Towards New Models of Teaching and Learning in Technology Enriched Mathematics Classrooms

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The introduction of technology resources into mathematics classrooms promises to create opportunities for enhancing students' learning through active engagement with mathematical ideas; however, little consideration has been given to the potential for technology to promote new pedagogies that foster a more investigative approach to learning. This paper draws on data from a three year longitudinal study of Australian secondary school classrooms to examine pedagogical issues in using technology resources in mathematics teaching – in particular, graphing calculators and overhead projection panels that allow screen output to be viewed by the whole class. Our research focuses on how technology contributes to students' understanding of mathematics, and changes the teacher's role in the classroom. The nature of these changes is analysed with the aid of classroom vignettes that illustrate different patterns of interaction between teachers, students, and technology – ranging from conservative use of technology as an adjunct to transmissive teaching methods, to the welcoming of technology as a liberating opportunity for students to experiment and conjecture.

Introduction

At the beginning of the last decade it was predicted that technologies such as computers and graphing calculators would have a major impact on the teaching and learning of secondary school mathematics (Barrett & Goebel, 1990; Demana and Waits, 1990). Some of these predictions were concerned with opportunities for enhancing student learning – for example, by enabling connections to be made between algebraic, graphical, and numeric representations of mathematical concepts. It was also anticipated that technology would bring about changes in the roles of teachers and students, in that teachers would act as facilitators of student discussion and collaborative exploration with peers (Heid, Sheets & Matras, 1990).

Research in mathematics education over the last decade has begun to address the nature of these new technologies and their effects on learning and teaching. Although overall findings concerning the predicted benefits for students' learning have been somewhat inconclusive (Lesmeister, 1996; Maldonado, 1998; Penglase & Arnold, 1996), many studies have reported that the use of technology has a positive effect on students' attitudes towards mathematics, understanding of function and graphing concepts, and spatial visualisation skills (Portafoglio, 1998; Weber, 1998; see also Penglase & Arnold, 1996). However, it seems that less is known about how the availability of technology, especially graphing calculators and their peripheral devices, has affected teaching approaches (Penglase & Arnold, 1996). While some studies have found changes in classroom dynamics leading to a less teacher centred and more investigative environment (e.g. Simonsen & Dick, 1997), it appears that negotiation of such a pedagogical shift is mediated not only by teachers' mastery of the technology itself, but also by their personal philosophies of mathematics and mathematics education (Simmt, 1997; Tharp, Fitzsimmons & Ayers, 1997).

The purpose of this paper is to consider some pedagogical issues in using graphing calculator technology in mathematics teaching, arising from a three year longitudinal study of secondary school classrooms. In particular we examine interactions between teachers and students, amongst students themselves, and between humans and technology, in order to investigate the extent to which different participation patterns provide opportunities for students to engage constructively and critically with mathematical ideas. The main focus of the paper is on teachers' and students' use of the graphing calculator and overhead projection unit (an LCD panel that allows the calculator screen to be projected for whole class viewing).

Emergent Uses of Technology in Learning Mathematics

Ramsden (1997) has acknowledged the impact of inherited traditions on the use of technology by referring to an instinct for teachers to begin by looking for electronic ways of doing familiar jobs previously done by textbooks and lectures. Similarly, Thorpe (1997) in examining teaching behaviours and attitudes towards technology found that computer technology was being used essentially to enhance preferred teaching methods: that is, the technology, although freely available, was utilised in a conservative way.

However Ramsden (1997) also observes the attraction of technology for educators who want to give their students power, as distinct from exercising it over them. Here he argues that while a technology cannot be used for a purpose that is patently unsuited to its design, *emergent* uses should be productively sought. These are uses that no one (including the designers) could have predicted, and the space of these is vast and unexplored. Shneiderman, Borkowski, Alavi and Norman (1998) describe settings in which teachers have evolved personal styles in using an elaborately fitted out "electronic classroom", involving the emergence of new patterns of interaction which, while retaining characteristics of personal styles, have in common a more collaborative approach. Each of these entailed combining the technology resources and human interaction to develop methods that were not obviously the precinct of the hardware design. One emergent property noted was the role technology played in changing the communication structure – by providing alternative and parallel channels for students to contribute to discussions and provide feedback, both privately (on individual computer screens), and publicly (on a class screen). This *equaliser* role was evidenced in the contributions of "quiet" students who would not participate in conventional classroom dialogue, but who eagerly shared comments through an electronic interface.

Emergence in the sense described here is a product of the interaction between human and technological agencies. In consequence it is not surprising that those who have been involved with some of the most innovative uses of technology are among the most definite in rejecting the teacher-replacement concept, the very antithesis of the concept of emergence:

> While technology can be wonderfully empowering for teachers and students, the relationship between human beings is still the heart of the educational process … key function of a university or school setting is to encourage the tie between teachers and students: technology can support and strengthen relationships, but never create or replace them. (Shneiderman, Borkowski, Alavi & Norman, 1998, p.24)

While the above discussion refers mainly to computer technology, the arguments concerning teaching and learning may be applied equally well to the use of more recent innovations such as graphing calculators and peripherals – although the portability of calculators adds another dimension in that students may own and attain intellectual intimacy with these devices.

The Research Study

The research reported here forms part of a three year (1998-2000) longitudinal study investigating the role of technology in facilitating students' exploration of mathematical ideas, and in mediating teacher-student and student-student interaction (see Galbraith, Goos, Renshaw & Geiger, 2000; Galbraith, Renshaw, Goos & Geiger, 1999; Goos, Galbraith, Renshaw & Geiger, 2000; Goos & Geiger, 1999). Data collection has involved four senior secondary mathematics classrooms, drawn from three co-educational schools (two government and one independent) in a large Australian city. Students participating in the study are in Years 11 and 12 (the two years of senior secondary schooling), and are taking mathematics subjects that prepare them for tertiary entrance. While the syllabuses for these subjects do not yet mandate the use of graphing calculators and computers, teachers are encouraged to make use of technology wherever appropriate.

At least one lesson every week is observed and videotaped, but more frequent classroom visits are scheduled if the teacher plans a technology intensive approach to the topic. Audiotaped interviews with individuals and groups of students are conducted at regular intervals to examine the extent to which technology contributes to students' understanding of mathematics, and how technology changes the teacher's role in the classroom. At the beginning and end of each year students also complete a questionnaire on their attitudes towards technology and its role in learning mathematics (see Geiger, 1998).

Analysis of technology-focused interactions has been framed by four metaphors we have developed to form a taxonomy of sophistication with which teachers and students work with technology. We draw on classroom observation and videotape data from 1998 and 1999 to illustrate these metaphors with respect to teacher roles.

Re-shaping Teacher Roles

In developing the potential of technology enriched learning the role of the teacher in making use of technology is clearly crucial. We theorise four roles for this interaction between teacher and technology.

Technology as Master. Here the teacher is subservient to the technology, and is able to employ only such features as are permitted either by limited individual knowledge, or force of circumstance. For example, pressure to be seen to use technology following "training" may result in implementation dominated by whatever basic skill has been acquired, without consideration of impact beyond the present (Stuve, 1997).

Technology as Servant. Here the user may be knowledgeable with respect to the technology, but uses it only in limited ways to support preferred teaching methods (Thorpe, 1997). That is, the technology is not used in creative ways to change the nature of activities in which it is used. For example, just as a graphing calculator can be restricted to the purpose of producing fast reliable answers to routine exercises, an overhead projection panel may be limited to providing a medium for a teacher to demonstrate output to the class.

Technology as Partner. Here the teacher has developed an "affinity" with both the class and the teaching resources available. Technology is used creatively in an endeavour to increase the power that students collectively exercise over their learning, rather than exercising it over them (Templer, Klug & Gould, 1998). This occurs both in the use of mathematically based technology (e.g. graphing calculators), for the purpose of enhancing individual prowess, and in the use of communications technology to enhance the quality of class learning through sharing, testing, and reworking mathematical understandings. For example, instead of functioning as a transmitter of teacher input, an overhead projection panel may be a vehicle for engendering otherwise non-existent participation (Shneiderman, Borkowski, Alavi & Norman, 1998) or act as a medium for the presentation and examination of alternative mathematical conjectures. A defining characteristic of this metaphor for technology is that "the locus of control never passes from user to machine" (Templer, Klug & Gould, 1998).

Technology as Extension of Self. This is the highest level of functioning, which may presently be only rarely in evidence. Here powerful and creative use of both mathematical and communications technology forms as natural a part of a teacher's repertoire as do fundamental pedagogical and mathematical skills. Writing courseware to support and enhance an integrated teaching program would be an example of operating at this level.

It can be noted that these levels of operating are not necessarily tied to the level of mathematics taught, nor the sophistication of technology available. Simple mathematics and basic technologies are sufficient to provide a context for highly creative teaching and learning.

Classroom Examples

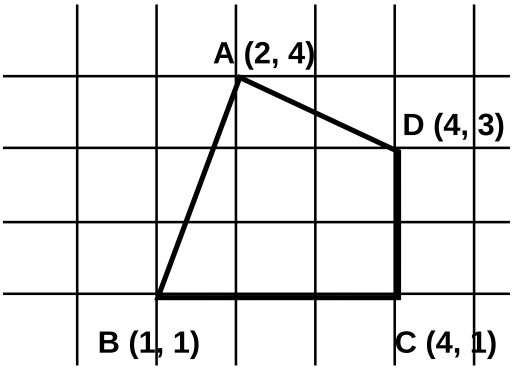
Use of the metaphors of *master, servant, partner,* and *extension of self* does not imply that teachers remain attached to a single mode of working with technology. In fact, some of our most interesting observations reveal variations on these themes – for example, where teachers may be in transition between different levels of operating, or where the levels themselves are instantiated in unexpected ways. Below we outline some classroom examples of instructional use of graphing calculators and overhead projection panels to illustrate these variations.

Master and Servant. One of the teachers participating in our study admits minimal expertise in using the graphing calculator, but counters his own lack of confidence by calling on a recognised student "expert" to demonstrate calculator procedures via the overhead projection panel. This student owes his expertise to having completed a "train the trainer" program offered by the calculator company. While the teacher lacks personal autonomy in the use of technology (suggesting that technology has the role of *master*), he nevertheless retains control of the lesson agenda through the medium of the student presenter (thus technology is used as a *servant*) – often to the extent of providing the mathematical commentary and explanations accompanying the student's silent display. Even when the student instructs the class on calculator keystrokes, the teacher's voice may still be heard in the student's articulation of carefully controlled, step-by-step procedures consistent with the teacher's preferred methods. Ultimate authority rests with the teacher, who remains reluctant to allow students to use technology to explore mathematical territory that is unfamiliar or outside the immediate lesson topic. Nevertheless, this teacher's actions could be interpreted as movement towards greater student participation, albeit through working with technology in *servant* mode.

Intelligent Servant. Another teacher with whom we work tends to use the calculator and OHP panel as conventional instructional devices – for example, the panel may be likened to an electronic blackboard for demonstrating calculator operations which students check against their own working. However, emergent uses of the technology in conjunction with other material resources are apparent. One simple example involves projecting the calculator display onto a whiteboard that simultaneously acts as a screen and a writing surface, thus enabling the teacher to interact with, highlight and modify aspects of the calculator's output by writing on the screen image projected onto the whiteboard.

An even more creative approach integrates technology with concrete aids, thus enhancing even further the graphing calculator's capacity for linking multiple representations of a concept. For example, in a Year 11 lesson on matrix transformations the teacher used transparent grid paper, plastic cut out polygons, and the overhead projector to physically demonstrate the results of matrix transformations (see Figure 1 for student worksheet).

Students then investigated further by placing their own polygons on grid paper and recording the coordinates of the vertices before and after transformation – with graphing calculators taking care of the matrix calculations. While the technology is still used to support the teacher's preferred approach involving hands-on activities, the calculators and projection panel are exploited in novel ways that retain effective features of more conventional instruction.



Place a plastic polygon on the grid so that each vertex lies on an integer co-ordinate position. This shape can be manipulated to any number of integer positions.
Consider each vertex as a vector;
e.g. A (2, 4) can be thought of as $\mathbf{a} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$.

From the list of 2 × 2 matrices below choose one. Apply this matrix to the vertex vectors and re-position your polygon to the new co-ordinates. Try and identify what each matrix transformation does geometrically. Follow up by trying to identify from the arithmetic elements of the matrices why your polygons were transformed in the way they were.

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad \mathbf{D} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

$$\mathbf{E} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad \mathbf{F} = \begin{pmatrix} \frac{1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \quad \mathbf{G} = \begin{pmatrix} \frac{-1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{-1}{2} \end{pmatrix} \quad \mathbf{H} = \begin{pmatrix} \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & \frac{-1}{2} \end{pmatrix}$$

Figure 1. *Matrix transformation task*

Creative Partnerships. Vignettes from two additional project classrooms illustrate how technology is having a liberating effect on both teachers and students. In the first classroom the teacher has basic but growing competence with graphing calculators, and is willing to try out calculator operations only partly understood. One of our visits to his Year 11 classroom prompted this teacher to use the overhead projection panel for the first time in a lesson situation. Students were set the task of investigating various transformations of the functions $y = x^2$, $y = \frac{1}{x}$, and $y = |x|$. Instead of using the projection panel to *control* the students' activities by requiring that they reproduce teacher-demonstrated transformations, the teacher shifted the locus of control to the students by assigning different functions to small groups and inviting them to use the OHP to present their findings to the whole class. Consequently the LCD panel acted as a communication medium that enabled students to explain and defend their own conjectures.

In our view, the liberating potential of the overhead projection panel represents one of the most significant emergent properties of technology in mathematics classrooms, as control is shared between machine and *student* users. Of course such presentations may parallel the teacher's actions if students simply display their solutions; however, we have observed instances of knowledge production and repair where partial solutions are shared and completed with whole class input, or previously unnoticed errors in the student-presenter's work are identified and corrected by peers. Such is the case in the classroom of a fourth teacher involved in the research project. This teacher is already an expert and innovative user of technology and fosters similar expertise in his students – not through detailed instructions on keystrokes, but by providing tasks that require students to use the calculator intelligently.

A brief vignette involving programming illustrates this point. The students (Year 12) were asked to program their calculators to find the angle between two three dimensional vectors $\mathbf{r_1} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ and $\mathbf{r}_2 = \begin{pmatrix} d \\ e \\ f \end{pmatrix}$, which is given by the formula

$$\vartheta = cos^{-1}\left(\frac{\mathbf{r_1} \cdot \mathbf{r_2}}{|\mathbf{r_1}||\mathbf{r_2}|}\right) = cos^{-1}\left(\frac{ad + be + cf}{\sqrt{a^2 + b^2 + c^2}\,\sqrt{d^2 + e^2 + f^2}}\right).$$

The teacher provided only minimal instruction in basic programming techniques, and expected individual students to consult more knowledgeable peers for assistance. Volunteers then demonstrated their programs via the overhead projection panel, and examined the wide variation in command lines that peers had produced (see Figure 2 for examples).

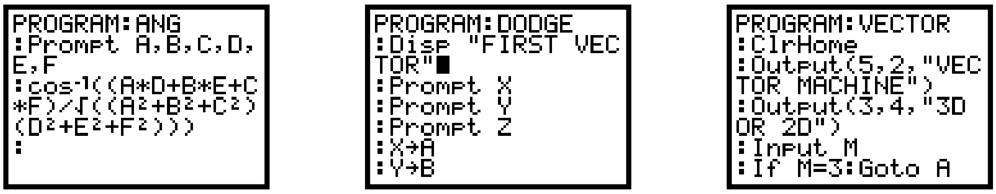


Figure 2. *Student programs for finding angle between vectors*

This public inspection of student work also revealed programming errors that were subsequently corrected by other members of the class. For example, the class disputed the answer obtained by executing the program shown in the first screen of Figure 3.

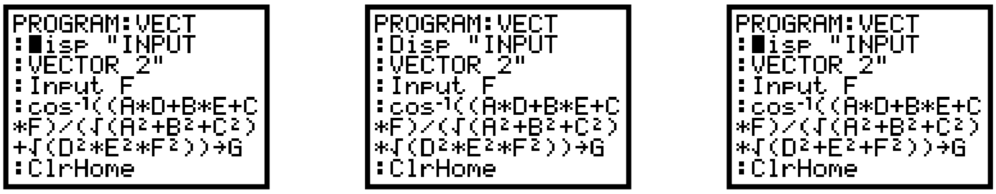


Figure 3. *Correcting errors in a student program*

His actions guided by the suggestions of fellow students, the presenter scrolled down through the program and replaced the plus sign with a multiplication sign between the two bracketed terms in the denominator (Figure 3, second screen). The amended program again produced an incorrect answer, and yet another change was suggested by students (Figure 3, third screen, replacing multiplication with addition signs in the second term of the denominator). The presenter executed the program once more, and the appearance of the correct answer was greeted with cheers and applause from his classmates.

A further emergent feature of the viewscreen as part of the mathematical practice of the classroom is its attraction for students who are initially reluctant to accept the social and cultural norms established by the teacher. Here we observed how one student consistently rejected the teacher’s invitations to discuss his thinking with peers, participate in whole class discussions, and generally take some responsibility for advancing his mathematical understanding. This situation changed when the student was persuaded to use the viewscreen to show the class an animated program he had created that depicted the adventures of mathematical objects as human-like characters. The enthusiastic and admiring response to his “movie” (and several “sequels”) was significant in drawing this student into the kind of mathematical discussion he had previously resisted, and he became a willing participant in subsequent technology-focused discussions.

Conclusion

In their review of research on the use of graphing calculators in mathematics education, Penglase and Arnold (1996) concluded that “approaches to teaching and learning which emphasise problem solving and exploration, and within which students actively construct and negotiate meaning for the mathematics they encounter, find in this new technology a natural and mathematically powerful partner” (p. 85). While the findings presented here suggest that this is the case in some of the classrooms participating in our research program, the relationship between technology usage and teaching/learning environments is not one of simple cause and effect. The metaphors of *master*, *servant*, *partner*, and *extension of self* are intended to capture some of the diversity of teacher and student interactions in technology rich classrooms.

It seems natural for teachers to use new technologies such as graphing calculators and overhead projection panels in ways that are consistent with preferred teaching methods. However, teaching with technology need not – and perhaps should not – simply be a matter of grafting this tool onto existing pedagogical practices. For example, likening the graphing calculator to a portable computer, or the overhead projection panel to an electronic blackboard, obscures important qualitative differences between old and new technologies and may limit the scope of what teachers and students are able to achieve in the classroom.

In contrast, research that seeks out *emergent* (i.e. unplanned, unanticipated) uses of technology reveals that the calculator and projection panel are not passive or neutral objects, since these technologies are actively re-shaping human interactions and interactions between humans and the technology itself. For example, even though the graphing calculator is designed as a personal mathematical tool it can facilitate social interaction and sharing of knowledge when students are invited to present their calculator work to the whole class via an overhead projection panel. When control of the discussion is handed over to students the panel is no longer simply a presentation device – instead it becomes a discourse tool that mediates interaction between students at a whole class level. This is a class wide form of collaborative inquiry that is facilitated by the public display and interrogation of mathematical ideas.

Introducing new mathematical and communication technologies into classrooms can change the ways that knowledge is produced. Implicit in these changes are a number of challenges for teachers, the most obvious of which involves becoming familiar with the technology itself. While this will remain a significant professional development focus (as Penglase & Arnold, 1996, point out), more attention needs to be directed to the inherent mathematical and pedagogical challenges in technology enhanced classrooms if the goal of a problem solving and investigative learning environment is to be realised. For example, placing graphing calculators in the hands of students gives them the power and freedom to explore mathematical territory that may be unfamiliar to the teacher; and for many teachers, this challenge to their mathematical expertise and authority is something to be avoided rather than embraced. Perhaps the most significant challenge for teachers lies in orchestrating collaborative inquiry so as to share control of the technology with students. The research reported in this paper has begun to consider such emergent uses of technology in re-shaping social interaction patterns in mathematics classrooms.

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