Factorize Polynomials Based on Pattern Recognition

Zeng Zhenbing Fu Hongguang

June 16, 1999

In this paper the authors introduce an algothm for factorizing polynomial through algrbraic expression pattern recognition and the forward search technique. As a classical method in artificial intelligence, forward search technique has been used by many people to produce human readable proofs (that is, some kind of proof with tradition-form-like precess) of geometric theorems with great success in recent years. This progress in automated theorem-proof is made partly due to the fact that pattern recoginition of geometric configurations can be done by a rather straight way of comparing the corresponding vertices and edges.

To extend this method to algebraic transformation, the main problem we need to solve is how to decide that two polynomials are same in abstract pattern in an apropiate meaning. Suppose that A is a given polynomial and $R[U \to V]$ is a rule for polynomial transformation (say, polynomial factorization) rule, if A is similar with U, then by applying the rule R we obtain a new polynomial $B := R[U \to V](A)$. Our work to overcome this difficulty is

(1): an algorithm to find an "abstract form" of a given polynomial, for example, if input $x^2 - 2 * x * y + y^2$, then output

$$([1]^2 - 2 * [1] * [2] + [2]^2, [1] = x, [2] = y);;$$

(2): an alrogithm to decide if two given polynomials are "similar in pattern", for example, if input $p_1 = x^2 * y^2 - z^4$, $p_2 = a^2 - b^2$, then output $(a = x * y, b = z^2)$, while if input $p_1 = x * y^2 - z^2$, $p_2 = a^2 - b^2$, then output nil;

Based on the above result and some selected rules of factorization fomulae, we have developed an algorithm for polynomial factorization. This algorithm, obviously, can be used to produce human readble transform process of factorization, as well as other algebraic transformation, as we have done in desining of computer aided instruction software "MathLab: Algebra I".