Use of CAR in Teaching of Mathematics and Computing

Nelson Wai, Chan
Department of Computing
The Hong Kong Polytechnic University
csnchan@comp.polyu.edu.hk

Abstract

This paper describes research into the framework of CAR and its methodology together with software such as Scientific Notebook in a teaching and learning problem-solving environment to support student investigation as well as to enhance student logical thinking. CAR is an acronym that stands for Component Analysis Relationship. It aims to motivate students how to subdivide a problem on hand into three components. These components are input component, mental model component and target system component. Analysis work can then be focused on the relationship among entries in each of the components. Examples of how to apply CAR methodology in some typical problem-solving activities are presented.

1 Introduction

Educating the next generation of college students is by no means an easy task. Both teacher and student in classroom are constantly faced with the challenge and problem of how the learning and teaching process can be enhanced. In fact, teachers are interested of how can students be best motivated in their studies. On the other hand, students are concerned of how to enhance the learning process, and most of all, how to improve their problem-solving skills [4]. But with the growth in student numbers and rigorous demand on subject syllabuses, time for lecture/tutorial has attenuated. The problem may become more acute, as nowadays a college student is normally required to take five or six subjects per semester. Thus the achievement of excellence in learning and teaching can no longer be sustained using unaided traditional lecture/tutorial approach. There have been already a good number of useful methodologies such as problem-based learning [2], computer-aided instructions, and so on. Most of these methods actually assist students in the process of learning. The critical factor for the success of these methodologies depends substantially on students' self-motivation on the understanding of teaching materials as well as their logical thinking. Yet students may find these methodologies not that effective if they do not possess these qualities.

To overcome these shortcomings we propose in this paper an innovative methodology CAR, which may be considered as an active learning tool. CAR is an acronym that stands for Component Analysis Relationship. It enables student to subdivide a problem on hand into a number of components using a top-down approach. These components are Input Component, Mental Model Component and Target System Component. The relationships among the various entities in each of the components can then be studied and analyzed using various interaction techniques.

The innovative use of CAR methodology in a problem-based environment is proposed and pioneered by the present author [1]. It actively supports student exploration and acquisition of scientific reasoning skills in students and integrates a number of component analysis techniques to assist in scaffolding scientific reasoning activity. More specifically, CAR together with Scientific Notebook aims to cultivate students' analytical mind in a problem-solving environment, develop their problem-solving skills, and enhance their logical thinking.

2 CAR Approach

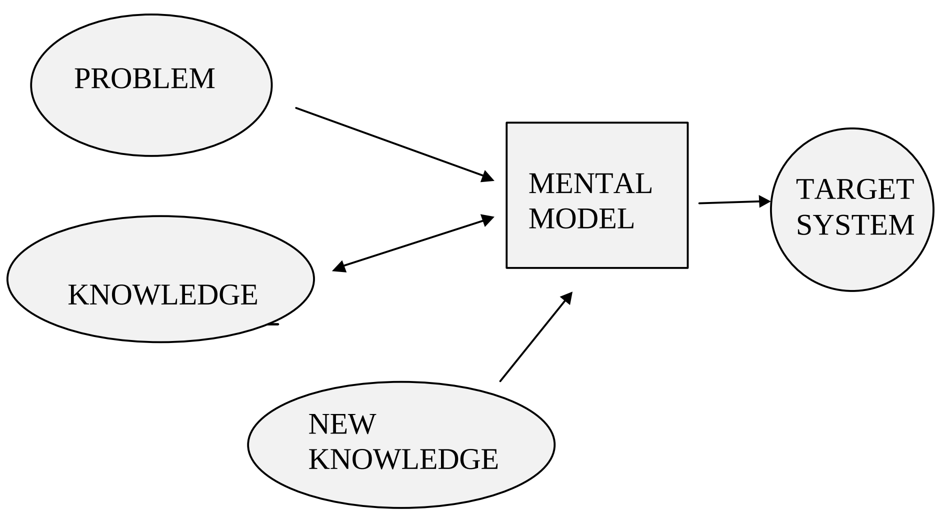

Figure 1. Concept of CAR methodology.

Figure 1 depicts the concept of CAR [1]. The overall structure of CAR is illustrated in Figure 2.

Problem

<Input Component> <Mental Model Component> <Target System Component>

- Input Component
- Target System Component
- Knowledge
- Interaction Technique
- New Knowledge

Figure 2. Structure of CAR

CAR is simple to use. It is best illustrated by considering an application to classroom problems in mathematics as well as in computing at high school or freshman level. Generally speaking, most of these problems are very structured, and their solutions only require a thorough understanding of the fundamentals concerned.

The first step of CAR is to decompose a problem on hand into three basic components using a top-down approach. These components are input component, mental model component and target system component [3]. The input component consists of all the given input specification of the said problem. The target system component will be the desired outcome or deliverable as required by the said problem. The mental model system component is composed of the input component, target system component, knowledge and interaction techniques. The second step consists of the analysis of various relationships among existing knowledge (i.e. definition, term, concept, required theory and/or algorithm), problem and perhaps any new information if required. Moreover, it may be considered as a conceptualization of the outcome, which is to be acted upon by various interaction techniques of existing knowledge. Use of new knowledge is necessary only if the existing knowledge is unable to construct the mental model system. There are a number of techniques associated with interactions, namely reasoning technique, evaluation technique, experimentation technique, and etc. In general, selection and application of these techniques will largely depend on the nature of a given problem. For example, reasoning technique is applied whenever a direct algorithm is known. On the other hand, experimentation technique can be applied if a direct algorithm is not known. In the case the problem can be tackled analytically, evaluation technique will normally be applied. The objective of the second step is to put emphasis on why and how the required knowledge is being rationalized and used.

3 Case Study

3.1 Let us consider a typical problem of finding the sum of the finite series:

$1/3 + 1/15 + 1/35 +$ ___ $+ 1/(4n^2-1)$

Step 1. Component Phase

The problem is divided into the three basic components:

Input Component:

The finite series: $1/3 + 1/15 + 1/35 +$ ___ $+ 1/(4n^2-1)$

Target System Component:

Sum of the given finite series

Mental Model Component:

Input Component

Target System Component

Knowledge:

Finite series

Mathematical induction

Interaction Technique:

Experimentation technique

New Knowledge:

nil

Step 2. Relationship Analysis Phase

The first attempt is to tabulate the sum S_n of the series having a finite number of n terms. This is accomplished in Scientific Notebook by defining S_n as a function of n by

$$S(n) = \sum_{k=1}^{n} 1/(4k^2 - 1)$$

and evaluate S(n) for n = 1, 2, 3, 4:

N	Series	$S_n = S(n)$
1	1/3	1/3
2	1/3 + 1/15	2/5
3	1/3 + 1/15 + 1/35	3/7
4	1/3 + 1/15 + 1/35 + 1/ 63	4/9

The above tabulation indicates that the sum S_n appears to be a fraction. In particular, it is interesting to observe that certain relationships seem to exist among the number of terms, n, in the finite series, the denominator D and the numerator N of S_n = N / D. This relationship analysis process can be best illustrated using the diagram:

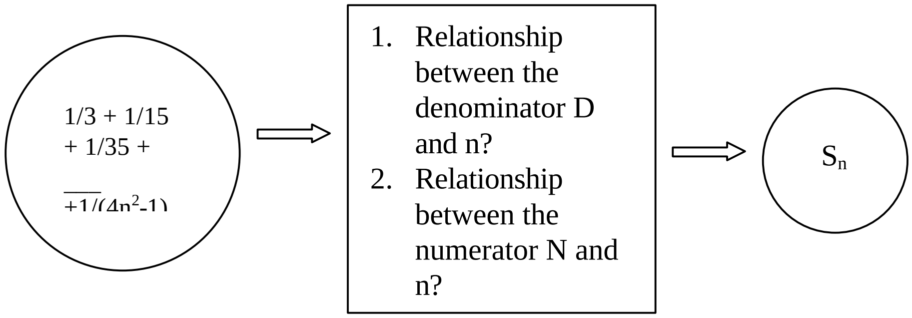

Figure 3. Relationship between the finite series and its sum S_n

By performing a simple analysis on the above relations, one can derive the following using the experimentation technique:

Entities	Relationship
N, n	N = n
D, n	D = 2n+1

As a result, the sum of the given finite series takes the following form:

$S_n = n / (2n + 1)$,

which is easily confirmed in Scientific Notebook by evaluating the sum S(n) using the “Simplify” command. Nevertheless, a mathematical proof of the above result using mathematical induction is still required.

Clearly, $S_1 = 1 / (2\_1 + 1)$. Assume $S_n = n / (2n + 1)$ true for n = k. Thus for n = k+1 we have

$S_{k+1} = S_k + 1 / (4 (k+1)^2 - 1)$

$= k / (2k + 1) + 1/(4(k+1)^2-1)$

$= (k + 1) /(2(k+1) +1)$.

This completes the induction and we can now conclude that

$S_n = n / (2n + 1)$.

3.2 Let us consider a second example of evaluating the sum of the finite series

$1^2 + 2^2 + 3^2 + \_\_\_ + n^2$

using the formula of summation by parts

$$\sum_{k=1}^{n} b_k (a_{k+1} - a_k) + \sum_{k=1}^{n} a_k (b_k - b_{k-1}) = a_{n+1} b_n - a_1 b_0 . \qquad (*)$$

Step 1. Component Phase

The three basic components for this example are:

Input Component:

$$1^2 + 2^2 + 3^2 + \_\_\_ + n^2 = \sum_{k=1}^{n} k^2$$

Target System Component:

Sum of the given finite series

Mental Model Component:

Input Component

Target System Component

Knowledge:

Finite series

Summation by parts formula

Interaction Technique:

Evaluation, experimentation and reasoning techniques

New Knowledge:

$$\sum_{k=1}^{n} k = n\,(n+1)\,/2$$

Step 2. Relationship Analysis Phase

We first consider the summation by parts formula (*) associated with the sequences $\{a_n\}$, $\{b_n\}$. This formula can be readily verified by expanding the sums of the left-hand side of (*) and noting that the result telescopes. The sum S_n of the given finite series is to be derived from the formula (*) by carefully making the appropriate choices of a_k, b_k, taking into account the general term of the series is of the form k^2. The relationship analysis process is summarized below:

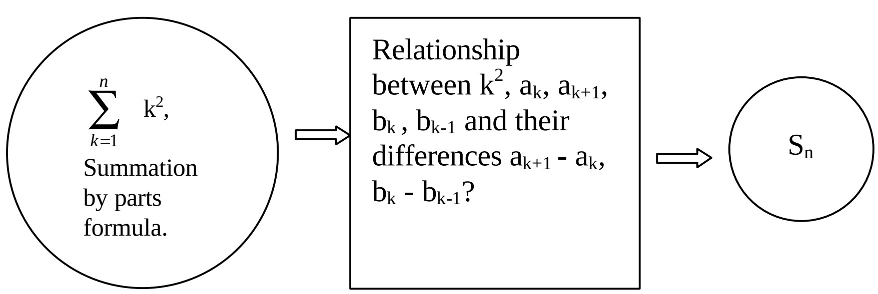

Figure 3. Relationship between the finite series and S_n

A careful analysis on the above relations enables us to derive the following choices for a_k , b_k , and their differences a_{k+1} - a_k, b_k - b_{k-1} using the reasoning, experimentation and evaluation techniques.

<u>Entities</u>	<u>Relationship</u>	
a_k, a_{k+1}, k^2	$a_k = k^2 \; {}^{-} k = k\,(k \; {}^{-}1)$,	$a_{k+1} - a_k = 2k$
b_k, b_{k-1}, k^2	$b_k = k$,	$b_k - b_{k-1} = 1$

Thus the summation by parts formula in this case yields

$$\sum_{k=1}^{n} k\,(2k\,) + \sum_{k=1}^{n} (k^2 - k\,)\,(1) = (n+1) \_\; n \;\_\; n \; {}^{-}0.$$

Using the fact that $\sum_{k=1}^{n} k = n\,(n+1)\,/\,2$ and some algebraic simplifications, we obtain

$$\sum_{k=1}^{n} k^2 = n\,(n+1)\,(2n+1)\,/\,6.$$

All the steps and calculations in the above discussion can be carried out using the various suitable commands available in Scientific Notebook.

4 Conclusions

The above examples explicitly illustrate the process of CAR thinking in a typical problem-solving activity. CAR is an active learning tool. It is based on the belief that without a thorough analysis of a problem and understanding of the fundamentals, a student is unlikely to solve a problem within a reasonable time, mainly because the solution of a problem generally involves a good number of theory, rules and algorithms. But with the CAR methodology, a student is able to identify the entities of a given problem and decompose them into the three manageable components. Coupled with the understanding of the fundamentals, he is then able to perform logical thinking, and eventually arrives at the solution of the given problem. The development of CAR is therefore built on the framework that learning by tackling a problem through relationship analysis via logical thinking should be emphasized, rather than learning by rote. A set of learning and teaching materials of certain selected topics in mathematics and computing (for example, Boolean algebra, the construction of a binary sequence detector using state graph, and the examples presented above) are being compiled using CAR approach. All these will be tested in a problem-solving environment to evaluate the feasibility of CAR methodology in teaching and learning.

Acknowledgments

This feasibility study is a funded project supported by the Learning and Teaching Committee of the Hong Kong Polytechnic University. The author would also like to express his sincere thanks to Mr. Julian Kwan for his participation and contribution in this project.

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