The Dynamic Plots for the Instant Energy Flow of the Transmitted Waves in the Total Internal Reflection of Electromagnetic Waves

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Abstract

Using Maple, the dynamic field plots for the instant energy flow of the transmitted waves is shown with the normal plane wave solution for the transmitted waves. The incidence angle is treated as the parameter so that once it outstrips the critical value, the spontaneous transition of the view of the transmitted waves from the normal refraction. to the total internal reflection is achieved. Thus, the rich physical characteristics of the transmitted waves in the total internal reflection are brought to light vividly.

1. The Transmitted Waves in the Total Internal Reflection

If light goes from a material with a greater index n_1 toward a material with a index n_2 ($n_1 > n_2$),according to refraction law

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{n_1}{n_2} \qquad (1)$$

the transmitting angle θ_t of the wave becomes 90° when the incident angle θ_i is equal to the critical angle θ_c given by

$$\sin\theta_c = \frac{n_2}{n_1} = n_{21} \qquad (2)$$

When θ_t is going greater than the critical angle θ_c, as is known, there is total internal reflection. Let us focus on the transmitted wave. Suppose the incident wave vector is on the x-z plane, the complex solution for the transmitted wave is expressed as

$$E = E_0 e^{j(k_z z + k_x x - \omega t)} \qquad (3)$$

According to the boundary conditions of the electric fields, there is $k_x = k\sin\theta_i$and $k = \frac{\omega}{c}n_1$, so

$$k_z^2 = \frac{\omega^2}{c^2}(1 - \frac{n_1^2}{n_2^2}\sin^2\theta_i) \qquad (4)$$

Thus when $\theta_i > \theta_c$, $k_z$is a pure imaginary, say $\pm i\kappa$. If k_z take the positive sign, the transmitted wave will have the form:

$$E = E_0 e^{-j\kappa}\ e^{j(k_x x - \omega t)} \qquad (5)$$

The wave amplitude drops off exponentially with the increasing transmitted distance z. However, in term of the theory of boundary problem of electromagnetic fields , the eq.(5) also is the solution of the boundary problem of the field equations, standing for some kind of reality.

2. Instant Energy Flow of the Transmitted Waves

With maple command linalg[curl], it is a straightforward piece of work to get the corresponding magnetic field and energy flow of the transmitted waves from the normal plane wave solution eq.(5)

$$H = -\frac{j}{\omega\mu}\nabla \times E \qquad (6)$$
$$S = E \times H \qquad (7)$$

and it is easily to prove that the average energy flow of the transmitted waves is zero:

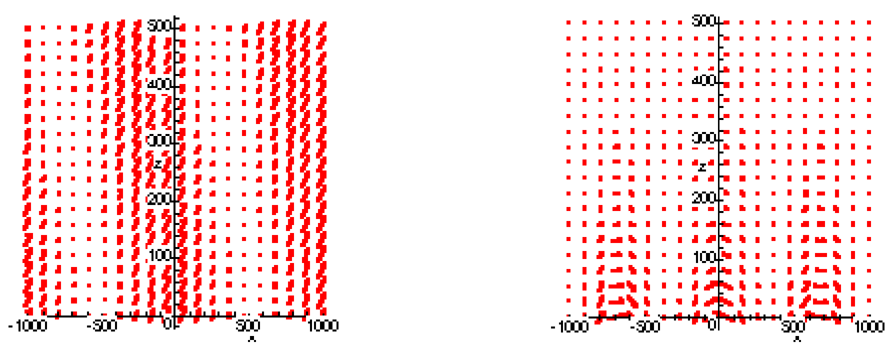
$$S_a = \frac{1}{2}\,\mathrm{Re}(E^* \times H)$$

suggesting the average effect of the total internal reflection. But how is the instant energy flow of the transmitted waves going? The plots[fieldplot] function is an ideal choice to show the internal characteristics of the transmitted waves.

However, since the eq.(3) is a general plane wave solution of the same boundary problem for the transmitted waves, whether normal reflection or total internal reflection is just the different result corresponding to different incident angle, in term of theory, it is more logical to show the physical images with the same program. Maple can do a remarkable job for that.

3. The Dynamic Plots of the Instant Energy Flow

There is no need to artificially treat the general plane wave solution for the transmitted waves, it is straightforward to put it into the maple plot program[1] . The maple command readstate can be used to prompt to input the incident angle. By this way, when the incident angle smaller than the critical angle is input, the plot shows the normal reflection, and once it outstrips the critical value, the spontaneous transition of the view of the transmitted waves from the normal refraction to the total internal reflection. would happen. Thus, the rich physical characteristics of the transmitted waves in the total internal reflection are brought to light vividly. The following are the views of the instant energy flows of the transmitted waves for the normal reflection and the total internal reflection respectively.



(a) $\theta_i \le \theta_c$ (b) $\theta_i \ge \theta_c$

Fig.1 the views of the instant energy flows

From the views shown in Fig.1(b) , it is clear that the instant energy flow for the transmitted waves of the total internal reflection possesses the characteristics as follows: (1) It drops off quickly with the increasing transmitted distance ; (2) During the period of half a cycle, the energy of the transmitted waves flows into the second material, and releases again to the reflective waves in the next half cycle. Thus the average energy flow density is zero; (3) The boundary guides some surface waves.

4. Conclusion

Owing to the maple functions capable of treating complex data and the flexible language of maple programming, the views of the instant energy flow for the transmitted waves are dynamically shown on the basis of the general plane wave solution of the boundary problem of electromagnetic fields. Different characteristic views of the transmitted waves spontaneously correspond with the incident angles, suggesting the unity of the theory.

References

[1] ISBN-O-387-94537-7, M.B.Monagan, K.O.Geddes, G.Labahn, S.Vorkoetter, Maple V Programming Guide, NY, USA, Springer,1995.