calculus Tertiary Entrance Examination (TEE) papers for 1996-1999, two years before and two years after the introduction of the technology to the examination. The analysis, which involved considering the difficulty and other characteristics of questions, brought to light an enhanced role for diagrams in solving problems upon the introduction of graphics calculators—including the use of graphs. In this paper we discuss characteristics of the 1996-1999 questions concerned with rectilinear motion and graphing of rational functions, looking in particular at the role of diagram.

Assessment in the Presence of Graphics Calculators

When technology is incorporated into the teaching of mathematics and becomes an expected tool, assessment practices need to be adapted to enable testing the usage of the tool as a means to carry out mathematics. Accordingly examination questions can be classified as technology *active*, *neutral* and *inactive* (Harvey, cited in Senk et al., 1997). Another typology relevant to graphics calculator usage, to which we refer in our analysis, is that the technology might have *no impact* on traditional questions or that it can *impact* by allowing alternative methods of solution which, in some instances, result in a question being *trivialised* (Jones & McCrae, 1996). Questions can also be designed especially for graphics calculator usage. These might include functions for graphical analysis that students could not manipulate manually, thus allowing testing of mathematical concepts in new ways (Anderson, Bloom, Mueller & Pedler, 1999). The aim of this paper is to discuss what impact the inclusion of graphics calculators as a teaching, learning and assessment aid has had on the nature of examination questions in the areas rectilinear motion and graphing of rational functions. In order to evaluate questions in the Calculus TEE (Mueller & Forster, 2000) we modified a scheme developed by Senk, Beckmann and Thompson (1997) for categorising assessment items from precalculus mathematics.

Graphics Calculators in TEE Calculus

Calculators without symbolic processing and the Hewlett Packard HP38G with limited symbolic capabilities are approved for TEE purposes. A non-prescriptive approach has been taken to setting out, other than for a general instruction to show all working in sufficient detail to allow answers to be readily checked. Specific instructions to use graphics calculators are not generally given. Because the text storage capacities differ between brands of calculator, four A4 pages (two sheets) of notes are allowed.

Research method

In establishing the characteristics of the Calculus TEE questions for 1996-1999, initially we independently coded them according to the scheme of Senk et al. (1997). Then, we fine-tuned the meaning of the categories for the context of the Calculus TEE and modified some of them. After modifying the scheme, we separately recoded the examination questions and then negotiated any differences. The role a diagram might play in answering a question depends on the ease with which such a diagram is accessible. If a graphics calculator is available to be used, a student might generate a graph on the technology, whereas without it drawing a graph could be impractical. When coding examination questions with respect to the category 'Role of diagram', we took into account the absence of technology for 1996 and 1997 and its presence for 1998 and 1999. For 'Technology' we coded its use whether it was available or not and use it to refer to graphics calculator capabilities that are over and above scientific calculator capabilities. Resulting categories to which we refer in this paper are given in Table 1

Table 1: Categories for analysing examination questions

Category		Description
Skill		
	Yes	Solution requires a well-known algorithm such as solving equations or inequalities or bisecting an angle. Item does not require translation between representations.
	No	No algorithm is generally taught for answering such questions, or item requires translation across representations.
Level		
	Low	A typical student in that course would use no more than three steps to solve.
	Other	A typical student in that course would use four or more steps to solve.
Reasoning		
	Yes	Item requires justification, explanation or proof or it is necessary to interpret the question before being able to start the answer
	No	No justification, explanation or proof is required. (By itself, 'Show your work is not considered reasoning.)
Realistic context		
	Yes	The item is set in a context outside of mathematics (e.g. art, fantasy, science, sports).
	No	There is no context outside mathematics.
Role of diagram		
	Interpret	A graph or diagram is given and must be interpreted to answer the question.
	Make	From some non-graphical representation (data, equation, verbal description) the student must make a graph or diagram.
	Assist	The use of a diagram or sketch would simplify a solution, but is not essential for obtaining the answer.
	None	No graphical representation is given or needed or a graph or diagram is given but is superfluous to answering the question.
Graphics calculator		
	Active	Use of the tool is necessary to obtain a solution or it greatly simplifies the work needed to get a solution.
	Neutral	It is possible to use the tool to obtain part or all the solution, but the question could be answered reasonably without the tool.
	Inactive	Use of the tool is not possible or is totally inappropriate.

Analysis of the 1996-1999 questions on rectilinear motion

The examination questions on rectilinear motion for 1996-1999 examinations are provided below and Table 2 shows the characteristics of the questions according to our scheme in Table 1.

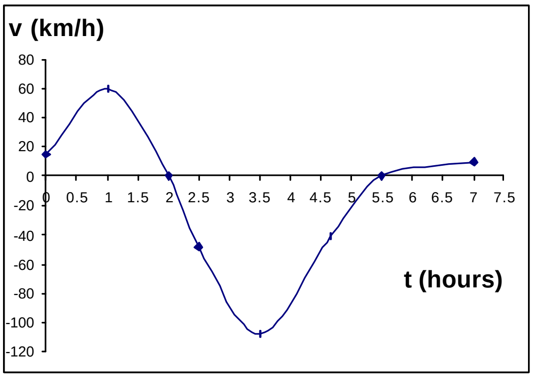
1996: A particle P_1 moves along the x-axis with constant acceleration of 2 metres per second. At time $t = 0$, P_1 has position 8 metres and velocity -2 metres per second. Another particle P_2 moves along the x-axis according to the equation $x_2(t) = 2t^2+4t+2$ where x_2 is the position of P_2 in metres at time t seconds.

(a) Show that P_1 and P_2 do not collide.
(b) Find the smallest distance between P_1 and P_2.
(c) Find the speeds of P_1 and P_2 at an arbitrary time t.
(d) In which interval do P1 and P2 move in the same direction?

1997: A body falls from rest in a uniform medium and its acceleration, $a(t)$, t seconds after it starts falling is given by $a(t) = (25/2)e^{-t/2}$. Find

(a) the velocity, $v(t)$, of the body for any t.
(b) $\lim_{t \to \infty} v(t)$.
(c) the distance the body falls in the first five seconds.

1998: The graph below shows the velocity of a truck travelling along a long straight road that runs east-west across the Nullarbor Plain.



The positive direction is east and t is in hours. At $t = 0$ the truck is 2000km due east of the town of Coonana and remains east of Coonana for the entire seven hour journey. The positions of the turning points on the graph are (1, 60) and (3·5, -108), and the points of inflection are (2·5, -48) and (4·5, -45). For what values of t

(a) does the truck turn around,
(b) is the acceleration positive,
(c) is the truck slowing down,
(d) is the displacement from the town at its absolute minimum and maximum values? Justify all your answers.

1999: A particle is moving along a straight line that runs in an east-west direction. Its position function $s(t)$ at time t is given by $s(t) = (t^2+1)/(t^4+1)$.

(a) Determine the velocity function of the particle.
(b) The particle is moving in an easterly direction when the velocity is positive. Use the graph of the velocity function to decide when the particle is moving in a westerly direction.
(c) Use the graph of the velocity function to determine the maximum speed of the particle and when it is attained.
(d) Calculate the position of the particle at the time when the maximum speed is attained.

The classifications 'Skill-no' (not algorithmic), 'Level-other' (four or more steps to solve) and 'Reasoning-yes' in Table 1 contribute to making a problem 'difficult'. On this basis, Table 2 suggests that the 1996 question on rectilinear motion would be found the most difficult of the four questions. Results in Table 3 support this conjecture. In addition, ranking in terms of students' low to high mean % marks (see Table 3) reflects the ranking in terms of percentage of 'difficult' characteristics for each question: (1996, 58%), (1998, 50%), (1999, 42%) and (1997, 33%).

Table 2: Characteristics of the 1996-1999 Calculus TEE questions on rectilinear motion

Description	Category	1996	1997	1998	1999
Skill-based	Yes	a, b, c	a, b, c	a, b	a, d
	No	d	-	c, d	b, c
Level	Low	c	a, b	a, b	a, b, d
	Other	a, b, d	c	c, d	c
Reasoning required	Yes	a, b, d	a, c	c, d	b, c
	No	c	b	a, b	a, d
Role of diagram	Interpret	-	-	a, b, c, d	b, c
	Make	-	-	-	b
	Assist	a, d	-	-	-
	None	b, c	a, b, c	-	a, d
Technology	Active	-	-	-	b, c
	Neutral	a, b, d	b	-	-
	Inactive	c	a, c	a, b, c, d	a, d

Table 3: Examination results for the population on the 1996-1999 questions on rectilinear motion

	1996	1997	1998	1999
% Mean mark	6.04(46%)	5.69(71%)	6.86(48%)	5.50(67%)
Standard deviation	4.15(32%)	2.23(28%)	3.74(27%)	2.10(26%)

The classification in Table 2 also indicates that the role of diagram increased across the four papers in the questions on rectilinear motion. Drawing a graph was practical and might have assisted the solution in only two part-questions in the questions for 1996-1997, whereas for 1998-1999 six part-questions involved interpretation or sketching a graph. However, we note that had technology been available it would have impacted on parts (a), (b) and (d) of the 1996 question without trivialising them. They lend themselves to a combined analytical-graphical approach, or to using a graph for checking.

In both the 1998 and 1999 questions the interpretation of a graph in terms of its relevance to rectilinear motion was allocated a substantial number of marks. The concepts to be tested included the notion of displacement, direction of movement, speed, and acceleration; all of which can be ascertained from the graph of the velocity function. In this aspect the questions differ from those of the previous two years where no interpretation was required. Between each other, the 1998 and 1999 questions differ in the means by which the graph is obtained. In the 1998 question the diagram was supplied, thus making the question technology inactive. For the 1999 question the students had to generate the graph themselves and then interpret it.

In summary, for the rectilinear motion questions in the Calculus TEE questions, there is not any pattern of increased or decreased difficulty upon introduction of graphics calculators, but there have been increased demands on students to interpret graphs, accompanied by increased mark allocation to graphical methods, away from analytic methods.

Analysis of the 1996-1999 questions on graphing of rational functions

The examination questions on graphing of rational functions for the 1996-1999 examinations are provided below.

1996. Let f be the function defined by $f(x) = \dfrac{-4x+5}{(x+2)(5-x)}$.

(a) State the poles of the function.

(b) Evaluate $\lim_{x \to +\infty} f(x)$.

(c) Evaluate $\lim_{x \to -\infty} f(x)$.

(d) Evaluate $\lim_{x \to 4} f(x)$.

(e) State the x and y intercepts.

(f) Show that there are no turning points and sketch the graph of $y = f(x)$, clearly labeling all the important features.

1997. If $f(x)=\dfrac{2x^2-x}{x^3-1}$,

(a) State the pole of the function.
(b) Evaluate $\lim_{x\to\infty} f(x)$.
(c) Evaluate $\lim_{x\to-\infty} f(x)$.
(d) State the x and y intercepts.
(e) The derivative is given by $f'(x)=\dfrac{-2x^4+2x^3-4x+1}{(x^3-)^2}$. The numerator of $f'(x)$ has exactly two roots. One of these roots is located at approximately 0.26, the other is near -1. Use the Newton-Raphson method to refine the root near -1 to two decimal places.
(f) Classify the critical points as a local maximum, minimum or points of inflection.
(g) Sketch the graph of f. Clearly label all the important features.

1998. If $f(x)=1-\dfrac{x}{(x-1)^2}$,

(a) Sketch the graph of $f(x)$, indicating all asymptotes and turning points.
(b) For each of the following initial values decide whether the Newton-Raphson method would lead to the left root, the right root or neither.
(i) $x_0 = 2$ (ii) $x_0 = -2$ (iii) $x_0 = 1/4$

1999. If $f(x)=\dfrac{x^2+3x-10}{x^2+x-6}$,

(a) state the domain of f,
(b) evaluate $\lim_{x\to 2} f(x)$,
(c) sketch the graph of f showing the intercepts, asymptotes and any other distinguishing features.

In contrast to the questions from 1998 and 1999, the questions from 1996 and 1997 on graphing were highly structured. Part questions, which relied on recognition of properties pertaining to the functions and involving some algebraic manipulation, led students item by item through features of the graphs and attracted part marks. In 1998, in the presence of graphics calculators, students were asked straight away to graph the function without guidance apart from a generic requirement to include possible asymptotes and turning points. However, we have suggested elsewhere (Mueller & Forster, 1999), on the basis of student interviews and perusal of examination scripts, that to successfully draw the graph students needed to interpret the graphical display on their calculator in the light of algebraic information available in the formula of the function. This in particular concerns the asymptotic behaviour and the location of poles. In 1999, two part questions warned students about features of the graph, as did the statement ‘other distinguishing features’ in part (c). Choosing to use a graphics calculator again meant students needed to integrate the graph on it with the properties in algebraic form--this time previously established. So, in 1998 and 1999, rather than using the properties to hand-plot the graph, they were needed to assist in the interpretation of the calculator graph.

The functions in 1998 and 1999 were “calculator-friendly” in the sense that there was no need for manipulation of a standard viewing window in order to obtain a first image of the graph and most features could be ascertained on the viewing screen in interaction with in-built calculator functions for locating extrema and roots (points of inflection were outside of the syllabus). The requirements as far as generation of a graph was concerned therefore were basic when it came to calculator skills and integration of information derived from the calculus.

Our classification in Table 4 of the questions according to our scheme in Table 1 provides another view of the difficulty (based on Skill-no, Level-other, Reasoning-yes) of them, as does student scores on them (see Table 5).

Table 4 : *Characteristics of the 1996-1999 Calculus TEE questions on graphing*

Description	Category	1996	1997	1998	1999
Skill-based	Yes	a, b, c, d, e	a, b, c, d, e, f	-	a, b
	No	f	g	a, b	c
Level	Low	a, b, c, d, e	a, b, c, d	b	a, b
	Other	f	e, f, g	a	c
Reasoning required	No	a, b, c, d, e	a, b, c, d, e	-	a, b
	Yes	f	f, g	a, b	c
Role of diagram	Interpret	-	-	-	
	Make	f	g	a	c
	Assist	-	-	b	b
	None	a, b, c, d, e	a, b, c, d, e, f	-	a
Technology	Active	f	e, f, g	a, b	c
	Neutral	b, c, d, e	a, b, c, d	-	a, b
	Inactive	a	-	-	-

Table 5: *Examination results for the population for the 1996-1999 questions on graphing of rational functions*

	1996	1997	1998	1999
Mean % mark for the question	82%	72%	74%	78%
Total marks available for the question	14	21	8	10

Tables 4 and 5 indicate that overall there has been little change in the degree of difficulty as we perceive it and as students experienced it in the Calculus TEE questions which ask students to graph a rational function. However the introduction of the technology has been accompanied by a lower mark allocation to the questions, explained by fewer or no lead-in parts to the graph, that is fewer marks were allocated to algebraic manipulation that has traditionally been a dominant feature of questions on graphing.

Discussion

The two topics discussed above indicate that there has not been any substantial change in the degree of difficulty of the questions on rectilinear motion and graphing rational functions upon the introduction of graphics calculators for the Calculus TEE. However, the range of strategies students have been called upon use has changed. For graphing rational functions, essentially linear solution processes like those necessary for successful solution of the 1996 and 1997 questions have been replaced by questions where the absence of implicit instruction forces upon the student a circular procedure (see Figure 1):

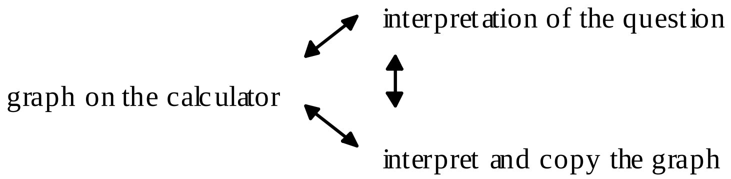


Figure 1: Process of producing a graphics calculator assisted answer

The process starts with entering the function as a character string into the calculator. At this stage a perceptive student might, after looking at the graph, adjust the scales, zoom in (literally and metaphorically) on features of the graph, while referring back to the wording of the question and the symbolic form of the function, and at the same time hand-draw the graph. This applies equally well to the questions on rectilinear motion which, in both 1998 and 1999, seem to have undergone a shift from straight algebraic manipulation and asking for knowledge of the relevant definitions to more interpretative skills being called upon. The question for each year, instead of testing algebraic methods that lead up to graphing, tested students' abilities at graphical interpretation. Here student errors, other than for entering expressions into the calculator, did not seem to be attributable to the technology but to difficulties with reading graphs in general (Forster & Mueller, 2000).

Another facet of assessment with technology is that a wider range of functions can be used than if students are restricted to algebraic methods. An example is the velocity function in the 1999 question on rectilinear motion. In exams without access to a calculator this function might have been the focus of a question on graphing, but not of one on rectilinear motion. However, depending on the concepts being tested, rather than having students generate their own graphs, testing might be just as credible or better with a graph that is supplied, as with the velocity-time graph that was provided in 1998. Another factor in assessment is that the influence of technology on students' answers to questions is potentially not restricted to direct use of the technology and this is one explanation for the more frequent use of graphs and diagrams in the examination – irrespective as to how they are obtained. Learning with graphics calculators might bring understanding to answering examination questions that is different to the understanding developed without technology—although evidence of this is mixed (Berger, 1998; Lauten, Graham & Ferrini-Mundy, 1994).

Conclusion

Being technology active is only one aspect, and not the most important one, in ensuring that assessment adjusts to accommodating graphics calculators. Of highest importance for teaching, the analysis here suggests that instruction needs to be aimed at students' developing their abilities at interpretation of graphs and at the integration of information coming from different representations. For example, it might be appropriate for testing students' understanding of the properties of a function to use functions whose salient features are not discernible on a standard viewing screen. For such a function it would be essential to combine information obtained algebraically to even be able to obtain a view of the graph. Graphical interpretation, in the experience of the Calculus TEE in Western Australia is taking a more important role upon introduction of the technology.

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